



# Formulae and Statistical Tables for GCE Mathematics and GCE Statistics

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For the new specifications for first teaching from September 2004

## GCE Mathematics

ADVANCED SUBSIDIARY MATHEMATICS (5361)  
ADVANCED SUBSIDIARY PURE MATHEMATICS (5366)  
ADVANCED SUBSIDIARY FURTHER MATHEMATICS (5371)

ADVANCED MATHEMATICS (6361)  
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## GCE Statistics

ADVANCED SUBSIDIARY STATISTICS (5381)  
ADVANCED STATISTICS (6381)

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## PURE MATHEMATICS

### Mensuration

$$\text{Surface area of sphere} = 4\pi r^2$$

$$\text{Area of curved surface of cone} = \pi r \times \text{slant height}$$

### Arithmetic series

$$u_n = a + (n-1)d$$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n[2a + (n-1)d]$$

### Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

### Summations

$$\sum_{r=1}^n r = \frac{1}{2}n(n+1)$$

$$\sum_{r=1}^n r^2 = \frac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^n r^3 = \frac{1}{4}n^2(n+1)^2$$

### Trigonometry – the Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

### Binomial Series

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N})$$

$$\text{where } \binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1.2}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{1.2\dots r}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

### Logarithms and exponentials

$$a^x = e^{x \ln a}$$

### Complex numbers

$$\{r(\cos \theta + i \sin \theta)\}^n = r^n (\cos n\theta + i \sin n\theta)$$

$$e^{i\theta} = \cos \theta + i \sin \theta$$

The roots of  $z^n = 1$  are given by  $z = e^{\frac{2\pi ki}{n}}$ , for  $k = 0, 1, 2, \dots, n-1$

## Maclaurin's series

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \dots + \frac{x^r}{r!} f^{(r)}(0) + \dots$$

$$e^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^r}{r!} + \dots \quad \text{for all } x$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^{r+1} \frac{x^r}{r} + \dots \quad (-1 < x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \dots \quad \text{for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + (-1)^r \frac{x^{2r}}{(2r)!} + \dots \quad \text{for all } x$$

## Hyperbolic functions

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\cosh^{-1} x = \ln \left\{ x + \sqrt{x^2 - 1} \right\} \quad (x \geq 1)$$

$$\sinh^{-1} x = \ln \left\{ x + \sqrt{x^2 + 1} \right\}$$

$$\tanh^{-1} x = \frac{1}{2} \ln \left( \frac{1+x}{1-x} \right) \quad (|x| < 1)$$

## Conics

	Ellipse	Parabola	Hyperbola	Rectangular hyperbola
Standard form	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$y^2 = 4ax$	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$xy = c^2$
Asymptotes	none	none	$\frac{x}{a} = \pm \frac{y}{b}$	$x = 0, y = 0$

## Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \frac{1}{2})\pi)$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

## Vectors

The resolved part of  $\mathbf{a}$  in the direction of  $\mathbf{b}$  is  $\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|}$

The position vector of the point dividing  $AB$  in the ratio  $\lambda : \mu$  is  $\frac{\mu\mathbf{a} + \lambda\mathbf{b}}{\lambda + \mu}$

$$\text{Vector product: } \mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta \hat{\mathbf{n}} = \begin{vmatrix} \mathbf{i} & a_1 & b_1 \\ \mathbf{j} & a_2 & b_2 \\ \mathbf{k} & a_3 & b_3 \end{vmatrix} = \begin{bmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{bmatrix}$$

If  $A$  is the point with position vector  $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$  and the direction vector  $\mathbf{b}$  is given by  $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ , then the straight line through  $A$  with direction vector  $\mathbf{b}$  has cartesian equation

$$\frac{x - a_1}{b_1} = \frac{y - a_2}{b_2} = \frac{z - a_3}{b_3} = \lambda$$

The plane through  $A$  with normal vector  $\mathbf{n} = n_1\mathbf{i} + n_2\mathbf{j} + n_3\mathbf{k}$  has cartesian equation

$$n_1x + n_2y + n_3z = d \quad \text{where } d = \mathbf{a} \cdot \mathbf{n}$$

The plane through non-collinear points  $A, B$  and  $C$  has vector equation

$$\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a}) + \mu(\mathbf{c} - \mathbf{a}) = (1 - \lambda - \mu)\mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$$

The plane through the point with position vector  $\mathbf{a}$  and parallel to  $\mathbf{b}$  and  $\mathbf{c}$  has equation

$$\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$$

## Matrix transformations

Anticlockwise rotation through  $\theta$  about  $O$ :  $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Reflection in the line  $y = (\tan\theta)x$ :  $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$

The matrices for rotations (in three dimensions) through an angle  $\theta$  about one of the axes are

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \quad \text{for the } x\text{-axis}$$

$$\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \quad \text{for the } y\text{-axis}$$

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{for the } z\text{-axis}$$

## Differentiation

$f(x)$	$f'(x)$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\tan kx$	$k \sec^2 kx$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cosh^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\frac{f(x)}{g(x)}$	$\frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

## Integration

(+ constant;  $a > 0$  where relevant)

$f(x)$	$\int f(x) dx$
$\tan x$	$\ln \sec x $
$\cot x$	$\ln \sin x $
$\operatorname{cosec} x$	$-\ln \operatorname{cosec} x + \cot x  = \ln\left \tan\left(\frac{1}{2}x\right)\right $
$\sec x$	$\ln \sec x + \tan x  = \ln\left \tan\left(\frac{1}{2}x + \frac{1}{4}\pi\right)\right $
$\sec^2 kx$	$\frac{1}{k} \tan kx$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\ln \cosh x$

**INTEGRATION FORMULAE CONTINUE OVER THE PAGE**

$$\frac{1}{\sqrt{a^2 - x^2}} \quad \sin^{-1}\left(\frac{x}{a}\right) \quad (|x| < a)$$

$$\frac{1}{a^2 + x^2} \quad \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$$

$$\frac{1}{\sqrt{x^2 - a^2}} \quad \cosh^{-1}\left(\frac{x}{a}\right) \text{ or } \ln\{x + \sqrt{x^2 - a^2}\} \quad (x > a)$$

$$\frac{1}{\sqrt{a^2 + x^2}} \quad \sinh^{-1}\left(\frac{x}{a}\right) \text{ or } \ln\{x + \sqrt{x^2 + a^2}\}$$

$$\frac{1}{a^2 - x^2} \quad \frac{1}{2a} \ln\left|\frac{a+x}{a-x}\right| = \frac{1}{a} \tanh^{-1}\left(\frac{x}{a}\right) \quad (|x| < a)$$

$$\frac{1}{x^2 - a^2} \quad \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right|$$

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

### Area of a sector

$$A = \frac{1}{2} \int r^2 d\theta \quad (\text{polar coordinates})$$

### Arc length

$$s = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \quad (\text{cartesian coordinates})$$

$$s = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt \quad (\text{parametric form})$$

### Surface area of revolution

$$S_x = 2\pi \int y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \quad (\text{cartesian coordinates})$$

$$S_x = 2\pi \int y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt \quad (\text{parametric form})$$

### Numerical integration

The trapezium rule:  $\int_a^b y dx \approx \frac{1}{2} h \{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$ , where  $h = \frac{b-a}{n}$

The mid-ordinate rule:  $\int_a^b y dx \approx h(y_{\frac{1}{2}} + y_{\frac{3}{2}} + \dots + y_{n-\frac{3}{2}} + y_{n-\frac{1}{2}})$ , where  $h = \frac{b-a}{n}$

Simpson's rule:  $\int_a^b y dx \approx \frac{1}{3} h \{(y_0 + y_n) + 4(y_1 + y_3 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2})\}$

where  $h = \frac{b-a}{n}$  and  $n$  is even



## Numerical solution of differential equations

For  $\frac{dy}{dx} = f(x)$  and small  $h$ , recurrence relations are:

$$\text{Euler's method: } y_{n+1} = y_n + hf(x_n); \quad x_{n+1} = x_n + h$$

For  $\frac{dy}{dx} = f(x, y)$ :

$$\text{Euler's method: } y_{r+1} = y_r + hf(x_r, y_r)$$

$$\text{Improved Euler method: } y_{r+1} = y_r + \frac{1}{2}(k_1 + k_2), \text{ where } k_1 = hf(x_r, y_r), k_2 = hf(x_r + h, y_r + k_1)$$

## Numerical solution of equations

The Newton-Raphson iteration for solving  $f(x) = 0$ :  $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

## MECHANICS

### Motion in a circle

Transverse velocity:  $v = r\dot{\theta}$

Transverse acceleration:  $\dot{v} = r\ddot{\theta}$

Radial acceleration:  $-r\dot{\theta}^2 = -\frac{v^2}{r}$

### Centres of mass

For uniform bodies

Triangular lamina:  $\frac{2}{3}$  along median from vertex

Solid hemisphere, radius  $r$ :  $\frac{3}{8}r$  from centre

Hemispherical shell, radius  $r$ :  $\frac{1}{2}r$  from centre

Circular arc, radius  $r$ , angle at centre  $2\alpha$ :  $\frac{r \sin \alpha}{\alpha}$  from centre

Sector of circle, radius  $r$ , angle at centre  $2\alpha$ :  $\frac{2r \sin \alpha}{3\alpha}$  from centre

Solid cone or pyramid of height  $h$ :  $\frac{1}{4}h$  above the base on the line from centre of base to vertex

Conical shell of height  $h$ :  $\frac{1}{3}h$  above the base on the line from centre of base to vertex

### Moments of inertia

For uniform bodies of mass  $m$

Thin rod, length  $2l$ , about perpendicular axis through centre:  $\frac{1}{3}ml^2$

Rectangular lamina about axis in plane bisecting edges of length  $2l$ :  $\frac{1}{3}ml^2$

Thin rod, length  $2l$ , about perpendicular axis through end:  $\frac{4}{3}ml^2$

Rectangular lamina about edge perpendicular to edges of length  $2l$ :  $\frac{4}{3}ml^2$

Rectangular lamina, sides  $2a$  and  $2b$ , about perpendicular axis through centre:  $\frac{1}{3}m(a^2 + b^2)$

**MOMENTS OF INERTIA FORMULAE CONTINUE OVER THE PAGE**

Hoop or cylindrical shell of radius  $r$  about axis:  $mr^2$

Hoop of radius  $r$  about a diameter:  $\frac{1}{2}mr^2$

Disc or solid cylinder of radius  $r$  about axis:  $\frac{1}{2}mr^2$

Disc of radius  $r$  about a diameter:  $\frac{1}{4}mr^2$

Solid sphere, radius  $r$ , about diameter:  $\frac{2}{5}mr^2$

Spherical shell of radius  $r$  about a diameter:  $\frac{2}{3}mr^2$

Parallel axes theorem:  $I_A = I_G + m(AG)^2$

Perpendicular axes theorem:  $I_z = I_x + I_y$  (for a lamina in the  $x$ - $y$  plane)

### General motion in two dimensions

Radial velocity  $\dot{r}$

Transverse velocity  $r\dot{\theta}$

Radial acceleration  $\ddot{r} - r\dot{\theta}^2$

Transverse acceleration  $r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{1}{r} \frac{d}{dt}(r^2\dot{\theta})$

### Moments as vectors

The moment about  $O$  of  $\mathbf{F}$  acting through the point with position vector  $\mathbf{r}$  is  $\mathbf{r} \times \mathbf{F}$

### Universal law of gravitation

$$\text{Force} = \frac{Gm_1m_2}{d^2}$$

## PROBABILITY and STATISTICS

### Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A) \times P(B | A)$$

$$P(A_j | B) = \frac{P(A_j) \times P(B | A_j)}{\sum_{i=1}^n P(A_i) \times P(B | A_i)}$$

### Expectation algebra

$$\text{Covariance: } \text{Cov}(X, Y) = E((X - \mu_x)(Y - \mu_y)) = E(XY) - \mu_x \mu_y$$

$$\text{Var}(aX \pm bY) = a^2 \text{Var}(X) + b^2 \text{Var}(Y) \pm 2ab \text{Cov}(X, Y)$$

$$\text{Product moment correlation coefficient: } \rho = \frac{\text{Cov}(X, Y)}{\sigma_x \sigma_y}$$

For independent random variables  $X$  and  $Y$

$$E(XY) = E(X)E(Y)$$

$$\text{Var}(aX \pm bY) = a^2 \text{Var}(X) + b^2 \text{Var}(Y)$$

## Discrete distributions

For a discrete random variable  $X$  taking values  $x_i$  with probabilities  $p_i$

Expectation (mean):  $E(X) = \mu = \sum x_i p_i$

Variance:  $\text{Var}(X) = \sigma^2 = \sum (x_i - \mu)^2 p_i = \sum x_i^2 p_i - \mu^2 = E(X^2) - \mu^2$

For a function  $g(X)$ :  $E(g(X)) = \sum g(x_i) p_i$

Standard discrete distributions:

Distribution of $X$	$P(X = x)$	Mean	Variance
Binomial $B(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	$np$	$np(1-p)$
Poisson $Po(\lambda)$	$e^{-\lambda} \frac{\lambda^x}{x!}$	$\lambda$	$\lambda$
Geometric $Geo(p)$ on $1, 2, \dots$	$p(1-p)^{x-1}$	$\frac{1}{p}$	$\frac{1-p}{p^2}$

## Continuous distributions

For a continuous random variable  $X$  having probability density function  $f(x)$

Expectation (mean):  $E(X) = \mu = \int x f(x) dx$

Variance:  $\text{Var}(X) = \sigma^2 = \int (x - \mu)^2 f(x) dx = \int x^2 f(x) dx - \mu^2 = E(X^2) - \mu^2$

For a function  $g(X)$ :  $E(g(X)) = \int g(x) f(x) dx$

Cumulative distribution function:  $F(x) = P(X \leq x) = \int_{-\infty}^x f(t) dt$

Standard continuous distributions:

Distribution of $X$	Probability density function	Mean	Variance
Uniform (Rectangular) on $[a, b]$	$\frac{1}{b-a}$	$\frac{1}{2}(a+b)$	$\frac{1}{12}(b-a)^2$
Normal $N(\mu, \sigma^2)$	$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$	$\mu$	$\sigma^2$
Exponential	$\lambda e^{-\lambda x}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$

## Sampling distributions

For a random sample  $X_1, X_2, \dots, X_n$  of  $n$  independent observations from a distribution having mean  $\mu$  and variance  $\sigma^2$

$\bar{X}$  is an unbiased estimator of  $\mu$ , with  $\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$

$S^2$  is an unbiased estimator of  $\sigma^2$ , where  $S^2 = \frac{\sum (X_i - \bar{X})^2}{n-1}$

For a random sample of  $n$  observations from  $N(\mu, \sigma^2)$

$$\frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0, 1)$$

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi_{n-1}^2$$

$$\frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} \sim t_{n-1} \quad (\text{also valid in matched-pairs situations})$$

If  $X$  is the observed number of successes in  $n$  independent Bernoulli trials in each of which the probability of success is  $p$ , and  $Y = \frac{X}{n}$ , then

$$E(Y) = p \quad \text{and} \quad \text{Var}(Y) = \frac{p(1-p)}{n}$$

For a random sample of  $n_x$  observations from  $N(\mu_x, \sigma_x^2)$  and, independently, a random sample of  $n_y$  observations from  $N(\mu_y, \sigma_y^2)$

$$\frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}} \sim N(0, 1)$$

$$\frac{S_x^2 / \sigma_x^2}{S_y^2 / \sigma_y^2} \sim F_{n_x-1, n_y-1}$$

If  $\sigma_x^2 = \sigma_y^2 = \sigma^2$  (unknown),

$$\text{then } \frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{S_p^2 \left( \frac{1}{n_x} + \frac{1}{n_y} \right)}} \sim t_{n_x+n_y-2} \quad \text{where } S_p^2 = \frac{(n_x-1)S_x^2 + (n_y-1)S_y^2}{n_x + n_y - 2}$$

## Correlation and regression

For a set of  $n$  pairs of values  $(x_i, y_i)$

$$S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n}$$

$$S_{yy} = \sum (y_i - \bar{y})^2 = \sum y_i^2 - \frac{(\sum y_i)^2}{n}$$

$$S_{xy} = \sum (x_i - \bar{x})(y_i - \bar{y}) = \sum x_i y_i - \frac{(\sum x_i)(\sum y_i)}{n}$$

The product moment correlation coefficient is

$$r = \frac{S_{xy}}{\sqrt{S_{xx} \times S_{yy}}} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\{\sum (x_i - \bar{x})^2\} \{\sum (y_i - \bar{y})^2\}}} = \frac{\sum x_i y_i - \frac{(\sum x_i)(\sum y_i)}{n}}{\sqrt{\left(\sum x_i^2 - \frac{(\sum x_i)^2}{n}\right) \left(\sum y_i^2 - \frac{(\sum y_i)^2}{n}\right)}}$$

Spearman's rank correlation coefficient is the product moment correlation coefficient between ranks

When there are no tied ranks it may be calculated using  $r_s = 1 - \frac{6 \sum d_i^2}{n(n^2 - 1)}$

The regression coefficient of  $y$  on  $x$  is  $b = \frac{S_{xy}}{S_{xx}} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$

Least squares regression line of  $y$  on  $x$  is  $y = a + bx$ , where  $a = \bar{y} - b\bar{x}$

## Analysis of variance

One-factor model:  $x_{ij} = \mu + \alpha_i + \varepsilon_{ij}$ , where  $\varepsilon_{ij} \sim N(0, \sigma^2)$

$$\text{Total sum of squares } SS_T = \sum_i \sum_j x_{ij}^2 - \frac{T^2}{n}$$

$$\text{Between groups sum of squares } SS_B = \sum_i \frac{T_i^2}{n_i} - \frac{T^2}{n}$$

Two-factor model (with  $m$  rows and  $n$  columns):  $x_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}$ , where  $\varepsilon_{ij} \sim N(0, \sigma^2)$

$$\text{Total sum of squares, } SS_T = \sum_i \sum_j x_{ij}^2 - \frac{T^2}{mn}$$

$$\text{Between rows sum of squares, } SS_R = \sum_i \frac{R_i^2}{n} - \frac{T^2}{mn}$$

$$\text{Between columns sum of squares, } SS_C = \sum_j \frac{C_j^2}{m} - \frac{T^2}{mn}$$

## Distribution-free (non-parametric) tests

Goodness-of-fit tests and contingency tables:  $\sum \frac{(O_i - E_i)^2}{E_i}$  is approximately distributed as  $\chi^2$

Wilcoxon signed rank test

$T$  is the sum of the ranks of observations with the same sign

Mann-Whitney test

$U = T - \frac{n(n+1)}{2}$  where  $T$  is the sum of the ranks of the sample of size  $n$

Kruskal-Wallis test

$$H = \frac{12}{N(N+1)} \sum_i \frac{T_i^2}{n_i} - 3(N+1)$$

where  $T_i$  is the sum of the ranks of a sample of size  $n_i$  and  $N = \sum_i n_i$

$H$  is approximately distributed as  $\chi^2$  with  $k-1$  degrees of freedom  
where  $k$  is the number of samples

**TABLE 1 CUMULATIVE BINOMIAL DISTRIBUTION FUNCTION**

The tabulated value is  $P(X \leq x)$ , where  $X$  has a binomial distribution with parameters  $n$  and  $p$ .

$p$	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	$p$	
$x$	$n=2$																		$x$	
0	0.9801	0.9604	0.9409	0.9216	0.9025	0.8836	0.8649	0.8464	0.8281	0.8100	0.7225	0.6400	0.5625	0.4900	0.4225	0.3600	0.3025	0.2500	0	
1	0.9999	0.9996	0.9991	0.9984	0.9975	0.9964	0.9951	0.9936	0.9919	0.9900	0.9775	0.9600	0.9375	0.9100	0.8775	0.8400	0.7975	0.7500	1	
2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	2	
$x$	$n=3$																		$x$	
0	0.9703	0.9412	0.9127	0.8847	0.8574	0.8306	0.8044	0.7787	0.7536	0.7290	0.6141	0.5120	0.4219	0.3430	0.2746	0.2160	0.1664	0.1250	0	
1	0.9997	0.9988	0.9974	0.9953	0.9928	0.9896	0.9860	0.9818	0.9772	0.9720	0.9393	0.8960	0.8438	0.7840	0.7183	0.6480	0.5748	0.5000	1	
2	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9995	0.9993	0.9990	0.9966	0.9920	0.9844	0.9730	0.9571	0.9360	0.9089	0.8750	2	
3				1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	3	
$x$	$n=4$																		$x$	
0	0.9606	0.9224	0.8853	0.8493	0.8145	0.7807	0.7481	0.7164	0.6857	0.6561	0.5220	0.4096	0.3164	0.2401	0.1785	0.1296	0.0915	0.0625	0	
1	0.9994	0.9977	0.9948	0.9909	0.9860	0.9801	0.9733	0.9656	0.9570	0.9477	0.8905	0.8192	0.7383	0.6517	0.5630	0.4752	0.3910	0.3125	1	
2	1.0000	1.0000	0.9999	0.9998	0.9995	0.9992	0.9987	0.9981	0.9973	0.9963	0.9880	0.9728	0.9492	0.9163	0.8735	0.8208	0.7585	0.6875	2	
3			1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9995	0.9984	0.9961	0.9919	0.9850	0.9744	0.9590	0.9375	3
4										1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	4	
$x$	$n=5$																		$x$	
0	0.9510	0.9039	0.8587	0.8154	0.7738	0.7339	0.6957	0.6591	0.6240	0.5905	0.4437	0.3277	0.2373	0.1681	0.1160	0.0778	0.0503	0.0313	0	
1	0.9990	0.9962	0.9915	0.9852	0.9774	0.9681	0.9575	0.9456	0.9326	0.9185	0.8352	0.7373	0.6328	0.5282	0.4284	0.3370	0.2562	0.1875	1	
2	1.0000	0.9999	0.9997	0.9994	0.9988	0.9980	0.9969	0.9955	0.9937	0.9914	0.9734	0.9421	0.8965	0.8369	0.7648	0.6826	0.5931	0.5000	2	
3		1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9995	0.9978	0.9933	0.9844	0.9692	0.9460	0.9130	0.8688	0.8125	3	
4						1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990	0.9976	0.9947	0.9898	0.9815	0.9688	4	
5											1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	5	
$x$	$n=6$																		$x$	
0	0.9415	0.8858	0.8330	0.7828	0.7351	0.6899	0.6470	0.6064	0.5679	0.5314	0.3771	0.2621	0.1780	0.1176	0.0754	0.0467	0.0277	0.0156	0	
1	0.9985	0.9943	0.9875	0.9784	0.9672	0.9541	0.9392	0.9227	0.9048	0.8857	0.7765	0.6554	0.5339	0.4202	0.3191	0.2333	0.1636	0.1094	1	
2	1.0000	0.9998	0.9995	0.9988	0.9978	0.9962	0.9942	0.9915	0.9882	0.9842	0.9527	0.9011	0.8306	0.7443	0.6471	0.5443	0.4415	0.3438	2	
3		1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9995	0.9992	0.9987	0.9941	0.9830	0.9624	0.9295	0.8826	0.8208	0.7447	0.6563	3	
4					1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9984	0.9954	0.9891	0.9777	0.9590	0.9308	0.8906	4	
5										1.0000	1.0000	0.9999	0.9998	0.9993	0.9982	0.9959	0.9917	0.9844	5	
6											1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	6	
$x$	$n=7$																		$x$	
0	0.9321	0.8681	0.8080	0.7514	0.6983	0.6485	0.6017	0.5578	0.5168	0.4783	0.3206	0.2097	0.1335	0.0824	0.0490	0.0280	0.0152	0.0078	0	
1	0.9980	0.9921	0.9829	0.9706	0.9556	0.9382	0.9187	0.8974	0.8745	0.8503	0.7166	0.5767	0.4449	0.3294	0.2338	0.1586	0.1024	0.0625	1	
2	1.0000	0.9997	0.9991	0.9980	0.9962	0.9937	0.9903	0.9860	0.9807	0.9743	0.9262	0.8520	0.7564	0.6471	0.5323	0.4199	0.3164	0.2266	2	
3		1.0000	1.0000	0.9999	0.9998	0.9996	0.9993	0.9988	0.9982	0.9973	0.9879	0.9667	0.9294	0.8740	0.8002	0.7102	0.6083	0.5000	3	
4				1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9988	0.9953	0.9871	0.9712	0.9444	0.9037	0.8471	0.7734	4	
5								1.0000	1.0000	1.0000	0.9999	0.9996	0.9987	0.9962	0.9910	0.9812	0.9643	0.9375	5	
6											1.0000	1.0000	0.9999	0.9998	0.9994	0.9984	0.9963	0.9922	6	
7													1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	7	
$x$	$n=8$																		$x$	
0	0.9227	0.8508	0.7837	0.7214	0.6634	0.6096	0.5596	0.5132	0.4703	0.4305	0.2725	0.1678	0.1001	0.0576	0.0319	0.0168	0.0084	0.0039	0	
1	0.9973	0.9897	0.9777	0.9619	0.9428	0.9208	0.8965	0.8702	0.8423	0.8131	0.6572	0.5033	0.3671	0.2553	0.1691	0.1064	0.0632	0.0352	1	
2	0.9999	0.9996	0.9987	0.9969	0.9942	0.9904	0.9853	0.9789	0.9711	0.9619	0.8948	0.7969	0.6785	0.5518	0.4278	0.3154	0.2201	0.1445	2	
3	1.0000	1.0000	0.9999	0.9998	0.9996	0.9993	0.9987	0.9978	0.9966	0.9950	0.9786	0.9437	0.8862	0.8059	0.7064	0.5941	0.4770	0.3633	3	
4			1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9996	0.9971	0.9896	0.9727	0.9420	0.8939	0.8263	0.7396	0.6367	4	
5							1.0000	1.0000	1.0000	1.0000	0.9998	0.9988	0.9958	0.9887	0.9747	0.9502	0.9115	0.8555	5	
6											1.0000	0.9999	0.9996	0.9987	0.9964	0.9915	0.9819	0.9648	6	
7												1.0000	1.0000	0.9999	0.9998	0.9993	0.9983	0.9961	7	
8														1.0000	1.0000	1.0000	1.0000	1.0000	8	





$p$	<b>0.01</b>	<b>0.02</b>	<b>0.03</b>	<b>0.04</b>	<b>0.05</b>	<b>0.06</b>	<b>0.07</b>	<b>0.08</b>	<b>0.09</b>	<b>0.10</b>	<b>0.15</b>	<b>0.20</b>	<b>0.25</b>	<b>0.30</b>	<b>0.35</b>	<b>0.40</b>	<b>0.45</b>	<b>0.50</b>	$p$
$x$	$n=14$																		$x$
<b>0</b>	0.8687	0.7536	0.6528	0.5647	0.4877	0.4205	0.3620	0.3112	0.2670	0.2288	0.1028	0.0440	0.0178	0.0068	0.0024	0.0008	0.0002	0.0001	<b>0</b>
<b>1</b>	0.9916	0.9690	0.9355	0.8941	0.8470	0.7963	0.7436	0.6900	0.6368	0.5846	0.3567	0.1979	0.1010	0.0475	0.0205	0.0081	0.0029	0.0009	<b>1</b>
<b>2</b>	0.9997	0.9975	0.9923	0.9833	0.9699	0.9522	0.9302	0.9042	0.8745	0.8416	0.6479	0.4481	0.2811	0.1608	0.0839	0.0398	0.0170	0.0065	<b>2</b>
<b>3</b>	1.0000	0.9999	0.9994	0.9981	0.9958	0.9920	0.9864	0.9786	0.9685	0.9559	0.8535	0.6982	0.5213	0.3552	0.2205	0.1243	0.0632	0.0287	<b>3</b>
<b>4</b>		1.0000	1.0000	0.9998	0.9996	0.9990	0.9980	0.9965	0.9941	0.9908	0.9533	0.8702	0.7415	0.5842	0.4227	0.2793	0.1672	0.0898	<b>4</b>
<b>5</b>				1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9985	0.9885	0.9561	0.8883	0.7805	0.6405	0.4859	0.3373	0.2120	<b>5</b>
<b>6</b>						1.0000	1.0000	1.0000	0.9999	0.9998	0.9978	0.9884	0.9617	0.9067	0.8164	0.6925	0.5461	0.3953	<b>6</b>
<b>7</b>									1.0000	1.0000	0.9997	0.9976	0.9897	0.9685	0.9247	0.8499	0.7414	0.6047	<b>7</b>
<b>8</b>											1.0000	0.9996	0.9978	0.9917	0.9757	0.9417	0.8811	0.7880	<b>8</b>
<b>9</b>												1.0000	0.9997	0.9983	0.9940	0.9825	0.9574	0.9102	<b>9</b>
<b>10</b>													1.0000	0.9998	0.9989	0.9961	0.9886	0.9713	<b>10</b>
<b>11</b>														1.0000	0.9999	0.9994	0.9978	0.9935	<b>11</b>
<b>12</b>															1.0000	0.9999	0.9997	0.9991	<b>12</b>
<b>13</b>																1.0000	1.0000	0.9999	<b>13</b>
<b>14</b>																		1.0000	<b>14</b>
$x$	$n=15$																		$x$
<b>0</b>	0.8601	0.7386	0.6333	0.5421	0.4633	0.3953	0.3367	0.2863	0.2430	0.2059	0.0874	0.0352	0.0134	0.0047	0.0016	0.0005	0.0001	0.0000	<b>0</b>
<b>1</b>	0.9904	0.9647	0.9270	0.8809	0.8290	0.7738	0.7168	0.6597	0.6035	0.5490	0.3186	0.1671	0.0802	0.0353	0.0142	0.0052	0.0017	0.0005	<b>1</b>
<b>2</b>	0.9996	0.9970	0.9906	0.9797	0.9638	0.9429	0.9171	0.8870	0.8531	0.8159	0.6042	0.3980	0.2361	0.1268	0.0617	0.0271	0.0107	0.0037	<b>2</b>
<b>3</b>	1.0000	0.9998	0.9992	0.9976	0.9945	0.9896	0.9825	0.9727	0.9601	0.9444	0.8227	0.6482	0.4613	0.2969	0.1727	0.0905	0.0424	0.0176	<b>3</b>
<b>4</b>		1.0000	0.9999	0.9998	0.9994	0.9986	0.9972	0.9950	0.9918	0.9873	0.9383	0.8358	0.6865	0.5155	0.3519	0.2173	0.1204	0.0592	<b>4</b>
<b>5</b>			1.0000	1.0000	0.9999	0.9999	0.9997	0.9993	0.9987	0.9978	0.9832	0.9389	0.8516	0.7216	0.5643	0.4032	0.2608	0.1509	<b>5</b>
<b>6</b>					1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9964	0.9819	0.9434	0.8689	0.7548	0.6098	0.4522	0.3036	<b>6</b>
<b>7</b>								1.0000	1.0000	1.0000	0.9994	0.9958	0.9827	0.9500	0.8868	0.7869	0.6535	0.5000	<b>7</b>
<b>8</b>											0.9999	0.9992	0.9958	0.9848	0.9578	0.9050	0.8182	0.6964	<b>8</b>
<b>9</b>												1.0000	0.9999	0.9992	0.9963	0.9876	0.9662	0.9231	<b>9</b>
<b>10</b>													1.0000	0.9999	0.9993	0.9972	0.9907	0.9745	<b>10</b>
<b>11</b>														1.0000	0.9999	0.9995	0.9981	0.9937	<b>11</b>
<b>12</b>															1.0000	0.9999	0.9997	0.9989	<b>12</b>
<b>13</b>																1.0000	1.0000	0.9999	<b>13</b>
<b>14</b>																	1.0000	1.0000	<b>14</b>
$x$	$n=20$																		$x$
<b>0</b>	0.8179	0.6676	0.5438	0.4420	0.3585	0.2901	0.2342	0.1887	0.1516	0.1216	0.0388	0.0115	0.0032	0.0008	0.0002	0.0000	0.0000	0.0000	<b>0</b>
<b>1</b>	0.9831	0.9401	0.8802	0.8103	0.7358	0.6605	0.5869	0.5169	0.4516	0.3917	0.1756	0.0692	0.0243	0.0076	0.0021	0.0005	0.0001	0.0000	<b>1</b>
<b>2</b>	0.9990	0.9929	0.9790	0.9561	0.9245	0.8850	0.8390	0.7879	0.7334	0.6769	0.4049	0.2061	0.0913	0.0355	0.0121	0.0036	0.0009	0.0002	<b>2</b>
<b>3</b>	1.0000	0.9994	0.9973	0.9926	0.9841	0.9710	0.9529	0.9294	0.9007	0.8670	0.6477	0.4114	0.2252	0.1071	0.0444	0.0160	0.0049	0.0013	<b>3</b>
<b>4</b>		1.0000	0.9997	0.9990	0.9974	0.9944	0.9893	0.9817	0.9710	0.9568	0.8298	0.6296	0.4148	0.2375	0.1182	0.0510	0.0189	0.0059	<b>4</b>
<b>5</b>			1.0000	0.9999	0.9997	0.9991	0.9981	0.9962	0.9932	0.9887	0.9327	0.8042	0.6172	0.4164	0.2454	0.1256	0.0553	0.0207	<b>5</b>
<b>6</b>				1.0000	1.0000	0.9999	0.9997	0.9994	0.9987	0.9976	0.9781	0.9133	0.7858	0.6080	0.4166	0.2500	0.1299	0.0577	<b>6</b>
<b>7</b>						1.0000	1.0000	0.9999	0.9998	0.9996	0.9941	0.9679	0.8982	0.7723	0.6010	0.4159	0.2520	0.1316	<b>7</b>
<b>8</b>								1.0000	1.0000	0.9999	0.9987	0.9900	0.9591	0.8867	0.7624	0.5956	0.4143	0.2517	<b>8</b>
<b>9</b>										1.0000	0.9998	0.9974	0.9861	0.9520	0.8782	0.7553	0.5914	0.4119	<b>9</b>
<b>10</b>											1.0000	0.9994	0.9961	0.9829	0.9468	0.8725	0.7507	0.5881	<b>10</b>
<b>11</b>												0.9999	0.9991	0.9949	0.9804	0.9435	0.8692	0.7483	<b>11</b>
<b>12</b>													1.0000	0.9998	0.9987	0.9940	0.9790	0.9420	<b>12</b>
<b>13</b>														1.0000	0.9997	0.9985	0.9935	0.9786	<b>13</b>
<b>14</b>															1.0000	0.9997	0.9984	0.9936	<b>14</b>
<b>15</b>																1.0000	0.9997	0.9985	<b>15</b>
<b>16</b>																	1.0000	0.9997	<b>16</b>
<b>17</b>																		1.0000	<b>17</b>
<b>18</b>																		1.0000	<b>18</b>

$p$	<b>0.01</b>	<b>0.02</b>	<b>0.03</b>	<b>0.04</b>	<b>0.05</b>	<b>0.06</b>	<b>0.07</b>	<b>0.08</b>	<b>0.09</b>	<b>0.10</b>	<b>0.15</b>	<b>0.20</b>	<b>0.25</b>	<b>0.30</b>	<b>0.35</b>	<b>0.40</b>	<b>0.45</b>	<b>0.50</b>	$p$				
$x$	$n=25$																		$x$				
<b>0</b>	0.7778	0.6035	0.4670	0.3604	0.2774	0.2129	0.1630	0.1244	0.0946	0.0718	0.0172	0.0038	0.0008	0.0001	0.0000	0.0000	0.0000	0.0000	<b>0</b>				
<b>1</b>	0.9742	0.9114	0.8280	0.7358	0.6424	0.5527	0.4696	0.3947	0.3286	0.2712	0.0931	0.0274	0.0070	0.0016	0.0003	0.0001	0.0000	0.0000	<b>1</b>				
<b>2</b>	0.9980	0.9868	0.9620	0.9235	0.8729	0.8129	0.7466	0.6768	0.6063	0.5371	0.2537	0.0982	0.0321	0.0090	0.0021	0.0004	0.0001	0.0000	<b>2</b>				
<b>3</b>	0.9999	0.9986	0.9938	0.9835	0.9659	0.9402	0.9064	0.8649	0.8169	0.7636	0.4711	0.2340	0.0962	0.0332	0.0097	0.0024	0.0005	0.0001	<b>3</b>				
<b>4</b>	1.0000	0.9999	0.9992	0.9972	0.9928	0.9850	0.9726	0.9549	0.9314	0.9020	0.6821	0.4207	0.2137	0.0905	0.0320	0.0095	0.0023	0.0005	<b>4</b>				
<b>5</b>		1.0000	0.9999	0.9996	0.9988	0.9969	0.9935	0.9877	0.9790	0.9666	0.8385	0.6167	0.3783	0.1935	0.0826	0.0294	0.0086	0.0020	<b>5</b>				
<b>6</b>			1.0000	1.0000	0.9998	0.9995	0.9987	0.9972	0.9946	0.9905	0.9305	0.7800	0.5611	0.3407	0.1734	0.0736	0.0258	0.0073	<b>6</b>				
<b>7</b>				1.0000	0.9999	0.9998	0.9995	0.9989	0.9977	0.9745	0.8909	0.7265	0.5118	0.3061	0.1536	0.0639	0.0216		<b>7</b>				
<b>8</b>					1.0000	1.0000	0.9999	0.9998	0.9995	0.9920	0.9532	0.8506	0.6769	0.4668	0.2735	0.1340	0.0539		<b>8</b>				
<b>9</b>						1.0000	1.0000	0.9999	0.9979	0.9827	0.9287	0.8106	0.6303	0.4246	0.2424	0.1148			<b>9</b>				
<b>10</b>							1.0000	0.9995	0.9944	0.9703	0.9022	0.7712	0.5858	0.3843	0.2122				<b>10</b>				
<b>11</b>								0.9999	0.9985	0.9893	0.9558	0.8746	0.7323	0.5426	0.3450				<b>11</b>				
<b>12</b>									1.0000	0.9996	0.9966	0.9825	0.9396	0.8462	0.6937	0.5000			<b>12</b>				
<b>13</b>										0.9999	0.9991	0.9940	0.9745	0.9222	0.8173	0.6550			<b>13</b>				
<b>14</b>											1.0000	0.9998	0.9982	0.9907	0.9656	0.9040	0.7878		<b>14</b>				
<b>15</b>												1.0000	0.9995	0.9971	0.9868	0.9560	0.8852		<b>15</b>				
<b>16</b>													0.9999	0.9992	0.9957	0.9826	0.9461		<b>16</b>				
<b>17</b>														1.0000	0.9998	0.9988	0.9942	0.9784	<b>17</b>				
<b>18</b>															1.0000	0.9997	0.9984	0.9927	<b>18</b>				
<b>19</b>																0.9999	0.9996	0.9980	<b>19</b>				
<b>20</b>																	1.0000	0.9999	0.9995	<b>20</b>			
<b>21</b>																		1.0000	0.9999	<b>21</b>			
<b>22</b>																			1.0000	<b>22</b>			
$x$	$n=30$																		$x$				
<b>0</b>	0.7397	0.5455	0.4010	0.2939	0.2146	0.1563	0.1134	0.0820	0.0591	0.0424	0.0076	0.0012	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	<b>0</b>				
<b>1</b>	0.9639	0.8795	0.7731	0.6612	0.5535	0.4555	0.3694	0.2958	0.2343	0.1837	0.0480	0.0105	0.0020	0.0003	0.0000	0.0000	0.0000	0.0000	<b>1</b>				
<b>2</b>	0.9967	0.9783	0.9399	0.8831	0.8122	0.7324	0.6487	0.5654	0.4855	0.4114	0.1514	0.0442	0.0106	0.0021	0.0003	0.0000	0.0000	0.0000	<b>2</b>				
<b>3</b>	0.9998	0.9971	0.9881	0.9694	0.9392	0.8974	0.8450	0.7842	0.7175	0.6474	0.3217	0.1227	0.0374	0.0093	0.0019	0.0003	0.0000	0.0000	<b>3</b>				
<b>4</b>	1.0000	0.9997	0.9982	0.9937	0.9844	0.9685	0.9447	0.9126	0.8723	0.8245	0.5245	0.2552	0.0979	0.0302	0.0075	0.0015	0.0002	0.0000	<b>4</b>				
<b>5</b>		1.0000	0.9998	0.9989	0.9967	0.9921	0.9838	0.9707	0.9519	0.9268	0.7106	0.4275	0.2026	0.0766	0.0233	0.0057	0.0011	0.0002	<b>5</b>				
<b>6</b>			1.0000	0.9999	0.9994	0.9983	0.9960	0.9918	0.9848	0.9742	0.8474	0.6070	0.3481	0.1595	0.0586	0.0172	0.0040	0.0007	<b>6</b>				
<b>7</b>				1.0000	0.9999	0.9997	0.9992	0.9980	0.9959	0.9922	0.9302	0.7608	0.5143	0.2814	0.1238	0.0435	0.0121	0.0026	<b>7</b>				
<b>8</b>					1.0000	1.0000	0.9999	0.9996	0.9990	0.9980	0.9722	0.8713	0.6736	0.4315	0.2247	0.0940	0.0312	0.0081	<b>8</b>				
<b>9</b>						1.0000	0.9999	0.9998	0.9995	0.9903	0.9389	0.8034	0.5888	0.3575	0.1763	0.0694	0.0214		<b>9</b>				
<b>10</b>							1.0000	1.0000	0.9999	0.9971	0.9744	0.8943	0.7304	0.5078	0.2915	0.1350	0.0494		<b>10</b>				
<b>11</b>								1.0000	0.9992	0.9905	0.9493	0.8407	0.6548	0.4311	0.2327	0.1002			<b>11</b>				
<b>12</b>									0.9998	0.9969	0.9784	0.9155	0.7802	0.5785	0.3592	0.1808			<b>12</b>				
<b>13</b>										1.0000	0.9991	0.9918	0.9599	0.8737	0.7145	0.5025	0.2923		<b>13</b>				
<b>14</b>											0.9998	0.9973	0.9831	0.9348	0.8246	0.6448	0.4278		<b>14</b>				
<b>15</b>												0.9999	0.9992	0.9936	0.9699	0.9029	0.7691	0.5722	<b>15</b>				
<b>16</b>													1.0000	0.9998	0.9979	0.9876	0.9519	0.8644	0.7077	<b>16</b>			
<b>17</b>														0.9999	0.9994	0.9955	0.9788	0.9286	0.8192	<b>17</b>			
<b>18</b>															1.0000	0.9998	0.9986	0.9917	0.9666	0.8998	<b>18</b>		
<b>19</b>																1.0000	0.9996	0.9971	0.9862	0.9506	<b>19</b>		
<b>20</b>																	0.9999	0.9991	0.9950	0.9786	<b>20</b>		
<b>21</b>																		1.0000	0.9998	0.9984	0.9919	<b>21</b>	
<b>22</b>																			1.0000	0.9996	0.9974	<b>22</b>	
<b>23</b>																				0.9999	0.9993	<b>23</b>	
<b>24</b>																					1.0000	0.9998	<b>24</b>
<b>25</b>																						1.0000	<b>25</b>

$p$	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	$p$	
$x$	$n=40$																			$x$
<b>0</b>	0.6690	0.4457	0.2957	0.1954	0.1285	0.0842	0.0549	0.0356	0.0230	0.0148	0.0015	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	<b>0</b>	
<b>1</b>	0.9393	0.8095	0.6615	0.5210	0.3991	0.2990	0.2201	0.1594	0.1140	0.0805	0.0121	0.0015	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	<b>1</b>	
<b>2</b>	0.9925	0.9543	0.8822	0.7855	0.6767	0.5665	0.4625	0.3694	0.2894	0.2228	0.0486	0.0079	0.0010	0.0001	0.0000	0.0000	0.0000	0.0000	<b>2</b>	
<b>3</b>	0.9993	0.9918	0.9686	0.9252	0.8619	0.7827	0.6937	0.6007	0.5092	0.4231	0.1302	0.0285	0.0047	0.0006	0.0001	0.0000	0.0000	0.0000	<b>3</b>	
<b>4</b>	1.0000	0.9988	0.9933	0.9790	0.9520	0.9104	0.8546	0.7868	0.7103	0.6290	0.2633	0.0759	0.0160	0.0026	0.0003	0.0000	0.0000	0.0000	<b>4</b>	
<b>5</b>		0.9999	0.9988	0.9951	0.9861	0.9691	0.9419	0.9033	0.8535	0.7937	0.4325	0.1613	0.0433	0.0086	0.0013	0.0001	0.0000	0.0000	<b>5</b>	
<b>6</b>		1.0000	0.9998	0.9990	0.9966	0.9909	0.9801	0.9624	0.9361	0.9005	0.6067	0.2859	0.0962	0.0238	0.0044	0.0006	0.0001	0.0000	<b>6</b>	
<b>7</b>			1.0000	0.9998	0.9993	0.9977	0.9942	0.9873	0.9758	0.9581	0.7559	0.4371	0.1820	0.0553	0.0124	0.0021	0.0002	0.0000	<b>7</b>	
<b>8</b>				1.0000	0.9999	0.9995	0.9985	0.9963	0.9919	0.9845	0.8646	0.5931	0.2998	0.1110	0.0303	0.0061	0.0009	0.0001	<b>8</b>	
<b>9</b>					1.0000	0.9999	0.9997	0.9990	0.9976	0.9949	0.9328	0.7318	0.4395	0.1959	0.0644	0.0156	0.0027	0.0003	<b>9</b>	
<b>10</b>						1.0000	0.9999	0.9998	0.9994	0.9985	0.9701	0.8392	0.5839	0.3087	0.1215	0.0352	0.0074	0.0011	<b>10</b>	
<b>11</b>							1.0000	0.9999	0.9996	0.9980	0.9125	0.7151	0.4406	0.2053	0.0709	0.0179	0.0032		<b>11</b>	
<b>12</b>								1.0000	0.9999	0.9957	0.9568	0.8209	0.5772	0.3143	0.1285	0.0386	0.0083		<b>12</b>	
<b>13</b>									1.0000	0.9986	0.9806	0.8968	0.7032	0.4408	0.2112	0.0751	0.0192		<b>13</b>	
<b>14</b>										0.9996	0.9921	0.9456	0.8074	0.5721	0.3174	0.1326	0.0403		<b>14</b>	
<b>15</b>										0.9999	0.9971	0.9738	0.8849	0.6946	0.4402	0.2142	0.0769		<b>15</b>	
<b>16</b>										1.0000	0.9990	0.9884	0.9367	0.7978	0.5681	0.3185	0.1341		<b>16</b>	
<b>17</b>											0.9997	0.9953	0.9680	0.8761	0.6885	0.4391	0.2148		<b>17</b>	
<b>18</b>											0.9999	0.9983	0.9852	0.9301	0.7911	0.5651	0.3179		<b>18</b>	
<b>19</b>											1.0000	0.9994	0.9937	0.9637	0.8702	0.6844	0.4373		<b>19</b>	
<b>20</b>												0.9998	0.9976	0.9827	0.9256	0.7870	0.5627		<b>20</b>	
<b>21</b>												1.0000	0.9991	0.9925	0.9608	0.8669	0.6821		<b>21</b>	
<b>22</b>													0.9997	0.9970	0.9811	0.9233	0.7852		<b>22</b>	
<b>23</b>													0.9999	0.9989	0.9917	0.9595	0.8659		<b>23</b>	
<b>24</b>													1.0000	0.9996	0.9966	0.9804	0.9231		<b>24</b>	
<b>25</b>														0.9999	0.9988	0.9914	0.9597		<b>25</b>	
<b>26</b>														1.0000	0.9996	0.9966	0.9808		<b>26</b>	
<b>27</b>															0.9999	0.9988	0.9917		<b>27</b>	
<b>28</b>															1.0000	0.9996	0.9968		<b>28</b>	
<b>29</b>																0.9999	0.9989		<b>29</b>	
<b>30</b>																1.0000	0.9997		<b>30</b>	
<b>31</b>																	0.9999		<b>31</b>	
<b>32</b>																	1.0000		<b>32</b>	

$p$	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	$p$
$x$	$n=50$																		$x$
0	0.6050	0.3642	0.2181	0.1299	0.0769	0.0453	0.0266	0.0155	0.0090	0.0052	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0
1	0.9106	0.7358	0.5553	0.4005	0.2794	0.1900	0.1265	0.0827	0.0532	0.0338	0.0029	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1
2	0.9862	0.9216	0.8108	0.6767	0.5405	0.4162	0.3108	0.2260	0.1605	0.1117	0.0142	0.0013	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	2
3	0.9984	0.9822	0.9372	0.8609	0.7604	0.6473	0.5327	0.4253	0.3303	0.2503	0.0460	0.0057	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000	3
4	0.9999	0.9968	0.9832	0.9510	0.8964	0.8206	0.7290	0.6290	0.5277	0.4312	0.1121	0.0185	0.0021	0.0002	0.0000	0.0000	0.0000	0.0000	4
5	1.0000	0.9995	0.9963	0.9856	0.9622	0.9224	0.8650	0.7919	0.7072	0.6161	0.2194	0.0480	0.0070	0.0007	0.0001	0.0000	0.0000	0.0000	5
6		0.9999	0.9993	0.9964	0.9882	0.9711	0.9417	0.8981	0.8404	0.7702	0.3613	0.1034	0.0194	0.0025	0.0002	0.0000	0.0000	0.0000	6
7		1.0000	0.9999	0.9992	0.9968	0.9906	0.9780	0.9562	0.9232	0.8779	0.5188	0.1904	0.0453	0.0073	0.0008	0.0001	0.0000	0.0000	7
8			1.0000	0.9999	0.9992	0.9973	0.9927	0.9833	0.9672	0.9421	0.6681	0.3073	0.0916	0.0183	0.0025	0.0002	0.0000	0.0000	8
9				1.0000	0.9998	0.9993	0.9978	0.9944	0.9875	0.9755	0.7911	0.4437	0.1637	0.0402	0.0067	0.0008	0.0001	0.0000	9
10					1.0000	0.9998	0.9994	0.9983	0.9957	0.9906	0.8801	0.5836	0.2622	0.0789	0.0160	0.0022	0.0002	0.0000	10
11						1.0000	0.9999	0.9995	0.9987	0.9968	0.9372	0.7107	0.3816	0.1390	0.0342	0.0057	0.0006	0.0000	11
12							1.0000	0.9999	0.9996	0.9990	0.9699	0.8139	0.5110	0.2229	0.0661	0.0133	0.0018	0.0002	12
13								1.0000	0.9999	0.9997	0.9868	0.8894	0.6370	0.3279	0.1163	0.0280	0.0045	0.0005	13
14									1.0000	0.9999	0.9947	0.9393	0.7481	0.4468	0.1878	0.0540	0.0104	0.0013	14
15										1.0000	0.9981	0.9692	0.8369	0.5692	0.2801	0.0955	0.0220	0.0033	15
16											0.9993	0.9856	0.9017	0.6839	0.3889	0.1561	0.0427	0.0077	16
17											0.9998	0.9937	0.9449	0.7822	0.5060	0.2369	0.0765	0.0164	17
18											0.9999	0.9975	0.9713	0.8594	0.6216	0.3356	0.1273	0.0325	18
19											1.0000	0.9991	0.9861	0.9152	0.7264	0.4465	0.1974	0.0595	19
20												0.9997	0.9937	0.9522	0.8139	0.5610	0.2862	0.1013	20
21												0.9999	0.9974	0.9749	0.8813	0.6701	0.3900	0.1611	21
22												1.0000	0.9990	0.9877	0.9290	0.7660	0.5019	0.2399	22
23													0.9996	0.9944	0.9604	0.8438	0.6134	0.3359	23
24													0.9999	0.9976	0.9793	0.9022	0.7160	0.4439	24
25													1.0000	0.9991	0.9900	0.9427	0.8034	0.5561	25
26														0.9997	0.9955	0.9686	0.8721	0.6641	26
27														0.9999	0.9981	0.9840	0.9220	0.7601	27
28														1.0000	0.9993	0.9924	0.9556	0.8389	28
29															0.9997	0.9966	0.9765	0.8987	29
30															0.9999	0.9986	0.9884	0.9405	30
31															1.0000	0.9995	0.9947	0.9675	31
32																0.9998	0.9978	0.9836	32
33																0.9999	0.9991	0.9923	33
34																1.0000	0.9997	0.9967	34
35																	0.9999	0.9987	35
36																	1.0000	0.9995	36
37																		0.9998	37
38																		1.0000	38

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**TABLE 2 CUMULATIVE POISSON DISTRIBUTION FUNCTION**

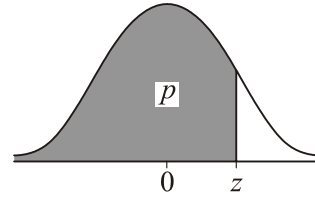
The tabulated value is  $P(X \leq x)$ , where  $X$  has a Poisson distribution with mean  $\lambda$ .

$\lambda$	<b>0.10</b>	<b>0.20</b>	<b>0.30</b>	<b>0.40</b>	<b>0.50</b>	<b>0.60</b>	<b>0.70</b>	<b>0.80</b>	<b>0.90</b>	<b>1.0</b>	<b>1.2</b>	<b>1.4</b>	<b>1.6</b>	<b>1.8</b>	$\lambda$
$x$															$x$
<b>0</b>	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066	0.3679	0.3012	0.2466	0.2019	0.1653	<b>0</b>
<b>1</b>	0.9953	0.9825	0.9631	0.9384	0.9098	0.8781	0.8442	0.8088	0.7725	0.7358	0.6626	0.5918	0.5249	0.4628	<b>1</b>
<b>2</b>	0.9998	0.9989	0.9964	0.9921	0.9856	0.9769	0.9659	0.9526	0.9371	0.9197	0.8795	0.8335	0.7834	0.7306	<b>2</b>
<b>3</b>	1.0000	0.9999	0.9997	0.9992	0.9982	0.9966	0.9942	0.9909	0.9865	0.9810	0.9662	0.9463	0.9212	0.8913	<b>3</b>
<b>4</b>		1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9986	0.9977	0.9963	0.9923	0.9857	0.9763	0.9636	<b>4</b>
<b>5</b>				1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9994	0.9985	0.9968	0.9940	0.9896	<b>5</b>
<b>6</b>							1.0000	1.0000	1.0000	0.9999	0.9997	0.9994	0.9987	0.9974	<b>6</b>
<b>7</b>										1.0000	1.0000	0.9999	0.9997	0.9994	<b>7</b>
<b>8</b>												1.0000	1.0000	0.9999	<b>8</b>
<b>9</b>														1.0000	<b>9</b>
$\lambda$	<b>2.0</b>	<b>2.2</b>	<b>2.4</b>	<b>2.6</b>	<b>2.8</b>	<b>3.0</b>	<b>3.2</b>	<b>3.4</b>	<b>3.6</b>	<b>3.8</b>	<b>4.0</b>	<b>4.5</b>	<b>5.0</b>	<b>5.5</b>	$\lambda$
$x$															$x$
<b>0</b>	0.1353	0.1108	0.0907	0.0743	0.0608	0.0498	0.0408	0.0334	0.0273	0.0224	0.0183	0.0111	0.0067	0.0041	<b>0</b>
<b>1</b>	0.4060	0.3546	0.3084	0.2674	0.2311	0.1991	0.1712	0.1468	0.1257	0.1074	0.0916	0.0611	0.0404	0.0266	<b>1</b>
<b>2</b>	0.6767	0.6227	0.5697	0.5184	0.4695	0.4232	0.3799	0.3397	0.3027	0.2689	0.2381	0.1736	0.1247	0.0884	<b>2</b>
<b>3</b>	0.8571	0.8194	0.7787	0.7360	0.6919	0.6472	0.6025	0.5584	0.5152	0.4735	0.4335	0.3423	0.2650	0.2017	<b>3</b>
<b>4</b>	0.9473	0.9275	0.9041	0.8774	0.8477	0.8153	0.7806	0.7442	0.7064	0.6678	0.6288	0.5321	0.4405	0.3575	<b>4</b>
<b>5</b>	0.9834	0.9751	0.9643	0.9510	0.9349	0.9161	0.8946	0.8705	0.8441	0.8156	0.7851	0.7029	0.6160	0.5289	<b>5</b>
<b>6</b>	0.9955	0.9925	0.9884	0.9828	0.9756	0.9665	0.9554	0.9421	0.9267	0.9091	0.8893	0.8311	0.7622	0.6860	<b>6</b>
<b>7</b>	0.9989	0.9980	0.9967	0.9947	0.9919	0.9881	0.9832	0.9769	0.9692	0.9599	0.9489	0.9134	0.8666	0.8095	<b>7</b>
<b>8</b>	0.9998	0.9995	0.9991	0.9985	0.9976	0.9962	0.9943	0.9917	0.9883	0.9840	0.9786	0.9597	0.9319	0.8944	<b>8</b>
<b>9</b>	1.0000	0.9999	0.9998	0.9996	0.9993	0.9989	0.9982	0.9973	0.9960	0.9942	0.9919	0.9829	0.9682	0.9462	<b>9</b>
<b>10</b>		1.0000	1.0000	0.9999	0.9998	0.9997	0.9995	0.9992	0.9987	0.9981	0.9972	0.9933	0.9863	0.9747	<b>10</b>
<b>11</b>				1.0000	1.0000	0.9999	0.9999	0.9998	0.9996	0.9994	0.9991	0.9976	0.9945	0.9890	<b>11</b>
<b>12</b>						1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9992	0.9980	0.9955	<b>12</b>
<b>13</b>								1.0000	1.0000	1.0000	0.9999	0.9997	0.9993	0.9983	<b>13</b>
<b>14</b>											1.0000	0.9999	0.9998	0.9994	<b>14</b>
<b>15</b>												1.0000	0.9999	0.9998	<b>15</b>
<b>16</b>													1.0000	0.9999	<b>16</b>
<b>17</b>														1.0000	<b>17</b>

$\lambda$	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0	11.0	12.0	13.0	14.0	15.0	$\lambda$
$x$															$x$
<b>0</b>	0.0025	0.0015	0.0009	0.0006	0.0003	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	<b>0</b>
<b>1</b>	0.0174	0.0113	0.0073	0.0047	0.0030	0.0019	0.0012	0.0008	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000	<b>1</b>
<b>2</b>	0.0620	0.0430	0.0296	0.0203	0.0138	0.0093	0.0062	0.0042	0.0028	0.0012	0.0005	0.0002	0.0001	0.0000	<b>2</b>
<b>3</b>	0.1512	0.1118	0.0818	0.0591	0.0424	0.0301	0.0212	0.0149	0.0103	0.0049	0.0023	0.0011	0.0005	0.0002	<b>3</b>
<b>4</b>	0.2851	0.2237	0.1730	0.1321	0.0996	0.0744	0.0550	0.0403	0.0293	0.0151	0.0076	0.0037	0.0018	0.0009	<b>4</b>
<b>5</b>	0.4457	0.3690	0.3007	0.2414	0.1912	0.1496	0.1157	0.0885	0.0671	0.0375	0.0203	0.0107	0.0055	0.0028	<b>5</b>
<b>6</b>	0.6063	0.5265	0.4497	0.3782	0.3134	0.2562	0.2068	0.1649	0.1301	0.0786	0.0458	0.0259	0.0142	0.0076	<b>6</b>
<b>7</b>	0.7440	0.6728	0.5987	0.5246	0.4530	0.3856	0.3239	0.2687	0.2202	0.1432	0.0895	0.0540	0.0316	0.0180	<b>7</b>
<b>8</b>	0.8472	0.7916	0.7291	0.6620	0.5925	0.5231	0.4557	0.3918	0.3328	0.2320	0.1550	0.0998	0.0621	0.0374	<b>8</b>
<b>9</b>	0.9161	0.8774	0.8305	0.7764	0.7166	0.6530	0.5874	0.5218	0.4579	0.3405	0.2424	0.1658	0.1094	0.0699	<b>9</b>
<b>10</b>	0.9574	0.9332	0.9015	0.8622	0.8159	0.7634	0.7060	0.6453	0.5830	0.4599	0.3472	0.2517	0.1757	0.1185	<b>10</b>
<b>11</b>	0.9799	0.9661	0.9467	0.9208	0.8881	0.8487	0.8030	0.7520	0.6968	0.5793	0.4616	0.3532	0.2600	0.1848	<b>11</b>
<b>12</b>	0.9912	0.9840	0.9730	0.9573	0.9362	0.9091	0.8758	0.8364	0.7916	0.6887	0.5760	0.4631	0.3585	0.2676	<b>12</b>
<b>13</b>	0.9964	0.9929	0.9872	0.9784	0.9658	0.9486	0.9261	0.8981	0.8645	0.7813	0.6815	0.5730	0.4644	0.3632	<b>13</b>
<b>14</b>	0.9986	0.9970	0.9943	0.9897	0.9827	0.9726	0.9585	0.9400	0.9165	0.8540	0.7720	0.6751	0.5704	0.4657	<b>14</b>
<b>15</b>	0.9995	0.9988	0.9976	0.9954	0.9918	0.9862	0.9780	0.9665	0.9513	0.9074	0.8444	0.7636	0.6694	0.5681	<b>15</b>
<b>16</b>	0.9998	0.9996	0.9990	0.9980	0.9963	0.9934	0.9889	0.9823	0.9730	0.9441	0.8987	0.8355	0.7559	0.6641	<b>16</b>
<b>17</b>	0.9999	0.9998	0.9996	0.9992	0.9984	0.9970	0.9947	0.9911	0.9857	0.9678	0.9370	0.8905	0.8272	0.7489	<b>17</b>
<b>18</b>	1.0000	0.9999	0.9999	0.9997	0.9993	0.9987	0.9976	0.9957	0.9928	0.9823	0.9626	0.9302	0.8826	0.8195	<b>18</b>
<b>19</b>		1.0000	1.0000	0.9999	0.9997	0.9995	0.9989	0.9980	0.9965	0.9907	0.9787	0.9573	0.9235	0.8752	<b>19</b>
<b>20</b>				1.0000	0.9999	0.9998	0.9996	0.9991	0.9984	0.9953	0.9884	0.9750	0.9521	0.9170	<b>20</b>
<b>21</b>					1.0000	0.9999	0.9998	0.9996	0.9993	0.9977	0.9939	0.9859	0.9712	0.9469	<b>21</b>
<b>22</b>						1.0000	0.9999	0.9999	0.9997	0.9990	0.9970	0.9924	0.9833	0.9673	<b>22</b>
<b>23</b>							1.0000	0.9999	0.9999	0.9995	0.9985	0.9960	0.9907	0.9805	<b>23</b>
<b>24</b>								1.0000	1.0000	0.9998	0.9993	0.9980	0.9950	0.9888	<b>24</b>
<b>25</b>										0.9999	0.9997	0.9990	0.9974	0.9938	<b>25</b>
<b>26</b>										1.0000	0.9999	0.9995	0.9987	0.9967	<b>26</b>
<b>27</b>											0.9999	0.9998	0.9994	0.9983	<b>27</b>
<b>28</b>											1.0000	0.9999	0.9997	0.9991	<b>28</b>
<b>29</b>												1.0000	0.9999	0.9996	<b>29</b>
<b>30</b>													0.9999	0.9998	<b>30</b>
<b>31</b>													1.0000	0.9999	<b>31</b>
<b>32</b>														1.0000	<b>32</b>

**TABLE 3 NORMAL DISTRIBUTION FUNCTION**

The table gives the probability,  $p$ , that a normally distributed random variable  $Z$ , with mean = 0 and variance = 1, is less than or equal to  $z$ .

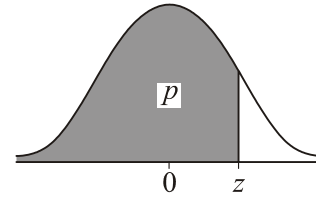


$z$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	$z$
<b>0.0</b>	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586	<b>0.0</b>
<b>0.1</b>	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535	<b>0.1</b>
<b>0.2</b>	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409	<b>0.2</b>
<b>0.3</b>	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173	<b>0.3</b>
<b>0.4</b>	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793	<b>0.4</b>
<b>0.5</b>	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240	<b>0.5</b>
<b>0.6</b>	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490	<b>0.6</b>
<b>0.7</b>	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524	<b>0.7</b>
<b>0.8</b>	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327	<b>0.8</b>
<b>0.9</b>	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891	<b>0.9</b>
<b>1.0</b>	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214	<b>1.0</b>
<b>1.1</b>	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298	<b>1.1</b>
<b>1.2</b>	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147	<b>1.2</b>
<b>1.3</b>	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774	<b>1.3</b>
<b>1.4</b>	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189	<b>1.4</b>
<b>1.5</b>	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408	<b>1.5</b>
<b>1.6</b>	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449	<b>1.6</b>
<b>1.7</b>	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327	<b>1.7</b>
<b>1.8</b>	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062	<b>1.8</b>
<b>1.9</b>	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670	<b>1.9</b>
<b>2.0</b>	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169	<b>2.0</b>
<b>2.1</b>	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574	<b>2.1</b>
<b>2.2</b>	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899	<b>2.2</b>
<b>2.3</b>	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158	<b>2.3</b>
<b>2.4</b>	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361	<b>2.4</b>
<b>2.5</b>	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520	<b>2.5</b>
<b>2.6</b>	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643	<b>2.6</b>
<b>2.7</b>	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736	<b>2.7</b>
<b>2.8</b>	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807	<b>2.8</b>
<b>2.9</b>	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861	<b>2.9</b>
<b>3.0</b>	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900	<b>3.0</b>
<b>3.1</b>	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929	<b>3.1</b>
<b>3.2</b>	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950	<b>3.2</b>
<b>3.3</b>	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965	<b>3.3</b>
<b>3.4</b>	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976	<b>3.4</b>
<b>3.5</b>	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983	<b>3.5</b>
<b>3.6</b>	0.99984	0.99985	0.99985	0.99986	0.99986	0.99987	0.99987	0.99988	0.99988	0.99989	<b>3.6</b>
<b>3.7</b>	0.99989	0.99990	0.99990	0.99990	0.99991	0.99991	0.99992	0.99992	0.99992	0.99992	<b>3.7</b>
<b>3.8</b>	0.99993	0.99993	0.99993	0.99994	0.99994	0.99994	0.99994	0.99995	0.99995	0.99995	<b>3.8</b>
<b>3.9</b>	0.99995	0.99995	0.99996	0.99996	0.99996	0.99996	0.99996	0.99996	0.99997	0.99997	<b>3.9</b>



**TABLE 4 PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION**

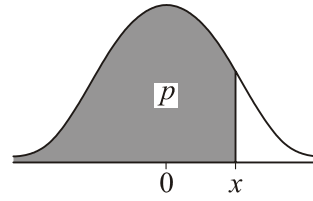
The table gives the values of  $z$  satisfying  $P(Z \leq z) = p$ , where  $Z$  is the normally distributed random variable with mean = 0 and variance = 1.



<i>p</i>	<b>0.00</b>	<b>0.01</b>	<b>0.02</b>	<b>0.03</b>	<b>0.04</b>	<b>0.05</b>	<b>0.06</b>	<b>0.07</b>	<b>0.08</b>	<b>0.09</b>	<i>p</i>
<b>0.5</b>	0.0000	0.0251	0.0502	0.0753	0.1004	0.1257	0.1510	0.1764	0.2019	0.2275	<b>0.5</b>
<b>0.6</b>	0.2533	0.2793	0.3055	0.3319	0.3585	0.3853	0.4125	0.4399	0.4677	0.4958	<b>0.6</b>
<b>0.7</b>	0.5244	0.5534	0.5828	0.6128	0.6433	0.6745	0.7063	0.7388	0.7722	0.8064	<b>0.7</b>
<b>0.8</b>	0.8416	0.8779	0.9154	0.9542	0.9945	1.0364	1.0803	1.1264	1.1750	1.2265	<b>0.8</b>
<b>0.9</b>	1.2816	1.3408	1.4051	1.4758	1.5548	1.6449	1.7507	1.8808	2.0537	2.3263	<b>0.9</b>
<i>p</i>	<b>0.000</b>	<b>0.001</b>	<b>0.002</b>	<b>0.003</b>	<b>0.004</b>	<b>0.005</b>	<b>0.006</b>	<b>0.007</b>	<b>0.008</b>	<b>0.009</b>	<i>p</i>
<b>0.95</b>	1.6449	1.6546	1.6646	1.6747	1.6849	1.6954	1.7060	1.7169	1.7279	1.7392	<b>0.95</b>
<b>0.96</b>	1.7507	1.7624	1.7744	1.7866	1.7991	1.8119	1.8250	1.8384	1.8522	1.8663	<b>0.96</b>
<b>0.97</b>	1.8808	1.8957	1.9110	1.9268	1.9431	1.9600	1.9774	1.9954	2.0141	2.0335	<b>0.97</b>
<b>0.98</b>	2.0537	2.0749	2.0969	2.1201	2.1444	2.1701	2.1973	2.2262	2.2571	2.2904	<b>0.98</b>
<b>0.99</b>	2.3263	2.3656	2.4089	2.4573	2.5121	2.5758	2.6521	2.7478	2.8782	3.0902	<b>0.99</b>

**TABLE 5 PERCENTAGE POINTS OF THE STUDENT'S *t*-DISTRIBUTION**

The table gives the values of  $x$  satisfying  $P(X \leq x) = p$ , where  $X$  is a random variable having the Student's  $t$ -distribution with  $\nu$  degrees of freedom.

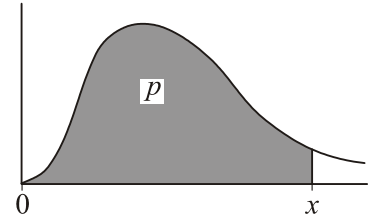


$p$	<b>0.9</b>	<b>0.95</b>	<b>0.975</b>	<b>0.99</b>	<b>0.995</b>
$\nu$					
<b>1</b>	3.078	6.314	12.706	31.821	63.657
<b>2</b>	1.886	2.920	4.303	6.965	9.925
<b>3</b>	1.638	2.353	3.182	4.541	5.841
<b>4</b>	1.533	2.132	2.776	3.747	4.604
<b>5</b>	1.476	2.015	2.571	3.365	4.032
<b>6</b>	1.440	1.943	2.447	3.143	3.707
<b>7</b>	1.415	1.895	2.365	2.998	3.499
<b>8</b>	1.397	1.860	2.306	2.896	3.355
<b>9</b>	1.383	1.833	2.262	2.821	3.250
<b>10</b>	1.372	1.812	2.228	2.764	3.169
<b>11</b>	1.363	1.796	2.201	2.718	3.106
<b>12</b>	1.356	1.782	2.179	2.681	3.055
<b>13</b>	1.350	1.771	2.160	2.650	3.012
<b>14</b>	1.345	1.761	2.145	2.624	2.977
<b>15</b>	1.341	1.753	2.131	2.602	2.947
<b>16</b>	1.337	1.746	2.121	2.583	2.921
<b>17</b>	1.333	1.740	2.110	2.567	2.898
<b>18</b>	1.330	1.734	2.101	2.552	2.878
<b>19</b>	1.328	1.729	2.093	2.539	2.861
<b>20</b>	1.325	1.725	2.086	2.528	2.845
<b>21</b>	1.323	1.721	2.080	2.518	2.831
<b>22</b>	1.321	1.717	2.074	2.508	2.819
<b>23</b>	1.319	1.714	2.069	2.500	2.807
<b>24</b>	1.318	1.711	2.064	2.492	2.797
<b>25</b>	1.316	1.708	2.060	2.485	2.787
<b>26</b>	1.315	1.706	2.056	2.479	2.779
<b>27</b>	1.314	1.703	2.052	2.473	2.771
<b>28</b>	1.313	1.701	2.048	2.467	2.763

$p$	<b>0.9</b>	<b>0.95</b>	<b>0.975</b>	<b>0.99</b>	<b>0.995</b>
$\nu$					
<b>29</b>	1.311	1.699	2.045	2.462	2.756
<b>30</b>	1.310	1.697	2.042	2.457	2.750
<b>31</b>	1.309	1.696	2.040	2.453	2.744
<b>32</b>	1.309	1.694	2.037	2.449	2.738
<b>33</b>	1.308	1.692	2.035	2.445	2.733
<b>34</b>	1.307	1.691	2.032	2.441	2.728
<b>35</b>	1.306	1.690	2.030	2.438	2.724
<b>36</b>	1.306	1.688	2.028	2.434	2.719
<b>37</b>	1.305	1.687	2.026	2.431	2.715
<b>38</b>	1.304	1.686	2.024	2.429	2.712
<b>39</b>	1.304	1.685	2.023	2.426	2.708
<b>40</b>	1.303	1.684	2.021	2.423	2.704
<b>45</b>	1.301	1.679	2.014	2.412	2.690
<b>50</b>	1.299	1.676	2.009	2.403	2.678
<b>55</b>	1.297	1.673	2.004	2.396	2.668
<b>60</b>	1.296	1.671	2.000	2.390	2.660
<b>65</b>	1.295	1.669	1.997	2.385	2.654
<b>70</b>	1.294	1.667	1.994	2.381	2.648
<b>75</b>	1.293	1.665	1.992	2.377	2.643
<b>80</b>	1.292	1.664	1.990	2.374	2.639
<b>85</b>	1.292	1.663	1.988	2.371	2.635
<b>90</b>	1.291	1.662	1.987	2.368	2.632
<b>95</b>	1.291	1.661	1.985	2.366	2.629
<b>100</b>	1.290	1.660	1.984	2.364	2.626
<b>125</b>	1.288	1.657	1.979	2.357	2.616
<b>150</b>	1.287	1.655	1.976	2.351	2.609
<b>200</b>	1.286	1.653	1.972	2.345	2.601
$\infty$	1.282	1.645	1.960	2.326	2.576

**TABLE 6 PERCENTAGE POINTS OF THE  $\chi^2$  DISTRIBUTION**

The table gives the values of  $x$  satisfying  $P(X \leq x) = p$ , where  $X$  is a random variable having the  $\chi^2$  distribution with  $\nu$  degrees of freedom.



$p$	<b>0.005</b>	<b>0.01</b>	<b>0.025</b>	<b>0.05</b>	<b>0.1</b>	<b>0.9</b>	<b>0.95</b>	<b>0.975</b>	<b>0.99</b>	<b>0.995</b>	$p$
$\nu$											$\nu$
<b>1</b>	0.00004	0.0002	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879	<b>1</b>
<b>2</b>	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597	<b>2</b>
<b>3</b>	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838	<b>3</b>
<b>4</b>	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860	<b>4</b>
<b>5</b>	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750	<b>5</b>
<b>6</b>	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548	<b>6</b>
<b>7</b>	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278	<b>7</b>
<b>8</b>	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955	<b>8</b>
<b>9</b>	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589	<b>9</b>
<b>10</b>	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188	<b>10</b>
<b>11</b>	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757	<b>11</b>
<b>12</b>	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300	<b>12</b>
<b>13</b>	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819	<b>13</b>
<b>14</b>	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319	<b>14</b>
<b>15</b>	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801	<b>15</b>
<b>16</b>	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267	<b>16</b>
<b>17</b>	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718	<b>17</b>
<b>18</b>	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156	<b>18</b>
<b>19</b>	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582	<b>19</b>
<b>20</b>	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997	<b>20</b>
<b>21</b>	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401	<b>21</b>
<b>22</b>	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796	<b>22</b>
<b>23</b>	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181	<b>23</b>
<b>24</b>	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559	<b>24</b>
<b>25</b>	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928	<b>25</b>
<b>26</b>	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290	<b>26</b>
<b>27</b>	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645	<b>27</b>
<b>28</b>	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993	<b>28</b>
<b>29</b>	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336	<b>29</b>
<b>30</b>	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672	<b>30</b>
<b>31</b>	14.458	15.655	17.539	19.281	21.434	41.422	44.985	48.232	52.191	55.003	<b>31</b>
<b>32</b>	15.134	16.362	18.291	20.072	22.271	42.585	46.194	49.480	53.486	56.328	<b>32</b>
<b>33</b>	15.815	17.074	19.047	20.867	23.110	43.745	47.400	50.725	54.776	57.648	<b>33</b>
<b>34</b>	16.501	17.789	19.806	21.664	23.952	44.903	48.602	51.996	56.061	58.964	<b>34</b>
<b>35</b>	17.192	18.509	20.569	22.465	24.797	46.059	49.802	53.203	57.342	60.275	<b>35</b>
<b>36</b>	17.887	19.223	21.336	23.269	25.643	47.212	50.998	54.437	58.619	61.581	<b>36</b>
<b>37</b>	18.586	19.960	22.106	24.075	26.492	48.363	52.192	55.668	59.892	62.883	<b>37</b>
<b>38</b>	19.289	20.691	22.878	24.884	27.343	49.513	53.384	56.896	61.162	64.181	<b>38</b>
<b>39</b>	19.996	21.426	23.654	25.695	28.196	50.660	54.572	58.120	62.428	65.476	<b>39</b>
<b>40</b>	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766	<b>40</b>
<b>45</b>	24.311	25.901	28.366	30.612	33.350	57.505	61.656	65.410	69.957	73.166	<b>45</b>
<b>50</b>	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490	<b>50</b>
<b>55</b>	31.735	33.570	36.398	38.958	42.060	68.796	73.311	77.380	82.292	85.749	<b>55</b>
<b>60</b>	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952	<b>60</b>
<b>65</b>	39.383	41.444	44.603	47.450	50.883	79.973	84.821	89.177	94.422	98.105	<b>65</b>
<b>70</b>	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215	<b>70</b>
<b>75</b>	47.206	49.475	52.942	56.054	59.795	91.061	96.217	100.839	106.393	110.286	<b>75</b>
<b>80</b>	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321	<b>80</b>
<b>85</b>	55.170	57.634	61.389	64.749	68.777	102.079	107.522	112.393	118.236	122.325	<b>85</b>
<b>90</b>	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299	<b>90</b>
<b>95</b>	63.250	65.898	69.925	73.520	77.818	113.038	118.752	123.858	129.973	134.247	<b>95</b>
<b>100</b>	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169	<b>100</b>





**TABLE 8 CRITICAL VALUES OF THE PRODUCT MOMENT CORRELATION COEFFICIENT**

The table gives the critical values, for different significance levels, of the product moment correlation coefficient,  $r$ , for varying sample sizes,  $n$ .

<b>One tail Two tail</b>	<b>10% 20%</b>	<b>5% 10%</b>	<b>2.5% 5%</b>	<b>1% 2%</b>	<b>0.5% 1%</b>	<b>One tail Two tail</b>
<i>n</i>						<i>n</i>
<b>4</b>	0.8000	0.9000	0.9500	0.9800	0.9900	<b>4</b>
<b>5</b>	0.6870	0.8054	0.8783	0.9343	0.9587	<b>5</b>
<b>6</b>	0.6084	0.7293	0.8114	0.8822	0.9172	<b>6</b>
<b>7</b>	0.5509	0.6694	0.7545	0.8329	0.8745	<b>7</b>
<b>8</b>	0.5067	0.6215	0.7067	0.7887	0.8343	<b>8</b>
<b>9</b>	0.4716	0.5822	0.6664	0.7498	0.7977	<b>9</b>
<b>10</b>	0.4428	0.5494	0.6319	0.7155	0.7646	<b>10</b>
<b>11</b>	0.4187	0.5214	0.6021	0.6851	0.7348	<b>11</b>
<b>12</b>	0.3981	0.4973	0.5760	0.6581	0.7079	<b>12</b>
<b>13</b>	0.3802	0.4762	0.5529	0.6339	0.6835	<b>13</b>
<b>14</b>	0.3646	0.4575	0.5324	0.6120	0.6614	<b>14</b>
<b>15</b>	0.3507	0.4409	0.5140	0.5923	0.6411	<b>15</b>
<b>16</b>	0.3383	0.4259	0.4973	0.5742	0.6226	<b>16</b>
<b>17</b>	0.3271	0.4124	0.4821	0.5577	0.6055	<b>17</b>
<b>18</b>	0.3170	0.4000	0.4683	0.5425	0.5897	<b>18</b>
<b>19</b>	0.3077	0.3887	0.4555	0.5285	0.5751	<b>19</b>
<b>20</b>	0.2992	0.3783	0.4438	0.5155	0.5614	<b>20</b>
<b>21</b>	0.2914	0.3687	0.4329	0.5034	0.5487	<b>21</b>
<b>22</b>	0.2841	0.3598	0.4227	0.4921	0.5368	<b>22</b>
<b>23</b>	0.2774	0.3515	0.4132	0.4815	0.5256	<b>23</b>
<b>24</b>	0.2711	0.3438	0.4044	0.4716	0.5151	<b>24</b>
<b>25</b>	0.2653	0.3365	0.3961	0.4622	0.5052	<b>25</b>
<b>26</b>	0.2598	0.3297	0.3882	0.4534	0.4958	<b>26</b>
<b>27</b>	0.2546	0.3233	0.3809	0.4451	0.4869	<b>27</b>
<b>28</b>	0.2497	0.3172	0.3739	0.4372	0.4785	<b>28</b>
<b>29</b>	0.2451	0.3115	0.3673	0.4297	0.4705	<b>29</b>
<b>30</b>	0.2407	0.3061	0.3610	0.4226	0.4629	<b>30</b>
<b>31</b>	0.2366	0.3009	0.3550	0.4158	0.4556	<b>31</b>
<b>32</b>	0.2327	0.2960	0.3494	0.4093	0.4487	<b>32</b>
<b>33</b>	0.2289	0.2913	0.3440	0.4032	0.4421	<b>33</b>
<b>34</b>	0.2254	0.2869	0.3388	0.3972	0.4357	<b>34</b>
<b>35</b>	0.2220	0.2826	0.3338	0.3916	0.4296	<b>35</b>
<b>36</b>	0.2187	0.2785	0.3291	0.3862	0.4238	<b>36</b>
<b>37</b>	0.2156	0.2746	0.3246	0.3810	0.4182	<b>37</b>
<b>38</b>	0.2126	0.2709	0.3202	0.3760	0.4128	<b>38</b>
<b>39</b>	0.2097	0.2673	0.3160	0.3712	0.4076	<b>39</b>
<b>40</b>	0.2070	0.2638	0.3120	0.3665	0.4026	<b>40</b>
<b>41</b>	0.2043	0.2605	0.3081	0.3621	0.3978	<b>41</b>
<b>42</b>	0.2018	0.2573	0.3044	0.3578	0.3932	<b>42</b>
<b>43</b>	0.1993	0.2542	0.3008	0.3536	0.3887	<b>43</b>
<b>44</b>	0.1970	0.2512	0.2973	0.3496	0.3843	<b>44</b>
<b>45</b>	0.1947	0.2483	0.2940	0.3457	0.3801	<b>45</b>
<b>46</b>	0.1925	0.2455	0.2907	0.3420	0.3761	<b>46</b>
<b>47</b>	0.1903	0.2429	0.2876	0.3384	0.3721	<b>47</b>
<b>48</b>	0.1883	0.2403	0.2845	0.3348	0.3683	<b>48</b>
<b>49</b>	0.1863	0.2377	0.2816	0.3314	0.3646	<b>49</b>
<b>50</b>	0.1843	0.2353	0.2787	0.3281	0.3610	<b>50</b>
<b>60</b>	0.1678	0.2144	0.2542	0.2997	0.3301	<b>60</b>
<b>70</b>	0.1550	0.1982	0.2352	0.2776	0.3060	<b>70</b>
<b>80</b>	0.1448	0.1852	0.2199	0.2597	0.2864	<b>80</b>
<b>90</b>	0.1364	0.1745	0.2072	0.2449	0.2702	<b>90</b>
<b>100</b>	0.1292	0.1654	0.1966	0.2324	0.2565	<b>100</b>

**TABLE 9 CRITICAL VALUES OF SPEARMAN'S RANK CORRELATION COEFFICIENT**

The table gives the critical values, for different significance levels, of Spearman's rank correlation coefficient,  $r_s$ , for varying sample sizes,  $n$ .

Since  $r_s$  is discrete, exact significance levels cannot be obtained in most cases. The critical values given are those with significance levels closest to the stated value.

<b>One tail Two tail</b>	<b>10% 20%</b>	<b>5% 10%</b>	<b>2.5% 5%</b>	<b>1% 2%</b>	<b>0.5% 1%</b>	<b>One tail Two tail</b>
<i>n</i>						<i>n</i>
<b>4</b>	1.0000	1.0000	1.0000	1.0000	1.0000	<b>4</b>
<b>5</b>	0.7000	0.9000	0.9000	1.0000	1.0000	<b>5</b>
<b>6</b>	0.6571	0.7714	0.8286	0.9429	0.9429	<b>6</b>
<b>7</b>	0.5714	0.6786	0.7857	0.8571	0.8929	<b>7</b>
<b>8</b>	0.5476	0.6429	0.7381	0.8095	0.8571	<b>8</b>
<b>9</b>	0.4833	0.6000	0.6833	0.7667	0.8167	<b>9</b>
<b>10</b>	0.4424	0.5636	0.6485	0.7333	0.7818	<b>10</b>
<b>11</b>	0.4182	0.5273	0.6091	0.7000	0.7545	<b>11</b>
<b>12</b>	0.3986	0.5035	0.5874	0.6713	0.7273	<b>12</b>
<b>13</b>	0.3791	0.4780	0.5604	0.6484	0.6978	<b>13</b>
<b>14</b>	0.3670	0.4593	0.5385	0.6220	0.6747	<b>14</b>
<b>15</b>	0.3500	0.4429	0.5179	0.6000	0.6536	<b>15</b>
<b>16</b>	0.3382	0.4265	0.5029	0.5824	0.6324	<b>16</b>
<b>17</b>	0.3271	0.4124	0.4821	0.5577	0.6055	<b>17</b>
<b>18</b>	0.3170	0.4000	0.4683	0.5425	0.5897	<b>18</b>
<b>19</b>	0.3077	0.3887	0.4555	0.5285	0.5751	<b>19</b>
<b>20</b>	0.2992	0.3783	0.4438	0.5155	0.5614	<b>20</b>
<b>21</b>	0.2914	0.3687	0.4329	0.5034	0.5487	<b>21</b>
<b>22</b>	0.2841	0.3598	0.4227	0.4921	0.5368	<b>22</b>
<b>23</b>	0.2774	0.3515	0.4132	0.4815	0.5256	<b>23</b>
<b>24</b>	0.2711	0.3438	0.4044	0.4716	0.5151	<b>24</b>
<b>25</b>	0.2653	0.3365	0.3961	0.4622	0.5052	<b>25</b>
<b>26</b>	0.2598	0.3297	0.3882	0.4534	0.4958	<b>26</b>
<b>27</b>	0.2546	0.3233	0.3809	0.4451	0.4869	<b>27</b>
<b>28</b>	0.2497	0.3172	0.3739	0.4372	0.4785	<b>28</b>
<b>29</b>	0.2451	0.3115	0.3673	0.4297	0.4705	<b>29</b>
<b>30</b>	0.2407	0.3061	0.3610	0.4226	0.4629	<b>30</b>
<b>31</b>	0.2366	0.3009	0.3550	0.4158	0.4556	<b>31</b>
<b>32</b>	0.2327	0.2960	0.3494	0.4093	0.4487	<b>32</b>
<b>33</b>	0.2289	0.2913	0.3440	0.4032	0.4421	<b>33</b>
<b>34</b>	0.2254	0.2869	0.3388	0.3972	0.4357	<b>34</b>
<b>35</b>	0.2220	0.2826	0.3338	0.3916	0.4296	<b>35</b>
<b>36</b>	0.2187	0.2785	0.3291	0.3862	0.4238	<b>36</b>
<b>37</b>	0.2156	0.2746	0.3246	0.3810	0.4182	<b>37</b>
<b>38</b>	0.2126	0.2709	0.3202	0.3760	0.4128	<b>38</b>
<b>39</b>	0.2097	0.2673	0.3160	0.3712	0.4076	<b>39</b>
<b>40</b>	0.2070	0.2638	0.3120	0.3665	0.4026	<b>40</b>
<b>41</b>	0.2043	0.2605	0.3081	0.3621	0.3978	<b>41</b>
<b>42</b>	0.2018	0.2573	0.3044	0.3578	0.3932	<b>42</b>
<b>43</b>	0.1993	0.2542	0.3008	0.3536	0.3887	<b>43</b>
<b>44</b>	0.1970	0.2512	0.2973	0.3496	0.3843	<b>44</b>
<b>45</b>	0.1947	0.2483	0.2940	0.3457	0.3801	<b>45</b>
<b>46</b>	0.1925	0.2455	0.2907	0.3420	0.3761	<b>46</b>
<b>47</b>	0.1903	0.2429	0.2876	0.3384	0.3721	<b>47</b>
<b>48</b>	0.1883	0.2403	0.2845	0.3348	0.3683	<b>48</b>
<b>49</b>	0.1863	0.2377	0.2816	0.3314	0.3646	<b>49</b>
<b>50</b>	0.1843	0.2353	0.2787	0.3281	0.3610	<b>50</b>
<b>60</b>	0.1678	0.2144	0.2542	0.2997	0.3301	<b>60</b>
<b>70</b>	0.1550	0.1982	0.2352	0.2776	0.3060	<b>70</b>
<b>80</b>	0.1448	0.1852	0.2199	0.2597	0.2864	<b>80</b>
<b>90</b>	0.1364	0.1745	0.2072	0.2449	0.2702	<b>90</b>
<b>100</b>	0.1292	0.1654	0.1966	0.2324	0.2565	<b>100</b>

**TABLE 10 CRITICAL VALUES OF THE WILCOXON SIGNED RANK STATISTIC**

The table gives the lower tail critical values of the statistic  $T$ .

The upper tail critical values are given by  $\frac{1}{2}n(n+1) - T$ .

$T$  is the sum of the ranks of observations with the same sign.

Since  $T$  is discrete, exact significance levels cannot usually be obtained.

The critical values tabulated are those with significance levels closest to the stated value.

The critical region includes the tabulated value.

<b>One tail</b>	<b>10%</b>	<b>5%</b>	<b>2.5%</b>	<b>1%</b>	<b>0.5%</b>
<b>Two tail</b>	<b>20%</b>	<b>10%</b>	<b>5%</b>	<b>2%</b>	<b>1%</b>
<i>n</i>					
<b>3</b>	0				
<b>4</b>	1	0			
<b>5</b>	2	1	0		
<b>6</b>	4	2	1	0	
<b>7</b>	6	4	2	0	0
<b>8</b>	8	6	4	2	0
<b>9</b>	11	8	6	3	2
<b>10</b>	14	11	8	5	3
<b>11</b>	18	14	11	7	5
<b>12</b>	22	17	14	10	7
<b>13</b>	26	21	17	13	10
<b>14</b>	31	26	21	16	13
<b>15</b>	37	30	25	20	16
<b>16</b>	42	36	30	24	19
<b>17</b>	49	41	35	28	23
<b>18</b>	55	47	40	33	28
<b>19</b>	62	54	46	38	32
<b>20</b>	70	60	52	43	37



**TABLE 11 CRITICAL VALUES OF THE MANN-WHITNEY STATISTIC**

The table gives the lower tail critical values of the statistic  $U$ .

The upper tail critical values are given by  $mn - U$ .

$U = T - \frac{n(n+1)}{2}$  where  $T$  is the sum of the ranks of the sample of size  $n$ .

Since  $U$  is discrete, exact significance levels cannot be obtained.

The critical values tabulated are those with significance levels closest to the stated value.

The critical region includes the tabulated value.

**One tail 5% Two tail 10%**

<i>m</i>	2	3	4	5	6	7	8	9	10	11	12
<i>n</i>											
2		0	0	0	0	1	1	1	2	2	2
3	0	0	1	1	2	3	3	4	5	5	6
4	0	1	2	3	4	5	6	7	8	9	10
5	0	1	3	4	5	7	8	10	11	12	14
6	0	2	4	5	7	9	11	12	14	16	18
7	1	3	5	7	9	11	13	15	18	20	22
8	1	3	6	8	11	13	16	18	21	24	26
9	1	4	7	10	12	15	18	21	24	27	30
10	2	5	8	11	14	18	21	24	28	31	34
11	2	5	9	12	16	20	24	27	31	35	39
12	2	6	10	14	18	22	26	30	34	39	43

**One tail 2.5% Two tail 5%**

<i>m</i>	2	3	4	5	6	7	8	9	10	11	12
<i>n</i>											
2				0	0	0	0	0	1	1	1
3			0	0	1	2	2	3	3	4	4
4		0	1	2	2	3	4	5	6	7	8
5	0	0	2	3	4	5	6	7	9	10	11
6	0	1	2	4	5	7	8	10	12	13	15
7	0	2	3	5	7	9	11	13	15	17	18
8	0	2	4	6	8	11	13	15	18	20	22
9	0	3	5	7	10	13	15	18	21	23	26
10	1	3	6	9	12	15	18	21	24	27	30
11	1	4	7	10	13	17	20	23	27	30	34
12	1	4	8	11	15	18	22	26	30	34	38

**TABLE 12 CONTROL CHARTS FOR VARIABILITY**

For range charts, multiply  $\sigma$  by the appropriate value of  $D$ .

For standard deviation charts, multiply  $\sigma$  by the appropriate value of  $E$ .

To obtain an estimate of  $\sigma$ , multiply the mean range by the appropriate value of  $b$ .

Normal distribution is assumed.

Sample size	$D_{0.999}$	$D_{0.975}$	$D_{0.025}$	$D_{0.001}$	$E_{0.999}$	$E_{0.975}$	$E_{0.025}$	$E_{0.001}$	$b$
<b>2</b>			3.170	4.654			2.24	3.29	0.8862
<b>3</b>	0.060	0.303	3.682	5.063	0.03	0.16	1.92	2.63	0.5908
<b>4</b>	0.199	0.595	3.984	5.309	0.09	0.27	1.76	2.33	0.4857
<b>5</b>	0.367	0.850	4.197	5.484	0.15	0.35	1.67	2.15	0.4299
<b>6</b>	0.535	1.066	4.361	5.619	0.20	0.41	1.60	2.03	0.3946
<b>7</b>	0.691	1.251	4.494	5.730	0.25	0.45	1.55	1.93	0.3698
<b>8</b>	0.835	1.410	4.605	5.823	0.29	0.49	1.51	1.86	0.3512
<b>10</b>	1.085	1.674	4.784	5.973	0.36	0.55	1.45	1.76	0.3249
<b>12</b>	1.293	1.884	4.925	6.096	0.41	0.59	1.41	1.69	0.3069

**TABLE 13 RANDOM NUMBERS**

2 9 9 2 7	6 6 1 8 7	8 0 7 8 4	3 7 5 4 2	6 2 4 4 6	1 3 4 8 1	7 2 7 3 0	4 8 5 1 1	4 2 3 1 5
6 2 5 0 6	2 2 7 8 0	3 0 7 2 0	7 9 3 3 8	6 8 3 5 8	6 2 7 6 5	3 3 4 0 1	8 2 7 5 8	4 2 9 2 9
8 3 7 5 2	1 0 6 6 4	1 2 1 9 3	8 8 7 6 6	7 6 7 6 3	9 0 9 7 7	4 6 8 8 1	5 9 0 8 9	3 9 6 4 8
7 5 7 0 3	2 7 5 2 2	7 9 5 0 4	0 6 6 6 2	2 5 4 6 8	9 2 4 0 7	1 9 6 2 6	6 1 1 7 3	5 2 7 9 3
9 9 6 1 7	5 9 1 2 0	3 3 5 5 4	3 2 9 0 4	9 5 3 1 2	6 1 7 6 3	6 8 8 6 8	9 4 1 7 9	7 3 4 4 2
1 7 4 9 0	6 4 9 0 0	1 2 6 9 0	9 5 4 7 4	5 3 8 4 9	6 4 7 9 1	3 5 8 4 3	4 4 8 3 2	0 1 2 9 6
0 2 3 8 4	1 6 6 8 0	7 6 6 3 7	4 2 4 3 7	2 7 9 9 4	2 4 7 1 8	0 9 5 6 6	4 3 8 2 1	8 9 3 1 5
4 4 0 3 1	5 1 6 6 8	8 5 9 0 7	2 2 6 8 3	0 6 1 1 9	2 5 3 6 0	3 5 4 8 0	9 1 3 3 4	0 1 5 2 2
9 4 0 5 8	3 6 4 6 6	9 9 7 1 7	5 7 6 5 1	0 2 5 1 2	9 8 7 8 5	8 6 4 9 1	7 6 8 1 2	1 0 3 2 4
8 8 7 8 3	7 7 1 2 7	9 5 7 8 3	4 0 6 6 6	8 2 5 3 9	8 4 2 2 4	9 4 3 5 4	4 1 9 7 9	3 2 8 2 3
3 3 3 8 0	4 7 4 4 4	0 2 9 3 6	5 7 3 0 3	3 1 4 5 8	2 8 6 6 9	2 2 5 3 8	6 6 8 8 4	3 8 3 7 0
9 5 1 9 8	4 1 6 8 4	8 9 0 6 6	1 7 9 6 3	3 9 0 4 2	5 0 7 9 1	4 4 6 8 3	1 5 1 3 4	1 9 9 0 9
0 3 1 8 3	6 2 7 0 6	6 5 5 3 1	4 7 7 6 7	4 2 3 4 7	5 1 8 9 9	3 3 5 8 2	2 8 0 9 8	4 3 1 6 8
2 6 6 2 3	0 0 5 5 0	5 2 3 2 9	9 0 2 9 2	3 7 5 0 8	9 7 3 1 0	9 2 0 4 9	4 7 3 6 5	8 0 2 4 2
5 7 9 2 9	1 4 2 9 0	0 8 1 1 8	9 5 4 7 3	9 1 5 8 6	5 8 9 5 3	7 4 9 9 8	7 3 9 5 0	5 4 6 6 2
4 9 1 0 3	9 2 1 5 0	7 8 2 1 1	2 7 7 6 2	1 8 1 3 5	4 3 4 7 9	6 1 6 9 8	7 7 7 6 8	0 0 2 2 3
9 8 6 3 4	3 1 8 7 0	5 6 8 3 9	6 0 4 7 8	6 2 1 2 9	8 7 1 4 9	6 0 2 4 0	0 9 0 7 9	3 8 5 6 7
9 0 5 9 3	7 6 2 4 8	6 0 3 7 9	9 8 2 0 4	5 9 2 5 4	5 1 6 1 6	4 1 0 9 1	1 1 8 1 8	1 1 0 0 1
6 8 6 0 4	9 0 2 9 8	3 8 5 9 5	5 2 0 4 8	9 5 1 3 7	7 3 3 6 3	5 3 3 0 7	3 7 9 1 4	2 7 9 0 3
7 2 7 1 1	4 3 4 4 1	8 7 1 0 8	8 2 1 5 5	4 3 6 5 0	8 1 9 6 7	5 6 3 4 8	1 9 8 7 8	7 5 8 1 3
0 8 1 9 3	0 5 3 0 2	1 1 3 5 2	4 8 3 6 9	5 5 7 3 1	8 1 1 5 8	2 1 0 3 7	2 9 5 3 4	9 8 0 7 4
7 4 8 2 9	5 1 6 9 5	5 1 6 8 2	9 7 6 6 0	9 7 1 1 0	6 9 5 4 0	6 9 7 7 6	2 2 7 3 6	5 4 6 3 5
2 5 9 0 0	8 1 2 6 5	2 5 3 3 9	4 3 8 7 5	3 8 5 6 3	4 3 5 3 0	3 6 2 8 9	7 8 8 1 0	1 8 9 5 9
4 2 4 1 7	5 0 1 0 6	2 4 7 5 2	9 4 6 6 4	1 1 6 1 1	0 5 7 2 0	7 7 0 9 1	9 6 3 3 8	6 8 5 0 7
3 2 2 9 1	5 7 6 5 3	4 2 1 3 5	3 6 4 4 0	7 9 4 2 7	1 1 6 6 0	1 5 6 6 6	5 5 6 8 2	2 5 4 4 9
2 4 0 9 6	5 7 4 1 9	3 5 6 1 1	9 1 1 7 9	5 1 4 6 4	9 4 2 8 4	9 2 4 4 9	9 7 3 4 7	2 2 1 8 4
5 0 3 4 4	2 2 8 2 4	0 9 1 9 3	9 8 7 7 1	3 0 9 6 3	0 2 8 7 6	9 7 6 7 1	5 6 3 9 7	9 1 6 7 7
7 6 6 7 2	5 2 8 7 2	4 8 6 1 0	3 1 3 1 4	2 1 5 4 5	2 3 6 0 1	1 8 2 7 8	9 3 5 3 0	0 2 1 4 2
5 0 3 8 5	7 0 1 1 2	2 6 8 9 7	0 0 0 7 7	0 4 8 0 3	9 8 3 2 6	8 8 9 3 3	1 7 7 1 0	7 5 7 5 0
6 4 2 2 2	9 5 9 2 0	8 0 5 3 4	5 5 0 9 0	0 4 1 0 5	0 1 4 1 5	1 1 3 7 6	2 0 7 0 9	7 8 8 8 7
1 1 6 0 2	0 6 8 9 1	0 7 9 2 4	4 2 9 5 9	7 3 1 2 4	3 6 8 3 0	7 0 5 5 9	5 5 7 3 9	7 3 1 9 1
8 7 9 6 2	9 2 0 7 1	1 3 4 0 5	0 5 0 5 7	8 5 9 4 7	7 3 0 4 3	9 4 2 0 8	5 2 8 2 9	8 8 2 7 2
4 1 5 9 5	0 7 6 1 1	3 6 6 4 6	7 0 8 6 3	5 7 7 9 7	8 2 0 3 3	1 9 2 3 6	7 4 6 0 8	1 4 3 2 4
3 4 9 1 7	5 8 0 3 8	4 7 2 3 0	3 8 8 1 7	7 0 6 0 5	6 2 7 7 1	0 2 8 5 1	2 3 1 9 5	2 0 2 0 4
5 0 6 2 2	7 6 1 3 3	5 4 0 6 5	3 4 0 5 5	1 3 9 6 1	0 7 6 0 4	3 0 2 6 0	9 2 2 4 0	4 0 7 3 6
1 4 4 2 2	5 8 2 8 2	7 3 6 7 3	0 4 5 3 5	0 3 5 5 7	4 0 0 3 6	8 5 4 7 5	1 6 0 2 1	7 7 1 7 3
4 8 2 5 4	7 1 0 4 3	4 4 9 4 2	1 2 2 5 2	5 9 5 5 7	5 3 0 1 3	2 6 1 7 0	2 1 9 8 0	1 8 5 8 2
5 9 3 2 2	6 5 2 5 1	8 4 3 7 9	0 5 9 8 5	4 5 7 6 5	3 8 3 4 9	6 8 6 6 1	1 8 1 2 9	2 9 3 3 8
0 4 2 2 4	1 9 5 9 3	7 2 5 5 4	5 4 2 3 9	4 4 8 7 0	3 8 7 2 6	5 1 2 9 7	8 2 4 1 2	6 5 7 9 9
1 7 2 6 4	4 1 1 5 4	1 6 0 1 9	7 0 4 8 1	9 7 7 1 6	5 3 1 8 5	5 3 9 0 1	8 9 0 3 6	0 1 2 5 3
0 9 6 3 2	0 7 1 8 2	7 8 1 1 1	1 9 2 5 3	1 2 4 1 4	7 3 4 9 6	2 4 0 9 0	5 4 9 7 4	4 8 9 4 1
5 4 2 8 2	7 4 6 2 6	4 0 8 6 6	9 1 3 7 1	4 4 5 8 9	3 1 4 7 8	5 8 8 4 2	7 1 9 6 1	3 8 4 8 7
8 0 2 0 7	4 3 4 9 7	3 7 0 7 9	5 3 9 7 4	2 0 2 4 1	6 2 5 7 6	1 5 6 6 0	6 8 4 0 5	5 7 9 8 2
9 3 8 9 9	9 4 3 0 9	5 6 7 3 2	5 9 8 5 8	2 8 4 5 7	7 4 5 4 6	4 5 4 2 4	9 2 4 9 6	7 1 0 3 5
4 6 8 6 9	5 2 2 8 4	0 0 0 0 0	4 2 5 5 4	5 8 7 7 0	8 3 4 5 8	5 8 4 2 5	6 0 9 5 6	2 1 5 9 5