RECOGNISING ACHIEVEMENT

GCE

# Mathematics

Advanced GCE **A2 7890 – 2**

Advanced Subsidiary GCE **AS 3890 – 2**

# OCR Report to Centres

# June 2013

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS / A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching / training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This report on the examination provides information on the performance of candidates which it is hoped will be useful to teachers in their preparation of candidates for future examinations. It is intended to be constructive and informative and to promote better understanding of the specification content, of the operation of the scheme of assessment and of the application of assessment criteria.

Reports should be read in conjunction with the published question papers and mark schemes for the examination.

OCR will not enter into any discussion or correspondence in connection with this report.

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## OCR REPORT TO CENTRES

# Overview – Pure

Several of the following reports on the Core Mathematics and Further Pure Mathematics units draw attention to questions where candidates are asked to show or prove particular results. There is concern from examiners that the response by many candidates to such requests is not as good as it should be. The introduction to Section 5 of the Specification booklet includes the following:

> ‘In all examinations candidates are expected to construct and present clear mathematical arguments, consisting of logical deductions and precise statements …’
>
> ‘… candidates are expected to understand the nature of a mathematical proof.’

Sometimes the result to be demonstrated is a standard proof to be found in any textbook on the relevant topic. Other results to be demonstrated may be specific to a particular question. Often, the result to be shown is to be used in subsequent parts of a question and, with the result given in the question, those subsequent parts become accessible to candidates who were unable to prove the result.

The following table shows the number of marks out of 72 given to requests where particular results were to be shown or proved in each of the last two series.

| Unit | January 2013 | May/June 2013 |
|---|---|---|
| 4721 | 1 | 0 |
| 4722 | 20 | 14 |
| 4723 | 12 | 9 |
| 4724 | 10 | 9 |
| 4725 | 16 | 14 |
| 4726 | 20 | 20 |
| 4727 | 24 | 14 |

These figures show that proofs and ‘Show that …’ requests form a significant part of the assessment of most units and it is in candidates’ interests to approach such requests with the necessary awareness of what is required. The reports stress the need to construct these proofs with care and thought, and with full details shown. Each request to show a particular result should prompt a candidate to be as persuasive and convincing as possible, as if explaining the mathematics involved to someone who is not quite as mathematically able as he or she is.

# 4721 Core Mathematics 1

**General Comments**

The large majority of candidates were very well prepared for this paper. Totals achieved spanned the entire range of marks available, although there were fewer at the extremes than in some previous series. The time allocation was clearly sufficient as there was no evidence of candidates failing to finish. Indeed, omissions were relatively rare even in the later questions.

Candidates are now clearly comfortable with the Printed Answer Book format. The use of additional sheets was very rare this series and the unnecessary use of graph paper is now almost non-existent. Many of those who did need to repeat a solution indicated so clearly, which was very helpful. Nonetheless, a significant minority of the candidates who make repeated attempts at a question did not indicate which solution was the attempt they wished to be marked. Centres should advise candidates to make their intentions clear.

The best candidates presented clear and compact solutions that demonstrated full understanding of the mathematics needed to answer the questions. Many candidates had both the knowledge of and the ability to apply processes efficiently and effectively. In particular, differentiating powers of $x$ and solving quadratics by an appropriate method continue to be approached very well, although failing to recognise 0 as a solution to a two-term quadratic equation was common, even among those few who relied only on the quadratic formula. What separated the very best was the additional ability not to rely solely on a process but to interpret questions in slightly unfamiliar forms or recognise easier ways to a solution; Q6(ii) and Q9(iii) as described below are cases in point. In addition, better candidates were more likely to justify their solutions clearly; lack of this skill often caused problems in Q10(iii).

**Comments on Individual Questions**

**1)(i)** Most candidates were successful with this easy starter, but a significant minority found it quite challenging. Most earned at least a method mark for correct surd manipulation of some kind, but the accuracy was more of a problem, with some arithmetic errors and also conceptual ones such as $4\times 3\sqrt{5}=7\sqrt{5}$ .

**(ii)** Around 85% of candidates were successful in rationalising the denominator. Where no credit was earned, this was usually due to a lack of understanding rather than arithmetical error with a significant minority appearing not to know how to rationalise the given expression. Simply rewriting it as $20(\sqrt{5})^{-1}$ was quite common.

**(iii)** This was generally less successful than parts (i) and (ii), with just under three-quarters of candidates earning the mark. Many of those who did not give the answer in the required form did at least understand the notation as $\left(\sqrt{5}\right)^3$ was often seen, but then simplified to $3\sqrt{5}$ .

**2)** This disguised quadratic was approached well by the vast majority of candidates, with about two-thirds achieving all five marks. It was very rare to see candidates leap straight to the quadratic formula with no attempt to find the cube roots at the end, which has been a problem with similar questions in the past. Most candidates opted to perform a substitution and then to factorise, although those who opted for a two-bracket approach using $x^3$ were also often successful. Some candidates who used the quadratic formula failed to deal accurately with $a = 8$, but most earned the first three marks with apparent ease. Thereafter a small number of candidates opted to cube rather than find cube roots, but the main loss of credit was due to lack of accuracy at the end. The assertion "you can't cube root a negative number" was seen regularly; $\sqrt[3]{\frac{1}{8}}=\pm\frac{1}{2}$ was less common but not rare. This confusion between cube and square roots clearly needs addressing as it let down an otherwise well-answered question where only the very lowest-scoring candidates made no progress at all.

**3)(i)** This was very well done, with over 90% of candidates securing all three marks despite the added difficulty of negative powers of $x$. Even candidates whose overall total was very low recognised and performed the routine of differentiation efficiently. Where errors did occur, these were usually in converting the original expression.

**(ii)** Again, this was very well done, with almost all candidates recognising the notation and differentiating again, usually successfully.

**4)(i)** The fact that the first digit was given in this "completing the square" question appeared to ease the difficulty somewhat, but this is still an area which many candidates find difficult with less than two-thirds achieving full marks. Identifying the value of $p$ was usually very well done; the problems usually occurred in the calculation of $q$, with both arithmetic problems, particularly with the squaring, and structural misunderstanding when the candidates failed to multiply by 3.

**(ii)** This question provided a follow through from the previous part which enabled many candidates with poor arithmetic to earn credit for their understanding of the relationship between the format and the graph. Many secured both marks as a result. Those who re-started by differentiation were usually less successful, again due to the difficulties with the fraction work.

**(iii)** Most candidates are familiar with the term discriminant and only a few erroneously used $\sqrt{b^2 - 4ac}$ . Around one in ten candidates substituted correctly but then made arithmetical errors. Commonly seen was $9^2 = 49$ and the subtraction 81 – 120 often resulted in 39 or ±49 or ±41.

**5)(i)** Graph sketching continues to prove challenging for many candidates. In this case, both the shape and the choice of quadrants proved demanding with fewer than 60% of candidates securing both marks. It would help if candidates were to equip themselves with a ruler to draw axes as perhaps their intention with asymptotic graphs would then be clearer. With regard to the shape, too many candidates drew an "L" shaped diagram with large sections parallel to, rather than approaching, the axes; this lost marks.

**(ii)** Transformations of this type still prove difficult for candidates, with $y = \frac{2}{x^2 + 5}$ the most common incorrect answer. Again, fewer than 60% provided a fully correct equation.

**(iii)** Mathematical description of transformations also continues to prove a challenge. Although around four in five candidates were successful in choosing the word stretch (many of those who did not used "squash" or similar colloquialisms), only about half of these could correctly describe its scale factor and direction. Many used incorrect language with references to "in/along/by/on/across the $y$ axis", rather than "parallel to the $y$ axis".

**6)(i)** Over two-thirds of candidates secured all three marks interpreting the given equation of a circle correctly. Marks were lost mainly due to sign errors, both in attempting to find the centre and in attempts to complete the square.

**(ii)** Candidates who drew a diagram usually recognised that the coordinates of $B$ could be found by simple addition or subtraction and were then usually successful in scoring both marks, especially as there was a follow-through from part (i). Those who tried to apply standard techniques involving Pythagoras' theorem and resulting quadratics were very rarely successful in finding either value.

**7)(i)** The negative $x$ coefficient increased the difficulty of this linear inequality so that only two-thirds of candidates secured both marks.

**(ii)** Less than half of candidates provided fully correct solutions to this quadratic inequality. Some failed to expand and rearrange initially and thus earned no credit. Most were able to complete both first stages accurately, but on reaching $2x^2 - 10x \le 0$ many "cancelled" $x$ and thus could get no further. Where both roots were found, choosing the correct region still proved difficult, with some choosing the "outside" and other candidates writing $x \le 0, x \le 5$.

**8)** This familiar question was an opportunity for candidates to demonstrate proficiency in some basic techniques. Around two-fifths secured all seven marks and the vast majority at least three marks. The main errors were arithmetical, usually sign errors, when finding the mid-point and/or when simplifying the final equation of the line. Most candidates both found the gradient of the given line and the associated negative reciprocal accurately. Other errors that lost marks were lack of attention to detail in the question; some lines went through $A$ or $B$ and using the negative reciprocal of the gradient of line segment $AB$ was common. Others failed to notice the instruction to use integer coefficients in the final answer.

**9)(i)** This graph proved much more accessible than the one in Q5, with around four-fifths of candidates securing four or five marks. Almost all realised it was a positive quadratic and many were successful in factorising to obtain the two $x$-intercepts, although there were some sign errors. Most realised the $y$-intercept was (0, –6) but a significant number used this as the turning point rather than using the $x$-intercepts to locate the turning point. Some candidates who were successful in part (ii) had the wisdom to return and correct an error in this part.

**(ii)** Many candidates did not understand the term "decreasing function" and instead gave the region between their roots found in part (i); indeed nearly half of candidates failed to score at all in this part. Those that approached by differentiation were usually more successful in finding the minimum point than those who completed the square.

**(iii)** Around three quarters of candidates secured the first three marks of this question, equating to 4, simplifying and solving to find the $x$ values of $P$ and $Q$. Again, factorisation was both the most appropriate and most frequent approach. Rather than then subtracting these values, many saw the word "distance", used the "distance formula" and re-substituted to find $y$ (not always getting 4) and then used Pythagoras' theorem. This led to a lot of unnecessary arithmetical difficulty.

**10)(i)** More than half of candidates secured all seven marks available for this question and many clear, compact solutions were seen. Others also scored highly producing solutions marred only by arithmetical error. The conceptual problems that arose came from difficulties in differentiating $kx$ or differentiating $k$ to give 1; most knew to set their derivative to zero and substitute $x = -3$. Only a few candidates substituted into either the original expression or its expanded form without any attempt at differentiation.

**(ii)** Most candidates found the second derivative and considered the sign at $x = -3$; only a few equated to zero in error. As this result was independent of $k$, this was by far the easiest route to success; candidates considering signs or using other methods rarely made any progress.

**(iii)** This proved, appropriately, the most challenging question on the paper with only about half of candidates making any progress. Even high scoring candidates rarely produced a complete solution as it was necessary to identify a point where both the gradient of the line was equal to the gradient of the curve and such that the point was on both the line and the curve. Most commonly, candidates equated their derivate to 9 and tried to solve the resulting quadratic. Often the solution $x = 0$ was ignored or omitted; others substituted their solution(s) into the line only, sometimes offering both solutions. Many tried to equate the line and curve but were unable to solve the resulting cubic equation. A few who equated using the original form of the equation noticed that $x - 1$ was a common factor and the resulting quadratic had a repeated root, thus implying tangency but this approach was rarely rigorously explained.

# 4722 Core Mathematics 2

**General Comments**

Candidates seemed to find this paper accessible, and the majority made an attempt at most, if not all, of the questions. There were very few response boxes that were left blank. As always, there were some questions (particularly Q8(ii)) where a number of candidates made multiple attempts at the question. Unless candidates indicate which attempt they wish to be marked, by crossing through all other attempts, it is the final attempt that will be marked even if that does not yield as much credit as other attempts.

Candidates should always ensure that they show enough working to justify their answers, being especially careful when trying to show a given result or provide a proof. Full credit will only be given where each step has been shown explicitly. If candidates decide to amend their working they must ensure that this is done consistently throughout the solution; in some cases it may be easier to write out the final solution afresh in its entirety. Examiners can only award method marks if there is detail of the actual method used; quoting a generic formula such as the cosine rule and then producing a numerical answer is not good enough. Examiners will want to know which values have been used in the formula and cannot be expected to deduce this from an incorrect answer only.

Candidates should ensure that they always read questions carefully and ensure that they are answering the question posed. This could include whether an area or a perimeter of a region is required, and whether the term or the sum of a sequence has been asked for. Candidates should also check whether exact or approximate answers are required, and what the required degree of accuracy is for the latter.

**Comments on Individual Questions**

**1)** This was a straightforward start to the paper and most candidates gained full marks on the question. Some candidates lost the final mark by failing to work to the required degree of accuracy throughout. Candidates are strongly advised to use exact values in their calculations rather than truncated decimals. Candidates also lost marks through using two strips of width 3, rather than the required three strips of width 2, but they were still able to gain some credit. There were very few candidates who first attempted integration, which is an improvement on previous series.

**2)(i)** Most candidates were able to correctly find the first angle though a few halved rather than doubled the result of $\sin^{-1} 0.8$. Finding the second angle proved more challenging with the most common error being to simply subtract their first answer from 180°. The fact that this resulted in a second angle that was smaller than the first did not seem to deter them. Whilst some candidates were able to use the symmetry of the $\sin\frac{1}{2}x$ graph to find the second angle, the more successful method was to find the possible solutions for $\frac{1}{2}x$ from the $\sin x$ graph and then double all the solutions.

**(ii)** This part of the question was better attempted, and most candidates scored full marks with ease. A few struggled to find the second angle, or lost the final mark through a lack of precision when rounding. Some candidates made life difficult for themselves by attempting to square both sides and use $\sin^2 x + \cos^2 x \equiv 1$, but this was very rarely done correctly. Potential pitfalls included forgetting to square the coefficient of 3, omitting to use both the positive and negative square roots and finally realising that only two of the four solutions were valid. At least this was a valid method, which could not be said for those candidates who started with $\sin x + \cos x = 1$.

**3)(i)** Candidates are becoming ever more adept at the binomial expansion, and the majority scored full marks on this question. As always, the most successful candidates made effective use of brackets. The most common error was to omit to square 5*x* in its entirety. Candidates usually used the correct binomial coefficients, but this was not always shown explicitly which made it difficult to award credit. A few candidates spoiled an otherwise correct answer by dividing through by a common factor.

**(ii)** This part of the question proved to be more challenging. The higher-scoring candidates were able to identify the required products and find the value of $c$ with ease. Other candidates could identify the two required products but then spoilt their answer with $8640 + 384c$ becoming $9024c$. Some gained one mark for identifying one of the two required products. However, a number of candidates struggled to get started, with the most common error being to find the sum or the product of $6x$ and $960x$.

**4)(a)** The vast majority of the candidates successfully integrated the given function to obtain at least two of the algebraic terms, although the constant term sometimes disappeared. A few candidates lost a mark by leaving d$x$ or the integral sign in their final answer, or by omitting $+ c$.

**(b)(i)** This question was also very well done, with the majority of candidates obtaining the correct, simplified, integral. A surprising minority of candidates lost a mark in this part of the question for omitting the $+ c$, even though it had been given in the previous part.

**(b)(ii)** Candidates tended to get either full marks or no marks on this question. The higher-scoring candidates were able to deal successfully with the upper limit of infinity, either by appreciating that the value of the integral would be zero or by assigning a letter to the upper limit and deducing the value of the integral as this letter tended towards infinity. A few candidates had the correct idea but struggled to express themselves mathematically and at times it was infinity that became zero rather than F(infinity). However, there were also a significant number of candidates who had no idea how to deal with the upper limit of infinity and simply ignored it, leading to a rather inconvenient negative sign in their square root.

**5)(i)** This question was very well answered, with the majority scoring full marks. Candidates quoted the relevant formulae accurately, and then work to an acceptable degree of accuracy. As always, a few candidates felt the need to work in degrees rather than radians; not only is this more long-winded but using rounded values can affect the accuracy of the final answer. A few candidates decided that the region specified was itself a sector of a different circle and pursued this method, seeming not fazed when their two radii then appeared to be of different lengths.

**(ii)** This was equally well done, with many concise and elegant solutions seen. The arc length was invariably correct, and candidates recognised the need to use the cosine rule though some struggled to evaluate this correctly, either by inserting imaginary brackets or by using the incorrect calculator mode. A surprising minority thought that $16 - 7 = 11$.

**6)(i)** The vast majority of candidates were able to gain full marks on this question. A few gained just one mark by finding the 30th term rather than the required sum of the first 30 terms.

**(ii)** Most candidates were able to gain some credit on this question, but only a few scored full marks. The sum of $N$ terms was usually quoted correctly and candidates could then make an attempt to rearrange it. A common error was the failure to reverse the direction of the inequality sign when multiplying or dividing by a negative number. Others started with an equality and then tried to justify the inequality sign at the end, which was not sufficient to gain the accuracy mark. Some candidates were unable to manipulate the indices, with $6 \times 1.3^N = 7.8^N$ being a fairly common error. When solving the given inequality, most candidates could use logarithms correctly to get a decimal answer, but did not then appreciate that the context of the question meant that $N$ had to be an integer value. Some candidates simply solved the given inequality and made no attempt to show where it had come from. Others solved the inequality and then tested their solution in the sum formula to justify it, without appreciating that they had not fully answered the question.

**7)(i)** This was very well answered with most candidates gaining full marks. As the answer was given candidates were expected to show sufficient detail in their method, including the use of limits. Some candidates were clearly aided by the answer being given and were able to go back and amend incorrect working, though they must ensure that this is done consistently throughout the entire solution.

**(ii)** This proved to be one of the most challenging questions on the paper. Whilst 25% of the candidates were able to provide accurate and concise solutions, 50% were unable to score any credit at all. Most candidates did seem to recognise that they needed to find the equation of the tangent, but many did not realise that differentiation was required to find the gradient. Of those who did find the correct equation, a number then just integrated this equation between 1 and 4, rather than appreciating that a different lower limit was required and that this would be given by the point of intersection of the tangent with the $x$-axis. Some of the more successful candidates drew a sketch graph and gave consideration to the area that was to be subtracted from the answer to part (i), but too many launched straight into calculations with seemingly no clear strategy.

**8)(i)** Most candidates were able to give the coordinates of the two required points of intersection. Many of the unsuccessful candidates had the correct idea but just gave the $y$-value rather than the required coordinates. The final part was not so well done. Most candidates were able to give an appropriate value for $a$, but many were less successful on $b$, with a negative value being the most common incorrect answer.

**(ii)** As in previous series, candidates seem to have a basic understanding of the rules of logarithms but struggle to apply them consistently and accurately throughout a convincing proof. Most candidates gained a mark for equating and introducing logarithms, and a second mark for using the power rule correctly at least once. The most common error by far was for $\log 4b^x$ to become $x\log 4b$, which meant that no further progress could be made. Some of the more successful solutions eliminated $b$ as the first step and simplified the resulting equation before introducing logarithms. Candidates must appreciate that a proof needs to be convincing throughout, which here included consistent use of bases, brackets being used correctly and sufficient detail being provided for each step. It was also noticeable that a number of candidates made multiple attempts at this question; they must recognise that it is the last attempt that will be marked unless they indicate otherwise by deleting unwanted attempts.

**9)(i)** This question was very well answered with the majority of candidates gaining full marks. The most common and most successful method was to use the remainder theorem. Long division was less common, and usually less successful.

**(ii)** This part of the question was also well answered. Many candidates used the factor theorem to demonstrate that $(2x + 1)$ was a factor, which was the expected approach. Some embarked straightaway on the long division and then used a remainder of 0 to justify that it was a factor and others just ignored this request completely. Most candidates could make an attempt to factorise $f(x)$. The most popular method was division, but the zero coefficient of $x^2$ caused problems for some. Synthetic division seems to be becoming more popular, but having to use $(x + 0.5)$ caused problems for some, especially with there being a mismatch when pairing it back up with the quadratic quotient. Coefficient matching and inspection tended to be more successful, if less common. Having found the quadratic quotient correctly, some made no attempt to factorise their quotient and others failed to write $f(x)$ in fully factorised form as requested.

**(iii)** Some candidates failed to spot the link between this part of the question and the previous one and a number started afresh in an attempt to solve the cubic, in some cases using the quadratic formula. However, most candidates did identify what was required and were able to gain some credit from finding at least one solution from $(2\cos\theta + 1)$, even if their other quadratic factors were incorrect. A number of candidates failed to gain full marks through leaving their solutions in degrees, through introducing additional incorrect solutions such as 0 and $2\pi$ or through having $\frac{5}{3}\pi$ rather than $\frac{4}{3}\pi$ as one of their solutions. Nevertheless, a pleasing number of fully correct solutions were seen.

# 4723 Core Mathematics 3

**General Comments**

This paper contained some routine requests, such as questions 1, 2, 4 and 5, that enabled most candidates to make some progress. Fewer than 1% of the candidates recorded a total mark for the paper of less than 10 out of 72 and fewer than 3% recorded a total mark of less than 20. There were also some challenging aspects that required some thought and care and it was pleasing to note that there were plenty of candidates up to the challenges presented by questions 6(ii), 6(iii), 8(ii)(b) and 9(ii). Full marks were recorded by several very capable candidates and 10% of the candidates recorded a mark of 65 or more out of 72. A lack of time did not seem to be a problem; indeed, some candidates embarked on unnecessarily long and involved solutions to parts of questions 8 and 9.
An aspect of examination technique that needs attention concerns effective communication. Questions 6(ii) and (iii) required candidates to explain results. Long-winded, wordy attempts usually missed the crucial points whereas thoughtfully constructed mathematical arguments were much more convincing. In question 8(ii)(b), candidates were required to find greatest and least values of an expression; good responses indicated at each stage what was being found whereas there were many instances of solutions where examiners were given no guidance at all as to what was going on.

**Comments on Individual Questions**

**1)** The two parts of this question were generally answered well although, in part (ii), the incorrect answer $\ln(4-3x)$ occurred quite often. The arbitrary constant was often missing from both parts.

**2)(i)** Most candidates were able to use a correct identity for $\cos 2\alpha$ and to reach an equation such as $9\sin^2\alpha = 4$. Many candidates did not conclude successfully. Some gave only the one answer $\sin\alpha = \frac{2}{3}$ and others offered $\sin\alpha = \sqrt{\frac{4}{9}}$ or $\sin\alpha = \pm\sqrt{\frac{4}{9}}$. Going further to find an angle or angles was not penalised in either part of this question.

**(ii)** Some candidates showed uncertainty at the outset but most were able to reach and solve the correct equation involving $\sec\beta$. Many candidates were then content to give the two answers $-\frac{1}{2}$ and 5. No justification for rejecting the former value was required but candidates were expected to make a clear and definite decision as to the value of $\sec\beta$. Some candidates did do a little work considering the possibility of $\cos\beta = -2$ but, often, the impossibility of solving this was not transferred into a final conclusion about the value of $\sec\beta$.

**3)(i)** This simple request proved troublesome for many candidates. The notation $\tan^{-1}(\frac{1}{2})$ seemed a problem for some and expressions involving $\tan(\tan^{-1}(\frac{1}{2}))$ were often noted. Some candidates tried to use particular lengths for the height and radius. Other candidates did not recognise the need for some elementary trigonometry and thought this part involved rates of change. Only 47% of candidates earned the two available marks.

**(ii)** Most candidates did differentiate to obtain $\frac{1}{4}\pi x^2$ but often the answer was then obtained by substituting $x = 8$ into this, thereby ignoring the rate at which liquid is being added to the container. Candidates with clear, accurate notation for the derivatives usually proceeded to reach the correct final answer but there were many instances where the calculation was $16\pi \div 14$ or $16\pi \times 14$ instead of the correct $14 \div (16\pi)$. There was some evidence of poor use of calculators when the answer 2.75 appeared – the result of calculating $14 \div 16 \times \pi$.

**4)** This question was answered very well and 63% of candidates recorded all 6 marks. The first term was usually differentiated correctly but there were a few more problems with the second term. Careless simplification often led to an expression $\frac{1}{(2x+1)^2}$ for those candidates using the quotient rule. Some candidates rewrote the expression as $4x(2x+1)^{-1}$; a few did not use the product rule and, for some others, there were errors as the chain rule was not used. The vast majority of candidates recognised the need to give an exact answer and there were few instances where candidates resorted to decimal approximations.

**5)(i)** The vast majority of candidates recognised that a translation and stretch were the transformations involved. Generally the details of the two transformations were correct if sometimes the use of language was not as precise as it might have been. As in previous series, use of terms such as 'move' and 'shift' instead of 'translate' meant that the accuracy mark was not earned. A minority of candidates used a stretch with scale factor $\frac{1}{2}$ parallel to the $x$-axis; this needs care with the translation and with the order of the two transformations and this care was often absent in such cases. Occasionally a third transformation was included, reflection in the $x$-axis, obviously the result of candidates recalling some aspect connected with the graphs of modulus functions.

**(ii)** There were some algebraic slips but, generally, candidates were able to find the two critical values of –6 and –2. Squaring both sides of the given inequality was the more usual method. There was more difficulty in deciding the correct set of values of $x$. Some candidates were able to write down the answer immediately but others had no method for determining the answer and conclusions often seemed somewhat haphazard. Candidates concluding with $-6 \le x \le -2$ did not earn the final mark and nor did candidates offering $x < -2, \quad x > -6$.

**6)(i)** There were few difficulties in finding the value of $A$ and 78% of the candidates duly recorded all four marks. Some candidates formed the correct expression but then went astray with the actual calculation. Others formed expressions in which the necessary brackets were not always present; if the correct answer followed, the marks were awarded but, often in such cases, the calculated answer was not 22.4.

**(ii)** A convincing answer to this part required some thought and planning; many candidates found it difficult to present a clear justification. Many did observe that $(3+x^2)^2 = 9+6x^2+x^4$ but there were also many inaccurate statements such as $\ln(9+6x^2+x^4)$ is the square of $\ln(3+x^2)$. Other candidates were content to offer generalisations such as $\ln x^n = n\ln x$.

**(iii)** This part proved challenging and many candidates could do no more than observe that 'ln and e' cancel each other or that 3e is approximately equal to 8. As in part (ii), candidates were prone to embark on lengthy and wordy explanations when a concise and considered mathematical approach would have been more convincing.

**7)(i)** Only 40% of the candidates earned this mark. A quite common response was $y \ge 3$, which did not earn the mark, and other attempts involved 0 or 7.

**(ii)** Many candidates found the correct expression for the inverse function but there was much uncertainty about the domain and range. Some candidates ignored these two requests and many others found it difficult to determine the range or to express it clearly; a statement saying that the range is all real numbers was perfectly adequate.

**(iii)** As in previous series, the iteration was usually carried out efficiently and accurately. However many candidates were guilty of not reading the question carefully and concluded with only $x = 3.168$, thereby losing the final mark.

**(iv)** Most candidates did earn the mark but, in many cases, more information was provided than was necessary. There were many references to reflection or to the curves being symmetrical about the line $y = x$ but, provided that *P* was recognised as the point of intersection of the two curves, the mark was awarded.

**8)(i)** This routine piece of work was answered well by most candidates with 73% of them earning the three marks. The fact that the expansion of $R\cos(\theta + \alpha)$ leads to a minus sign between the two terms confused some candidates and there were sign errors; some candidates concluded with $\sqrt{20}\cos(\theta - 26.565°)$. A value of 4.47 for *R* was accepted here but candidates are always advised to choose exact values or values to more than 3 significant figures when further work is dependent on the values.

**(ii)(a)** Many candidates had no difficulty in finding the two angles although some earlier lack of accuracy occasionally meant that the two answers were not the correct angles of $21.3°$ or $286°$. Some candidates found the first angle correctly but then wrongly subtracted that answer from $360°$ to claim a second angle. A few candidates provided four answers, one in each of the four quadrants.

**(ii)(b)** This proved to be a challenging request and many candidates made little or no significant progress. Some started by expanding $25 - (4\cos\theta - 2\sin\theta)^2$, a step that led into some involved trigonometry but no progress with the particular request. Two quite popular greatest and least values were 21 and 9, obtained by substituting, respectively, $\theta = 90°$ and $\theta = 0°$. Candidates realising that the result from part (i) needed to be used were able to make more progress although some claimed a greatest value of 45; others believing that the required values would be obtained by taking $\cos(\theta + \alpha)$ to be –1 and then +1 ended up with greatest and least values both being 5. Finding the smallest positive value of $\theta$ associated with the two values also proved difficult; in particular the fact that the angle associated with the least value of 5 comes from $\cos(\theta + \alpha) = -1$ eluded many.

**9)(i)** The majority of candidates had no difficulty in realising that equating the first derivative to zero was needed. With the answer $\ln 3$ given in the question, solutions were expected to be sufficiently detailed. Many candidates failed to earn the final mark because their solutions went immediately from $x = \frac{1}{2}\ln 9$ or $x = \frac{\ln 9}{2}$ to $x = \ln 3$.

**(ii)** Most candidates were able to make some progress with this question but only 14% of candidates succeeded in recording full marks. The integration was usually carried out accurately but a few used incorrect limits such as $\ln 3$ and 16. It was quite common for the limit 0 to be ignored when evaluating the area under the curve. A considerable problem for many involved the term $9(\ln 3)^2$. This was often carelessly written as $9\ln 3^2$ and then 'simplified' to $9\ln 9$ or $18\ln 3$. Perhaps it was this awkward term that prompted many to resort to decimal approximations.

A variety of approaches for finding the shaded area were seen. One that was wrong was to treat the equation of *PQ* as the tangent to the curve at *P*. The most successful approach was to find the area of the trapezium between *PQ* and the *x*-axis. A similar approach involved the areas of triangle and rectangle but there was more scope for sign errors for those adopting this. An alternative approach, successful occasionally, involved finding the equation of *PQ* and integrating this; again, for some, an awkward-looking gradient for the line was the motive for moving to decimal approximations.

Candidates who earned all eight marks usually provided clear and succinct solutions, where the steps were briefly described and where appropriate simplifications were carried out as the solution progressed.

# 4724 Core Mathematics 4

**General Comments**

While there were some very well presented scripts, there were also examples where the work was much more difficult to follow. In some instances it would seem likely that poor presentation had an impact on the quality and accuracy of solutions submitted. Candidates should be encouraged to prepare a plan of action in their heads before presenting their solution. Care should also be taken to ensure that the correct question is answered in the correct space in the Printed Answer Book.

**Comments on Individual Questions**

**1)** This was generally answered well though a few candidates did not show the format of the partial fractions and expected the examiner to know the meaning of the *A*, *B* and *C* in their solution.

**2)** This was a relatively straightforward question but two specific errors occurred. The first could have been forecast: the differentiation of $\ln(3x)$ as $\frac{1}{3x}$; the other, perhaps not so predictable, involved the integration (at the second stage) of $\frac{x^9}{9}.\frac{1}{x}$. Here examiners were looking, first of all, to see if candidates were integrating an expression of the form $kx^8$. Even the correct simplification at that stage was often followed by the incorrect result of $\frac{1}{72}x^9$. However, it can be said that the technique of 'integration by parts' was generally known.

**3)** Most candidates were able to form the appropriate equations but too many omitted the first negative sign in the second equation; this was considered to be due to carelessness rather than to misreading. The solution of these equations, with the associated inconsistency, was often not performed systematically but the vast majority, working carefully, fulfilled their objectives. However, one aspect was not clearly shown; examiners were frequently informed that the direction vectors were not equal or were not multiples – but these direction vectors were often not defined.

**4)** This relatively simple-looking question did test a number of useful features: the differentiation of $\cos 2x$, the solution of $\sin 2x = \cos x$ and, finally, the solution of $\sin x = \frac{1}{2}$ for $0 < x < \pi$. The majority of candidates passed the first test but failed the second and third. Most divided each side of the equation by $\cos x$ without considering the possibility of $\cos x$ being 0 and a similar number forgot that $\frac{5}{6}\pi$ was also a solution of the equation $\sin x = \frac{1}{2}$.

Although the $\sin 2x$ formula was generally used correctly at the end of the first main stage, it was surprising how many decided to use the double-angle formula for $\cos 2x$ at the beginning; unfortunately the derivative of $\cos^2 x$ often proved more problematic than that of $\cos 2x$.

**5)(i)** This was shown successfully by most although, as the answer was given, special notice was taken of each stage and for some $(1-\tan x)(1+\tan x) = 1+\tan^2 x$ was sometimes in evidence. This earned the method mark (for knowing the approach to be taken) but not the accuracy mark.

**(ii)** The obvious errors were made here and the correct multiples of $\ln(\sec 2x)$ or $\ln(\cos 2x)$ were frequently missing. The logarithmic work was usually well done.

**6)** The process of integration by substitution was well known and most candidates managed to get to the first stage of needing to integrate $\frac{u-1}{u^2}$. Although most had an idea of what to do, the integration of $-\frac{1}{u^2}$ proved harder than expected. The place where the majority fell down was at the end, when they forgot to re-substitute; perhaps candidates were more used to substitution being used with a definite, rather than indefinite, integral.

**7)(i)** In general, the basic work was done well but it was obvious that candidates were more used to finding the angle between two lines (and giving the acute version) than finding a specific angle in a geometrical diagram. In the context of this question there was only one angle, and it happened to be obtuse and it was the obtuse angle only which was accepted. Some of those obtaining it may have been lucky in the directions of the vectors used, though others would have realised the situation. Those using the cosine rule did not get involved with the dual possibility.

**(ii)** This tended to be universally well done, the only exception lying with those candidates who just said that the scalar products were zero, rather than demonstrating it numerically.

**(iii)** Those who had noticed carefully the information obtained from parts (i) and (ii) generally did well in this part. Both the obtuse and the corresponding acute angle were accepted but it was curious to note that either the $\frac{1}{2}$ or the $\sin\theta$ were frequently missing from the area of the base triangle.

**8) (i)** The section in the specification topic "First Order Differential Equations" requires candidates to formulate a simple statement involving a rate of change as a differential equation. This process is by no means understood by the majority of candidates – and particularly not in this case where inverse proportion was involved. Many of the candidates thought that the constant should be inverse, ie $\frac{1}{k}$, which would have worked through satisfactorily had they not thought that the inverse aspect was now fulfilled and they could just write $\frac{1}{k}\sqrt{r}$.

The separation of variables and subsequent integration were performed reasonably. At the end, many candidates failed to express *r* in terms of *t*, as directed.

**(ii)** Most understood what to do, gaining the method mark, and a few obtained the correct result.

**9)(i)** In general, apart from the derivative of $\frac{1}{t}$ being $\ln t$ in some cases, the differentiation was handled competently. The question asked for the answer to be simplified and many alternatives were accepted – though not fractions with negative powers involved in numerator and denominator.

**(ii)** The stationary point was relatively easy to find; having found *t*, the question directed candidates to find *x* and *y*. It was hoped that this would focus attention on the value of *x*, as is normal in the classifying of stationary points. However, some considered points on either side of the critical value of *t*, not realising that this would not indicate which side of the stationary point they were considering.

**(iii)** Apart from careless errors, this part of the question was well done.

**10)(i)** The required relationship had been given so, as with all such similar questions, the solutions were examined closely to see, firstly, if the method of expansion was known and, secondly, how accurately it was carried out. Any slight error in accuracy was penalised.

**(ii)** It was not immediately obvious just what the suitable value of $x$ was, but a fair number obtained $x = 0.1$ and substituted into the given expansion.

**(iii)** The required given identity was not difficult to prove, but most made heavy weather of it; simple stages, showing how $\frac{1}{x}$ and the negative sign were produced, were needed. Whether the identity had or had not been proved, it could then be used to produce the required expansion.

**(iv)** A simple explanation was required indicating why the value of $x$ used in part (ii) would be unsuitable if used with the expansion in part (iii). Accepted reasons included the fact that $-63100$ would be the result or that the substitution of a positive/small value of $x$ would give a negative/large value, which could not be an approximation to $\frac{100}{729}$.

# 4725 Further Pure Mathematics 1

**General Comments**

All questions on the paper proved accessible to candidates and completely correct solutions to all questions were seen. Most candidates found the space in the printed answer booklet sufficient for their solutions and very few candidates wrote their answers in the wrong space.
Most candidates were able to produce correct solutions to several questions, thus demonstrating a sound knowledge of a good range of syllabus topics.

Many candidates lost marks through simple arithmetic and algebraic errors when the required method was clearly understood. There was no evidence of candidates being under time pressure, so more careful checking may have enabled candidates to discover these basic errors.

**Comments on Individual Questions**

**1)** Most candidates found the value of *a* correctly, the most common errors were using $\tan^{-1}(\frac{1}{6}\pi)$ or $\frac{3}{a}=\tan(\frac{1}{6}\pi)$ . Candidates usually earned some or all of the marks for the remaining requests, with the notation for the modulus and the complex conjugate understood by the majority of candidates. The most frequent error was giving the conjugate as $-3-\sqrt{3}\text{i}$ .

**2)(i)** A row matrix was found by the majority of candidates, with only simple arithmetic errors occurring in a few cases. A few gave the answer as a column matrix, while others obtained a $1\times1$ matrix such as (30).

**(ii)** A significant minority found **BC** or thought that **CB** was a 1 × 1 matrix. Of those who found a $2\times2$ matrix, only a few made odd arithmetic slips and most candidates found their determinant correctly and deduced singularity or non-singularity correctly. The most common error was to state non-singularity having found the determinant to be zero.

**3)** Most candidates showed knowledge of the method required to find the square roots of a complex number and made good progress with the question. The main errors were either to give only one root, or more than two roots, eg $\pm(2\sqrt{5}\pm3\text{i})$ . A few found the correct values of *x* and *y*, but did not write their answers as complex numbers.

**4)** This was one of the less well-attempted questions. A significant minority thought that the induction step required the addition of $\mathbf{M}^k$ and **M**. Many failed to show sufficient working either in establishing the validity when $n = 1$, or in obtaining the elements in $\mathbf{M}^{k+1}$. Centres should remind candidates of this, as well as the need for a clear statement of the induction conclusion.

**5)** The three standard series were given correctly by most candidates. A fair proportion then expanded each term before attempting to factorise and any sign error meant that their expression would not factorise easily. Even those who used a common factor of $n(n + 1)$ still made sign or arithmetic slips, again often obtaining an expression that could not be factorised.

**6)(i)** This part proved quite testing, with many candidates not realising the form of answer that was required. Some gave cartesian forms for *C* and *l*.

**(ii)** Candidates who made some progress with part (i) usually scored some marks in part (ii). The most frequent error was the omission of the third inequality $\arg(z-3\text{i})\le\frac{1}{2}\pi$ .

**7)(i)** The majority of candidates obtained the correct matrix. Those with a wrong matrix usually failed to show any working; a sketch of the unit square under the required transformation might have helped.

**(ii)** This part was done slightly better than part (i), but the above comment also applies to this part.

**(iii)** The most common error here was to multiply the matrices in the wrong order.

**(iv)** Most candidates were able to describe correctly the transformation represented by their answer to part (iii).

**8)** In using the symmetric functions for the given cubic, many candidates made sign errors. Those candidates who used the correct substitution usually answered the question totally correctly. However the incorrect substitution, $x = u + 1$, was seen on quite a few occasions. Some candidates used a mixture of the two methods, but usually managed to score some of the marks.

**9)(i)** This part was answered correctly by the majority of candidates. Marks were lost by those who did not show sufficient correct working to justify the numerator in the answer given in the question; $3r + 2 - 3r - 1 = 3$ was frequently seen.

**(ii)** Some candidates tried to use the standard series results in the denominators. The majority used the method of differences, but many stopped at the $n$th term rather than the $2n$th term, and a good proportion did not deal with the extra factor of 3, even though the answer is given in the question.

**10)(i)** The determinant was found and equated to zero by most candidates, with only minor arithmetic slips being seen.

**(ii)** Although most candidates knew the various stages that are required when finding the inverse of a $3 \times 3$ matrix, often each stage of the working was labelled as $\mathbf{A}^{-1}$. This was not penalised, but centres should try to get candidates to annotate their work in a more precise way. Having found an inverse matrix, a number of candidates then did not attempt to use the inverse to solve the equations and others incorrectly post-multiplied by the inverse. Many made simple algebraic or arithmetic slips at this stage, or omitted the determinant completely.

# 4726 Further Pure Mathematics 2

**General Comments**

This paper was less straightforward than the January 2013 paper and provided a challenge in some areas to most candidates. The mean mark was lower than that in January, although still a little higher than has been usual over the past few years.

Comments have been made before on the problem of the "show that..." questions. Such a style is often used when the answer is required for a following part of the question. Candidates who are unable to complete one part can, using the answer given, start the next part from the right place. It does pose problems, however, for candidates who need to demonstrate conclusively with correct algebra that they have worked through to the right answer with no errors. There were particular places in this paper where there were difficulties which are discussed below.

**Comments on Individual Questions**

**1)** There were many good solutions to this first question and a variety of methods for obtaining the correct answer. Many candidates did not include the '$\mathrm{d}\theta$' in their integral and, as a result, failed to complete the full substitution. Some failed to convert the limits and so gave an incorrect answer. Others converted back unnecessarily.

**2)(i)** The vast majority of candidates scored full marks on this part, either from squaring the exponential forms of the hyperbolic functions or by using the difference of squares.

**(ii)** Only a very few ignored the first part and the majority generally found the solution of the quadratic equation and obtained the result for $x$.

**3)(i)** A significant proportion failed to use the chain rule in the original differentiation and consequently made little progress. Many candidates who made this error did not understand that what they were doing was not going to produce the result required and was involving them in very complicated algebra. Other common errors included not writing down the derivative of $\tanh^{-1} x$ correctly, even though it is in the formula book.

In order to obtain the correct result for $\mathrm{f}''(x)$ it was necessary to find $\mathrm{f}'(x)$. There were some who followed an incorrect route but still came up with the correct $\mathrm{f}'(x)$, presumably by working backwards from the given $\mathrm{f}''(x)$. This, of course, was not creditworthy.

An alternative method was to rewrite f($x$) in terms of natural logarithms. For those who could cope with the algebra this often produced a shorter and neater solution.

**(ii)** Those candidates who wrote down the general form of Maclaurin's expansion first, rarely omitted the 2 from the denominator of the third term. Several candidates who were unsuccessful in proving the results in part (i) worked backwards from the given result and were able to pick up the marks in this part.

**4)** There were many good complete solutions seen. Most used the expected split of $\cos^{n-1} x \cos x$ as the starting point for their attempt at integration by parts. It was common for sign errors to occur at the first stage. Some tried an approach involving $\cos^{n-2} x \cos^2 x$ but often failed to get very far, though some reached the correct conclusion. Another common error involved starting with a $\cos^n x \times 1$ split; again such attempts quickly came to a halt.

The problem with the early sign error meant that a second "error" had to be made to put it right, either inadvertently or deliberately. This of course is of no benefit as the original error will have been penalised. It appeared that many candidates, however, on realising that the result was not going to come out, checked back over their work and found their error.

**(ii)** A significant majority of candidates obtained a correct answer for this part, even though the answer to part (i) was not obtained. Common errors were to give $I_1 = -1$ or to have a careless slip in one of the fractions.

**5)(i)** This question was completed well although some candidates forgot to include the suffixes in their final statement.
This was another "show that..." problem. A number of candidates jumped to the given answer too soon to demonstrate that the answer was obtained through correct working; some candidates lost marks needlessly as a result.

**(ii)** A significant number of candidates only gave $x_2$ and $x_3$. A handful wrote down values as far as $x_6$ or even further before concluding with the value of $\alpha$.

**(iii)** There were many poor answers in this part with incorrect diagrams and insufficient explanations. Some drew chords, demonstrating a misunderstanding of how the Newton-Raphson method works. Those who drew a tangent at $x = 0$ did not always then show the vertical to the curve. In fact several seemed to think that the vertical would not ever reach the curve, interpreting the diagram as the finite limit of the curve. Some correctly referred to the first tangent being the wrong side of the turning point.

**6)(i)** Many seemed to be only expanding the given summation. Examiners needed to be convinced that what was being written down was not just the given answer. At the very least, candidates were expected to describe the areas as being the width multiplied by the height for each rectangle. A diagram usually helped. Many candidates who had problems here then gave up on the rest of the question.

**(ii)** This part was nearly always correct. An error for some was to leave the lower limit $r = 0$ alone, thus creating an extra rectangle.

**(iii)** There were many good attempts but plenty of opportunity for slips. The most common misunderstanding was that $U - L$ gave a single term, usually ln(ln6). Common slips included ln(ln…) becoming just ln… or totally disappearing, the difference being written the wrong way round, the right-hand side appearing as 0.01 or the final answer as being given as $n > 1468$.

**7)(i)** The concepts of this question were well understood by most candidates. The horizontal asymptote was occasionally given as $y = 0$ or not at all. A few candidates gave both $y = 0$ and $y = 1$.

**(ii)** Most candidates used the quotient rule accurately. Those who went down the partial fractions route were more likely to slip up with the algebra.
Those who made algebraic mistakes usually ended up with complicated roots. Students should be encouraged to check their previous work for accuracy in such circumstances as one small error can lead to the loss of several marks.

**(iii)** If part (i) was correct then this was mostly answered accurately. Candidates must be careful to read instructions carefully, though. Too many candidates lost a mark for not expressing their answer clearly as coordinates.

**(iv)** There seemed to be too much reliance on the use of graphical calculators here. Even when candidates had part (i) completely correct they often did not give a clear enough sketch to convince the examiner that they understood the nature of the graph as $x$ tends to plus or minus infinity. Only a few candidates stated the coordinates of the point where the graph crossed the $y$ axis. This should be understood as a key feature of the graph.

**8)(i)** This was mostly answered correctly. The most common error was to state that $x = r\sin\theta$. Candidates who drew a quick sketch to prompt them did not make this error.

**(ii)** Those candidates who made brief supporting comments such as “a cardiod” etc enabled the examiner to award full marks when their sketch was not fully convincing. The second mark was for the accuracy of the curve at the pole. A number of sketches did not include the pole. Some candidates who did include the pole did not draw their graph in a way that made the behaviour at the pole clear.

**(iii)** This was a challenging question for all but the highest scoring candidates.
For six marks, candidates should be aware that explicit working needs to be seen. Some candidates appeared to be using a calculator to complete the integration. Those candidates who made a sketch were able to refer to this in their working and made the most progress. Students should be encouraged to provide a sketch even if the question does not ask for it explicitly.

# 4727 Further Pure Mathematics 3

**General Comments**

Overall this paper was found to be of a very similar standard to those of recent years, maybe just slightly more straightforward. Most candidates were able to attempt all questions, and the time available appeared to be sufficient. Differential equations and the easier questions on vectors and group work were, once again, topics on which most candidates scored well, together with the standard proof of a multiple-angle identity.

There is still a problem for many candidates with questions where demonstration or proof is required; too many do not provide essential rigour in their answers. Candidates should learn that the answer to any such question is expected to conclude with the final answer in the form requested. It is never enough, when tackling a 'show that' question, to merely repeat words from the information given or to simply quote a general definition.

As usual there was a varied mix of abilities in the cohort with a good number of very able candidates who have been well prepared, but also a number who have only limited knowledge of the full spectrum of the specification.

**Comments on Individual Questions**

**1)** This question was generally answered well. It was very rare this series to see poor vector notation, which was pleasing.

**(i)** Occasional arithmetic errors were in evidence. Some candidates omitted '**r** =' but, surprisingly, the most common error was a transcription error of the sign from candidates' own work when moving from direction vector to equation of line.

**(ii)** Those who used a vector product generally had little problem in obtaining the correct equation, with only occasional arithmetic errors. However, those who tried to set up simultaneous equations often were unable to correctly work through the algebra and in many cases gave up part way through their solution. A few candidates lost the last mark by not giving a cartesian form of the equation.

**2)** The first three parts of this question were done well but part (iv) less well. However here, as elsewhere, candidates lost marks though not giving sufficiently thorough justifications. This is especially important in group questions since candidates have to persuade the reader that they are actually considering the set given in the question.

**(i)** Virtually all candidates correctly formed the group table, and most knew the group axioms. A few evidently thought that any group has to be commutative. However the main cause of dropped marks was an inability to be convincing in demonstration. This was especially true when candidates addressed closure. A few candidates decided that closure required each row to contain every element of the set. More candidates simply did not convince the examiner that they were relating their conclusion to $G$, giving instead a general definition of closure or simply saying "it's closed".

**(ii)** A common error here was to state that the identity had the same order as the other elements.

**(iii)** The main weakness observed in this part was poor set notation (which was not penalised this time). Candidates should be encouraged to use braces { } for sets.

**(iv)** This was a question where the need for rigour appeared to be lost on many candidates. The arguments presented were often only partial. For instance candidates often compared the orders of elements, but did not fully justify claims that 3 (for example) has order 4. A few seemed to want to treat the element 3 as corresponding in *G* and *H*.

**3)** This was generally answered well with many gaining full marks and many others losing only the last mark for failing to take the cube root in order to find $y$, having found $y^3$ correctly. Many candidates were able to see that, after the substitution, the left hand side of the equation became a perfect differential if the equation was multiplied throughout by $x$, thus circumventing the need to use the formal process for finding an integrating factor. Pleasingly, only a handful omitted the constant of integration, although there were some whose algebraic manipulation was poor after the inclusion of this constant.

**4)** This question was a good differentiator, with each part producing a balanced spread of marks.

**(i)** Poor quality sketches were evident from many candidates, with many omissions of the key features necessary on an Argand diagram. This was often linked with limited evidence being given for establishing that the triangle was equilateral, or a failure to conclude arguments. Good answers often used clear notation to show how candidates were, for instance, finding the length of AB.

**(ii)** This was generally done well with many completely correct solutions given. A number were, however, only able to express the result in cartesian rather than polar form. Exponential form, cis form and polar coordinate form were all acceptable, though candidates should be made aware that neither $3\cos(-\frac{1}{3}\pi) + 3\mathrm{i}\sin(-\frac{1}{3}\pi)$ nor $3(\cos(\frac{1}{3}\pi) - \mathrm{i}\sin(\frac{1}{3}\pi))$ is standard polar form, and these were penalised.

**(iii)** Candidates who had given part (ii) in polar form were usually able to use this to gain at least the first two marks. Since the question starts "Hence", those who tried to solve the question by means of a binomial expansion were only able to access the final mark; they were rarely successful in doing this.

**5)** Given the fact that this question required standard steps, it was disappointing that the majority were unable to gain full marks. Almost all candidates knew how to approach the problem, but the following types of error hampered progress:

- incorrect solution to the auxiliary equation
- incorrect trial functions for the particular integral with $kx\mathrm{e}^{-x}$ and $kx^2\mathrm{e}^{-x}$ common
- lack of care when differentiating the general solution
- complementary function and final answer given in complex form.

Multistep questions of this nature should be treated with extra care, since an early error can result in the loss of several marks by changing the level of difficulty of the follow-through work. Centres may wish to draw candidates' attention to the specification where it lists the (three) possible trial functions that candidates have to recall – the form of others is **always** given when required.

**6)(i)** This question was nearly always correctly answered. Where candidates dropped marks it was usually due to basic arithmetic or sign (transcription) errors.

**(ii)** Answers were usually correct, though some candidates gave, as their final answer, the angle between line and normal rather than between line and plane.

**(iii)** Various different methods were successfully used although those candidates who used the formula (from MF1) were most likely to get a complete solution. A common error was to use $PQ = 2$ rather than $PQ\sin\theta = 2$. Others who used a correct method considered only one possible solution rather than two, often ignoring the modulus symbols which they had earlier used.

**7)** This question proved challenging with only a few candidates demonstrating the ability to form cogent, logical arguments. Candidates need to positively show their reasoning rather than leaving the examiner to intuit it.

**(i)** This was generally poorly answered. There was seldom reference to commutativity – candidates cannot simply assume that $(ab)^6 = a^6b^6$. Amongst those who did show that $(ab)^6 = e$, there was seldom **explicit** consideration of why *ab* did not have order 2 or 3. Very few candidates gave a full answer to this.

**(ii)** It was rare to see a full and concise reasoned solution using least common multiple. Quite a few candidates intuited that *ac* (or *abc*) has order 18, but not many justified this properly. Many that did either computed all powers up to the 18th, or computed the factor powers, often without referencing Lagrange's theorem.

**8)(i)** Although many excellent solutions were seen, a number failed to give sufficient working to truly <u>show</u> the given expression to be correct. These candidates commonly neglected to quote de Moivre's theorem at the start of their argument, or omitted the full binomial expansion.

On occasions candidates crossed out terms as they transferred them from one line of their argument to the next. Although this may help these candidates keep track of terms, it does undermine their argument and could, potentially, be penalised. This practice should be discouraged. Terms should only be crossed out when they cancel out and even then it is important that these terms remain visible.

A few tried to start from $\cos\theta = \frac{1}{2}(z + z^{-1})$, but hardly any made progress with this approach.

**(ii)** Many were confused about exactly what was required in this part and in part (iii). Candidates could not gain credit here for working out the surd value of the root and then using their calculators to find this in the form cos *α*. Of those who correctly considered solutions to $\cos 5\theta = 0$, many failed to rule out the value of $\theta = \frac{1}{2}\pi$, despite this not being a valid solution to the given quartic equation. A small number also failed to give solutions in the form $\cos\alpha$, appearing to believe that the angles themselves were the roots of the quartic.

**(iii)** The majority gained the B1 mark, sometimes from working in part (ii), but relatively few gave a satisfactory justification for picking the correct value for $\cos\frac{1}{10}\pi$. Since parts (ii) and (iii) both use the "hence" request, full marks could not be gained unless candidates' arguments over the two parts convinced the examiner that a credible line of reasoning had been followed. A few candidates tried to use sum and/or product of roots rules but this did not help.

# Overview – Mechanics

**General Comments**

Candidates showed a consistent thoroughness in their preparation for these examinations, with a generally high standard of achievement. Routine questions were despatched competently, and any loss of marks was likely to be the result of carelessness rather than ignorance.

However, questions which possessed a greater degree of novelty left some candidates unable to decide which fundamental mechanics principles were relevant. If these questions also incorporated subtleties of sign, the associated pure mathematics would often prove challenging. The reports on M1 (Q6ii), M2 (Q7i) and M3 (Q7iii) all draw attention to this.

The more challenging questions in each unit which serve to differentiate between the most able candidates are likely to place demands on candidates' knowledge and competence in pure mathematics as well as testing knowledge of mechanical ideas. However, when marking scripts, it often seems that some candidates relax after thinking through the mechanics, and shed marks in the equations and calculations which follow.

The loss of marks through such mistakes may sometimes then be compounded as early errors may render subsequent parts of a problem insoluble. In some cases a wider knowledge of alternative ways to tackle a problem can lead to solutions to a later part which do not rely on the answer to an earlier part; this was often the case in M1 this series. Candidates should definitely not assume that questions are always structured in a way which makes the final parts dependent on what has been calculated earlier.

# 4728 Mechanics 1

**General Comments**

Candidates were well prepared for this unit, and nearly all could score a significant number of marks throughout the paper. Many quoted standard *suvat* formulae correctly. It was only with parts (ii) and (iii) of the final three questions that many candidates encountered serious difficulty. Lower-scoring candidates might struggle with the second or third parts of the earlier questions.

Many marks were lost through candidates not paying due regard to signs in their work, or not giving both answers when a question asked for two quantities (for example Q5 (i), Q6 (ii)).

**Comments on Individual Questions**

**1)** Though the configuration and motion of the particles was elementary, it was necessary for candidates to consider the second collision before the first. Only a few avoided starting with the first collision, but nearly all corrected their approach after one or two lines of work.

Possibly the commonest source of error was the creation of equations in which candidates multiplied two speeds together, having lost track of which numbers were masses, which velocities.

Many candidates, who had found the correct value of the speed of $Q$ in part (i), realised when doing part (ii) that there was no reason to work out the “after” speed of $P$, but that the answer could be found immediately from “after” momentum of the particle.

**2)** A few candidates had a clearly expressed sense of which direction was positive for velocity, acceleration and displacement. Each part could be tackled either from the position of projection or from the top of the motion. Which was the candidate’s intention was sometimes unclear and might change from part to part.

In part (iii) the answer required was exactly 5.88. Candidates who evaluated 1.23 + 0.539 (and calculated the speed of the particle after it had fallen from a position of rest at its greatest height) would get 5.89, and so lost an accuracy mark because of their premature approximation.

**3)** Parts (i)(a) and (i)(b) could be done independently of each other, and this meant that an error in either need not cause a loss of marks in the other. The candidates who found the resultant first, with their working contained in the answer space for 3(i)(a) were expected to repeat their working, or at least show the result, in the space reserved for 3(i)(b)in the answer booklet.

Similarly the answers to parts (ii)(a) and (ii)(b) could be found independently.

Very many good solutions to this question were seen, with parts (a) done before parts (b), or after parts (b), or separately from each other. Complex solutions involving the sine and cosine rules were often seen, and these necessarily required parts (b) to be done before parts (a).

**4)** Many fully correct solutions were seen. The most frequent error was not realising in part (ii) that 300 m was the entire journey distance while both accelerating and moving with constant speed. Thus “300/30 = 10, 10 + 5 = 15 seconds” was the most common mistake seen. Very many candidates expressed their work in a very informal way, their solutions consisting predominantly of numbers (120, 180, 6, 11) without much explanation of what they meant.

Candidates could directly find in part (iii) the distance while decelerating (from $0^2 = 30^2 - 2\times 6s$ ), and so those who first calculated a time while decelerating needed to use that time to find a distance before becoming eligible for any mark.

**5)** 5(i) was well attempted by nearly all, with many correct solutions seen. However, it was common for candidates to lose the last mark by giving a value for $d$ but not giving the length of the plane.

5(ii) and 5(iii) were often omitted, and only the best candidates knew how to answer each part. In 5(iii) a version of Newton's Second Law ($4m = 6$) was sometimes offered as the solution.

In nearly all cases when (iii) was correctly answered, candidates used their answer to part (ii). However, a few chose a different route, namely $(mg)^2 = (4m)^2 + 6^2$. The unfamiliar way in which this topic (component of force perpendicular to an inclined plane) was tested led to a rash of unexpected errors. The two most frequent were resolving 6, rather than the weight, and using sine function, rather than cosine.

**6)** In part (i), nearly all candidates adopted the correct methods, differentiating twice. Loss of marks mostly arose from leaving out the negative signs when giving the velocity and the acceleration for $t = 0$.

In part (ii) about half of all candidates based their answer for minimum velocity on $v = 0$. Candidates who correctly used $a = 0$ to find the correct value of $t$ might sometimes mis-calculate the corresponding value of $x$, but were more likely than not to give the corresponding velocity, instead of the speed as requested. There were also a significant proportion of scripts where only one of the two required quantities was found. (Some mark-worthy responses were based on candidates finding two values of $t$ which had the same value of $v$. Candidates would then find the average these two values. As $v$ is a quadratic function of $t$, the method was valid, and was marked as such.) About 10% of scripts contained a fully correct solution to Q6(ii).

Part (iii) was frequently left out by candidates who had used $v = 0$ in part (ii), while others simply quoted the value found previously, which (if correct) would gain full marks. A significant number of solutions foundered because candidates could not solve accurately the quadratic equation $0.18t^2 - 0.9t - 0.24 = 0$. If the initial step in a solution was to convert the coefficients to integers, this was likely to yield $18t^2 - 9t - 24 = 0$.

**7)** Part (i) was routine, though some solutions involved the mass of $P$ and were given no credit.

The best candidates were able to answer part (ii) correctly. The main error was the omission of one of the four terms from the Newton's Second Law equation for $P$.

Part (iii) involved many candidates in more work than was necessary, as most tried to find both an upper and a lower bound for $\mu$. The part of the solution best attempted was the normal component of the force between $B$ and the plane, and the least successful was the magnitude of the frictional force between the two. That a component of the weight of $P$ is included in the former but not the latter was the major difficulty.

# 4729 Mechanics 2

**General Comments**

On the whole, most candidates seemed well prepared for this examination. As should be expected, the last two questions were usually found to be the hardest, though there were some candidates who had success in these, after having problems earlier in the examination.

As stated in previous reports, candidates should present a good quality force diagram which will aid their understanding of the requests made. Labelling these diagrams appropriately will also be of benefit and preferably not using '*F*' for unknown forces, as examiners can not always be sure which force candidates are referring to, it may be friction or just a general force.

**Comments on Individual Questions**

**1)** (i) The majority of candidates answered this question well. Only a minority attempted to use kinetic energy to find the loss in potential energy.

(ii) This part was not answered well by a significant number of candidates. It was common to see the initial velocity of the particle being ignored in the energy equation and candidates finding a new wrong speed of $12.5\,\mathrm{m\,s^{-1}}$. Some candidates then thought they could deal with the initial speed by adding it to their wrong value.

**2)** Generally part (i) was answered well by candidates. Examiners were pleased when candidates gave a clear argument that, at constant speed, resistance was equal to driving force.

Part (ii), in which the driving force had to be calculated using Newton's 2nd Law, was done very well although it was common to see the weight component (or even the acceleration term) missing from the resolution parallel to the plane. The majority were able to multiply their driving force by the speed to find a power.

**3)** This question proved a good source of marks for some candidates, but others lost marks because they were confused by the idea of an arc, and preferred to make it into a lamina. In part (i), it was very common to see candidates only find the horizontal component of the force at the hinge and not consider that there might be some component to prevent the object from falling.

There were many poor attempts seen to part (i). The majority approached this by considering the components of the weight, but, usually, only considered the moment of one component when in fact both components have a moment about the hinge. The less common approach of using the perpendicular distance to the line of action of the weight also proved to be demanding.

**4)** This problem on centre of mass was answered extremely well by many candidates and was an excellent source of marks for many. Part (i) saw a few errors mainly due to inconsistencies when taking moments about a particular axis. Another common error was to treat the shell as a uniform solid. Some decided to take the volume of the shapes into consideration even though the mass of the individual components were given in the question. Part (ii) was again answered well with the most common errors being the omission of $g$ from moments/resolution equations or incorrectly using the distance found in part (i).

**5)** It is surprising how many used the string in part (i), and omitted it in part (ii). Nevertheless, there were many correct solutions seen to both parts of the question. Those who were unsuccessful frequently presented confused diagrams. Forces labelled $F$ often appeared in a number of different directions within the same diagram.

**6)** In part (i), most candidates knew which principles to apply, and their attempts to apply them usually gained marks. The momentum equation usually began correctly, though the coefficient of restitution equation was less reliably correct. A major source of error was in multiplying out brackets. Many failed to distribute the 4 to all terms when expanding $4(1 - e + e^2)$; $e(4 - u)$ often became $4e - 4u$.

In (ii), many good arguments were presented leading to $e = ½$, using completing the square, the discriminant of $e^2 - e + (1 - k) = 0$, or differentiation. As the earlier parts of the question had presented so many opportunities for error, the correct answer to the loss in kinetic energy was not usually obtained. Many had a good effort with KE, though often only one particle was considered.

Those who chose to deal with particle $A$, in part (iii) maximised their chances of gaining the full four marks since they were not relying on their own answers from part (i). Sign errors were common which lead to answers of $e = 1.2$ and $-0.2$, both of which should have been a clue that something had gone wrong in this part. Those who had not simplified their answers for $m$ and $u$ frequently found higher order equations to solve for $e$.

**7)** This question proved to be the most challenging for many candidates. On the whole in part (i) many did score at least two of the first three marks for finding the horizontal and vertical components of the initial velocity, but due to sign errors many incorrectly thought that the vertical component of the final velocity was 14sin(20) rather than $-14\sin(20)$. Some candidates had correct equations, but they should be reminded that just stating that $u = 15.9$ is inadequate for an answer that is given in the question.

In part (ii), the question was designed so that candidates could answer the request, by consideration of energy, without using (i). Only a minority used this approach, with the majority using, mostly successfully, constant acceleration.

In part (iii), as in part (i), sign errors with vertical components lost marks for many, though there were frequent good attempts to solve simultaneous equations. Nearly everyone who had attempted part (iii) managed at least a follow-through mark in part (iv), though some candidates used 15.9 m $s^{-1}$ instead of 14 m $s^{-1}$.

# 4730 Mechanics 3

**General Comments**

Many of the candidates for this unit were very competent and very well prepared; these candidates found most of the paper well within their grasp, but still found Question 7 challenging. However, a small number of candidates struggled with the whole paper, seeming to be unprepared.

The early part of the paper proved to be a gentle test of various topics, with very few candidates making more than an occasional slip. Question 5 proved inaccessible to a small number of candidates, while it seemed routine and fairly easy to most. The work on Question 6 was generally of a high standard, with only a few candidates showing little or no grasp of the topic. Question 7 was where the higher scoring candidates were really challenged, with many of them dropping marks on parts (i), (iii) and (iv).
The presentation of the scripts was extremely good in many cases, and generally acceptable. Most of the candidates who used additional paper did so since they genuinely needed to make an extra attempt at a question.

**Comments on Individual Questions**

**1** Almost all candidates were able to do this question correctly. Many of them based their working on the length $d$ given in the question; others used the length of one string or the other and added on either 0.4 m or 0.6 m at the end. A small number of candidates successfully worked with $x_1$ and $x_2$ as the lengths of the two strings. A few candidates used $x$ for the length of each string, and failed to distinguish between them; other wrong answers included use of energy.

**2(i)** Almost all candidates found the velocity correctly, though there were a very small number of scripts where a formula was quoted incorrectly. A small number of candidates omitted to find the time taken for the particle to travel from $A$ to $B$.

**(ii)** Almost all candidates did this correctly, though some made an error finding the speed of the particle on leaving $A$. A small number of candidates worked successfully in terms of a speed of $2.8e$ and did not need to find the speed explicitly.

**(iii)** Most candidates gained both marks for this, though some, including those who had earlier errors in speeds, gained only M1.

**3(i)** Most candidates gained full marks. Some candidates had the force in the opposite direction, some ignored the need for an arbitrary constant after the integration, and a small number left their answer as $0.2v = \ldots$ .

**(ii)** Almost all candidates gained M1 for putting their answer to part (i) equal to 0.8. The A1 mark was only awarded to those who gained the correct answer having used a correct equation from part (i).

**(iii)** Most candidates integrated their answer to part (i), with just a few losing a mark by failing to establish that the arbitrary constant was 0, and went on to solve a quadratic equation to give the two times required. A small number of candidates went back to the acceleration equation, used $v\dfrac{dv}{dx}$ for acceleration and failed to do the question correctly.

**4(i)** This question was done extremely well. The errors that did occur were in taking the direction of motion of $A$ in one direction in the momentum equation and the other in Newton's experimental law; in omitting mass from one or more term of the momentum equation or including it in Newton's experimental law, and in making a slip in the solution of the two simultaneous equations.

**(ii)** Almost all candidates correctly stated or used 0.8 as the velocity of $B$ perpendicular to the line of centres. Those who had part (i) correct usually arrived at 21.4° for the direction of motion of $B$ after impact, though not all then found the angle turned through correctly. The method mark for finding the direction of $B$ after the impact was available for those who had made errors in part (i), and so was the final follow through mark for finding the angle turned through.

**5(i)** This proved to be very straightforward for a large number of candidates who had been well trained and knew exactly what to do. Since the answer was given as a surd the final mark was not awarded to those who expressed sin ($\pi/6$) as a decimal in their working. However, quite a number of candidates made errors; some got the PE at $A$ and $B$ the other way round, and so had a negative quantity for $v^2$; others made a sign error in using $F = ma$; and some candidates had the particle start at $T$ so that $\pi/6$ did not appear in the energy equation.

**(ii)** Candidates who did part (i) correctly usually got this part right. Some who made errors in part (i), or made little attempt at part (i), also got this right by showing full correct working here.

**(iii)** This proved a difficulty to some candidates, with some working out $g\cos \text{TOB}$, others not using an angle, and others not being able to work out $\sin \text{TOB}$ correctly.

**6(i)** This was usually correct, with almost all candidates managing to give sufficient detail of their working to convince examiners that they were right.

**(ii)** Many candidates produced almost exactly the complicated equation shown in the mark scheme, with a few using equivalent trigonometric expressions and just a few making slips with the angles. Most of these then went on to give a correct final answer. A small number of candidates first worked out the components of the force acting on $AB$ at $B$. Full credit was available for this method though most of those using it made sign errors. A small number of candidates did not appreciate that having two rods meant that the moment terms for $F$, $R$ and $2W$ had separate length parts within them.

**(iii)** There were many good concise solutions to this part, and also quite a lot of complicated rambling solutions. In general, the longer the solution the more chance there was of an error creeping in. A significant number of candidates were sufficiently confident to work throughout in surds and leave the answer as $\dfrac{3\sqrt{3}}{13}$.

**7(i)** While many candidates did this completely correctly, quite a large number of candidates only established that the particle is in equilibrium at a point 1 m below $O$, while others only established that the particle performs SHM without any consideration of the centre of motion.

**(ii)** This was generally done successfully, with candidates either working in terms of the extension of the string or the distance below of $P$ below $O$.

**(iii)** Almost all candidates found this part difficult, with only a few succeeding completely. Most candidates realised that the motion was partly free fall and partly under SHM. However, not all candidates realised that the free fall was for a distance of 0.8 m, and quite a number failed to make any use of a standard SHM equation for displacement in terms of time. Correct solutions were seen based on $x = 0.6\cos 7t$ and $x = 0.6\sin 7t$ and, less often, $x = 0.6\cos(7t + \varepsilon)$ and $x = 0.6\sin(7t + \varepsilon)$. However, using a correct formula does not guarantee success – the candidate has to use it for the right portion of the motion. In this question starting at $x = -0.2$ caused many candidate to make errors. Very few candidates used any form of diagram of what was happening, which could have made it easier for them to be successful.

**(iv)** Most candidates realised that they needed to use a standard SHM equation here, but many had difficulty finding the correct time to use with it. This time, of course, depended on the approach they used, which had either been decided in part (iii), or was shown in this part by a fresh start.

# 4731 Mechanics 4

**General Comments**

The work on this paper was generally of a very high standard, with most candidates demonstrating a sound understanding of the principles of mechanics covered in this unit. Candidates seemed to be particularly confident when using calculus to find a centre of mass, and when using energy considerations to investigate stability of equilibrium. Topics which presented a challenge for a significant number of candidates were using relative velocities to find closest distances, and finding the force acting at an axis of rotation. The majority of candidates appeared to have sufficient time in which to complete the paper.

**Comments on Individual Questions**

**1)** This question, on constant angular acceleration, was almost always answered correctly.

**2)** The techniques for finding the centre of mass of a rod with variable density were very well understood, and were usually carried out accurately. Some candidates made errors with the integration of $\sqrt{x/a}$ and $x\sqrt{x/a}$ .

**3)** Most candidates found the moment of inertia of the solid of revolution correctly. A factor of ½ was sometimes omitted (or replaced by ⅓), and several candidates integrated $x^2y^2$ instead of $\frac{1}{2}y^4$. Careless integration sometimes led to an incorrect power of $a$.

**4)** In part (i)(a) the relative velocity was usually found correctly, either by applying the cosine and sine rules to a velocity triangle or by finding components. A fairly common error was using the sine rule to obtain the largest angle in the triangle as 87°, when it is actually 93°.
In part (i)(b) many candidates used an incorrect angle, or confused sine and cosine, when finding the shortest distance.
In part (ii)(a) the course for closest approach was quite well understood, and most candidates obtained the correct bearing.
Part (ii)(b) was quite often omitted, and again there was a lot of confusion in finding the shortest distance.

**5)** In part (i) very many candidates found the moment of inertia of the rod $AB$ about $O$ as the moment of inertia of $AB$ about $A$ plus (mass) × $AO^2$. This gives the correct value (since $AO$ is perpendicular to $AB$) but it cannot be accepted (without further justification) as a valid derivation when the answer has been given.
In part (ii) most candidates used the equation of rotational motion to derive the given result. A few did it by differentiating the energy equation.
Part (iii) was well answered, although many candidates did not indicate at what stage they had shown that the motion is approximately simple harmonic.
In part (iv) there were some attempts to use constant acceleration formulae, but most candidates produced an energy equation involving kinetic energy, potential energy and the work done by the couple. The potential energy term was quite often incorrect, and sign errors were reasonably frequent.

**6)** In part (i) most candidates derived the given expression for the total potential energy correctly, although many candidates experienced some difficulty with the trigonometry involved.
In parts (ii) and (iii) the use of the given expression to find the positions of equilibrium and investigate their stability was very well understood.

**7)** In part (i) the moment of inertia of the lamina was almost always found correctly.
In parts (ii) and (iii) the methods for finding the angular speed and angular acceleration were well understood and usually applied accurately.
In part (iv) there were many mistakes in forming the equations of motion to find $R$ and $S$. These included sign errors, omission of the mass, omission of the radius ($5a$), omission of the weight term and confusion between sine and cosine. Sometimes the radial and transverse accelerations were interchanged.

# Overview – Probability & Statistics

As usual there is much excellent work seen on these units, with many candidates able to carry out calculations correctly and with attention to detail. Statistics is, however, about more than just calculations. Questions that require understanding are often much less well answered. It is clear that many candidates think they have learnt and understood the ideas if they can copy out a phrase, sentence or list from a textbook or previous mark scheme. This is not the case; papers are set specifically to test whether candidates can go beyond that. Such regurgitation does not usually gain credit and should be discouraged. Specific instances are dealt with in more detail in the unit reports that follow.

This year Examiners did not give credit to conclusions of hypothesis tests of the form “there is significant evidence that the null hypothesis is correct”. No hypothesis test ever gives such evidence; conclusions such as “there is insufficient evidence that the null hypothesis is false” (duly contextualised) is to be preferred. This was flagged in last year’s report. Most candidates do now give conclusions in context and with a wording that does not suggest over-assertiveness; here phrases such as “insufficient evidence” or significant evidence” are to be encouraged.

Each year these reports emphasise the need for candidates to use the book of Mathematical Tables and Formulae more accurately. This need is still apparent.

# 4732 Probability & Statistics 1

**General Comments**

Most candidates were able to answer a reasonable proportion of the questions with some success. Some parts contained relatively non-standard requests (eg 1(iii), 2, 3(ii), 4(iii), 8(ii) and 9) and some candidates could not handle the slightly different approaches that were needed. In particular, the question on Spearman's rank correlation coefficient (2) was not the usual "turn the handle" calculation but required some thought. As a result fewer candidates than usual scored high marks on this question. The scenario in question 7 is a well-known one, which features in many text books, but many candidates found it difficult to find their way through it.

The questions that required an answer given in words were fairly well attempted, except for question 4(iii) which was not well understood on the whole.

A few candidates lost marks by premature rounding or by giving their answer to fewer than three significant figures without having previously given an exact or a longer version of their answer (for example in question 1(ii) or 5(i)). It is important to note that although an intermediate answer may be rounded to three significant figures, this rounded version should not be used in subsequent working. The safest approach is to use exact figures (in fraction form) or to keep intermediate answers correct to several more significant figures.

Few candidates appeared to run out of time.

In order to understand more thoroughly the kinds of answers which are acceptable in the examination context, centres should refer to the published mark scheme.

**Use of statistical formulae and tables**

The formula booklet, MF1, was useful in questions 2(i), 5 and 7 (for binomial tables).

In question 5 a few candidates quoted their own (incorrect) formulae for $r$ and/or $b$, rather than using the ones from MF1. Some thought that, eg, $S_{xy} = \Sigma xy$ or $\Sigma x^2 = (\Sigma x)^2$. Others tried to use the less convenient versions, $r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\{\Sigma(x-\bar{x})^2\}\{\Sigma(y-\bar{y})^2\}}}$ and $b = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$ from MF1.
All these candidates misunderstood the Σ notation. For example they interpreted $\Sigma(x-\bar{x})^2$ as $(\Sigma x - \bar{x})^2$.

In question 2(i), $\Sigma d^2$ was sometimes misinterpreted as $(\Sigma d)^2$ and the formula was sometimes misquoted as $\frac{6\times\Sigma d^2}{n(n^2-1)}$ or $\frac{1-6\times\Sigma d^2}{n(n^2-1)}$ or $1-\frac{6\times\Sigma d^2}{n}$ or $1-\frac{6\times\Sigma d^2}{n^2(n-1)}$.

In question 7, some candidates' use of the binomial tables showed that they misunderstood the entries as individual, rather than cumulative, probabilities.

It is worth noting again, that candidates would benefit from direct teaching on the proper use of the formula booklet, particularly in view of the fact that text books give statistical formulae in a variety of versions. Much confusion could be avoided if candidates were taught to use exclusively the versions given in MF1 (except in the case of $b$, the regression coefficient). They need to understand which formulae are the simplest to use, where they can be found in MF1 and also how to use them.

**Use of calculator functions**

Increasingly nowadays, calculators can provide answers using statistical functions, binomial functions etc, without the need to quote a formula and to substitute values into it. The problem here is that if candidates write down only an answer, they can only score either full marks or no marks, with no possibility of gaining any credit for partially correct working. In most cases, the use of such functions saves very little time and it is advisable to show working instead. However, if candidates wish to use these functions they should input all the relevant data twice in order to check their answer.

It should also be noted that there are sometimes questions in which a correct answer without working may not gain full marks.

**Comments on Individual Questions**

**1)(i)** Most candidates answered this part well. A few omitted one or two digits, but the most common error was misalignment. Many candidates did not appear to appreciate that the shape of a stem-and-leaf diagram is important. The lengths of the leaves show, at a glance, the general shape of the distribution of the data. Hence the alignment of the figures is important. For example, many candidates showed the 3rd digit in the first row clearly aligned with the 4th digit in one or more of the other rows. Also when candidates made an error and crossed out a digit, this usually resulted in misalignment when the correct digit was inserted. A few omitted the key.
A small number drew a box-and-whisker diagram instead of a stem-and-leaf diagram.

**(ii)** Most candidates answered this part correctly, although arithmetical errors in finding the mean were not uncommon. Also, for the median, a few candidates found the 9th value instead of the mean of the 9th and 10th. Follow-though from an incorrect diagram in part (i) was allowed for the median.

**(iii)** Many candidates correctly identified the 49 and 53 as possibilities for the incorrect value, but some gave incorrect replacements, most commonly 50 and 54. A few gave answers that suggested that they did not understand what a median is.
Some candidates understood the instruction "Give two possibilities for the incorrect length …" to mean "Give two possible explanations for incorrect measuring". This gave rise to answers such as "The snakes moved while being measured."

**2)(i)** Some candidates ranked the marks in the order "smallest first" (presumably working on auto-pilot), and the grades in alphabetical order. This means that one set of ranks is in reverse order and thus leads to the incorrect answer of –0.3 instead of 0.3. Others just made errors in their ranking. A few ranked the students 1, 2, 3, 4, 5 instead of the marks. As usual, a few used the original marks together with ranks for the grades, obtaining impossible values for $r_s$.

Incorrect versions of the formula, most commonly $\frac{6\times\Sigma d^2}{n(n^2-1)}$ and $\frac{1-6\times\Sigma d^2}{n(n^2-1)}$, were sometimes seen. A few candidates used the formula for the PMCC on the ranks, which is a valid method, but is long and gives more opportunities for arithmetical errors than Spearman's formula.

**(ii)** This unusual question was well answered by many. A few just reversed Judge 1's ranks: 4, 3, 2, 1.

**3)(i)** Most candidates answered this part well, although all the usual errors were seen, such as dividing $\Sigma x^2 p$ by 4 (or by 16), omitting to subtract $\mu^2$ or subtracting just $\mu$. Those who attempted $\Sigma(x-\mu)^2 p$ usually made errors.

**(ii)** Some candidates failed to write down the possible sets of values for $X_1$, $X_2$ and $X_3$ as requested, giving, for example, just 5, 7, 7. Some of these nevertheless obtained the correct answer for the probability, although many gave $\frac{1}{3}$. A good number of candidates ignored the fact that the values of $X$ were given in the table and made long lists of irrelevant combinations adding up to 19. A few attempted a binomial calculation such as ${}^7C_7 \times 0.1^7 \times 0.9^0$. Some candidates gave the possible values of $X_1$, $X_2$ and $X_3$, but then just gave the given value of $P(X_1 = 7)$, ie 0.1, ignoring the condition that the total has to be 19.

**(iii)** Many recognised that this is a binomial calculation. Some found P(5) instead of P(4). Others omitted the binomial coefficient. Some candidates omitted this part altogether, perhaps not understanding what "Use an appropriate formula . . ." meant. (This wording was intended to encourage candidates to show their working rather than just to write down an unsupported answer, obtained from the binomial function on their calculator).

**4)(i)** Many candidates gave the correct mean, but many gave 5.13 for the standard deviation (or even 0.74 for the mean). Some divided 0.74 by 10 before adding it to 5. Others confused this question with questions involving finding the mean and standard deviation of two groups combined. These candidates tried to find $\Sigma x^2$ by working backwards from $\sigma = 0.13$ and from there they tried to find the new standard deviation.

**(ii)** Some candidates found the unweighted mean of the two means or simply added the two means.

**(iii)** There was some confusion here. Some candidates considered only the mean of all 25 people, stating that it was very close to the true mean and was therefore "consistent" in some sense. Some gave general answers such as "It is untrue because they are just guessing." Others only compared the means of the two groups, correctly noting that one was nearer to the true mass than the other. A few appreciated that the standard deviation represents the consistency of a group's guesses, then compared the standard deviations of the groups and gave a fully correct answer, although a few thought that a higher standard deviation means greater consistency.

**5)(i)** This part was well answered. A few candidates made errors such as those mentioned above. Some candidates used the statistical functions on their calculator and gave an answer with no working. As noted above, this is a risky strategy, resulting in either full marks or no marks. Candidates who wish to use this method should be advised to carry out the whole calculation a second time, as a check. The safest approach is to show the substitutions into the relevant formulae for $S_{xy}$ etc and for $b$ and $r$.

**(ii)** A good number of candidates answered correctly either by pointing out that correlation does not imply causation or that other factors may be involved. Many, however, stated (wrongly) that the reason why the given statement is wrong is that the data in the table show that the progression is sometimes up and sometimes down.

**(iii)(a)** Some candidates misread the question and found the equation of the (more usual) $y$ on $x$ line. However, the majority read the question correctly and attempted to find the equation of the $x$ on $y$ line. But many of these failed to be consistent in their intention. Some found the gradient ($b'$) correctly but then reverted to the method for $y$ on $x$, attempting $a' = \bar{y} - b' \times \bar{x}$ , which is incorrect. Still others found $b'$ and $a'$ correctly but wrote their final answer as $y = b'x + a'$ instead of $x = b'y + a'$

**(iii)(b)** This was well answered, with follow-through from incorrect equations allowed. A few candidates substituted for $x$ instead of $y$.

**6)** Candidates generally found all three parts of this question difficult, whether they attempted to use combinations or multiplication of probabilities. Those who chose the first method sometimes used, for example, ${}^5C_5$ when a choice of fewer than 5 letters out of 5 was required. Others used permutations, which are not appropriate here since order is not an issue. Those using probabilities in parts (ii) and (iii) often used numerators of 1 instead of 2, 3 or 4.

**(i)** Some candidates found $\frac{1}{5}\times\frac{1}{4}$ but did not multiply by 2. Others found $\frac{2}{5}\times\frac{1}{4}$ but then multiplied by 2. Some assumed "with replacement", giving $\frac{2}{5}\times\frac{1}{5}$. Some who used combinations placed ${}^5C_2$ in the numerator rather than the denominator, for example, $\frac{{}^2C_2}{5!}$. Also $\frac{2}{{}^5C_2}$, $\frac{2!}{{}^5C_2}$ and $\frac{\frac{2}{5}}{{}^5C_2}$ were often seen, instead of the correct $\frac{{}^2C_2}{{}^5C_2}$.

**(ii)** Some candidates correctly found $\frac{1}{5}\times\frac{3}{4}\times\frac{2}{3}$ or $\frac{2}{5}\times\frac{3}{4}\times\frac{2}{3}$ but either failed to multiply by the appropriate constant (6 or 3 respectively) or multiplied $\frac{1}{5}\times\frac{3}{4}\times\frac{2}{3}$ by 3 instead of 6. Many of those attempting to use combinations or permutations wrongly included 5! in their working somewhere. Examples of half–correct solutions were $\frac{2\times{}^3C_2}{5!}$ and $\frac{{}^3C_2}{{}^5C_3}$. Some candidates placed ${}^5C_3$ in the numerator rather than the denominator. Others used a "with replacement" method, sometimes using the formula for a binomial probability.

**(iii)** Those who used probabilities tended to use a rather long method, finding P($B$) + P($XB$) + P($XXB$) + P($XXXB$) instead of the more straightforward $1-\frac{1}{5}$ or $4\times\frac{1}{5}$. Those using combinations sometimes put ${}^5C_4$ in the numerator instead of the denominator, and/or 5! in the denominator, such as $\frac{{}^5C_4}{5!}$ or $\frac{1}{5!}$. A better (although still incorrect) attempt was $\frac{1}{{}^5C_4}$, which ignores the fact that the "$B$" could be on any one of the four cards chosen.

**7)** A few misread the question, using $n$ = 15 throughout, instead of both $n$ = 30 and $n$ = 15.

**(i)(a)** The problem here was not generally the use of the binomial distribution but the need to see through the context and recognise that a binomial calculation was appropriate. In this part there was a distinct advantage to using tables rather than the formula (which is a much longer method in this case). Some candidates chose to use the formula and made arithmetical errors. Using either method, some candidates used B(30, 0.95) instead of B(30, 0.05). Others used the correct distribution but, in order to find $P(X < 2)$, they found $1 - P(X = 1)$ instead of $1 - P(X \le 1)$. Some omitted the binomial coefficients.

**(i)(b)** Few candidates were able to work their way through this part correctly. Some candidates found $P(Y \ge 1) = 1 - P(Y = 0 \text{ or } 1)$ or $P(Y \ge 1) = 1 - P(Y = 1)$. Some candidates were close to being correct, but found $P(X > 2) + P(X = 2) \times P(Y = 0)$ instead of $P(X > 2) + P(X = 2) \times (1 - P(Y = 0))$. Many complicated matters by using the same letter ($X$) for the number of defective mugs in the second sample as well as in the first.

**(ii)** This is an example of a common, "two-layered" type of question, requiring the use of a previously obtained figure as the value of $p$ in a geometric (or binomial) calculation. Candidates would benefit from being taught to look out for such questions. Many candidates used the correct geometric structure but with $p$ = 0.05 or with a probability equal to their answer to part (i)(a) instead of (i)(b). Others attempted to use a binomial distribution instead of a geometric.

**8)(i)** Many candidates correctly found $\frac{12}{27}\times\frac{10}{26}\times\frac{5}{25}$ but either failed to multiply by 6 or multiplied by an incorrect number, such as 3 or 4 or 12. Some added the three fractions instead of multiplying. A few added 12, 10 and 5 incorrectly, and so started with a denominator of, eg, 25 instead of 27. Some did the question "with replacement".

**(ii)** Many candidates were able to form an algebraic term such as $0.4 \times \frac{x}{50}$ or $\frac{2}{5} \times p$, but most then either equated this term alone to 0.54 or added it to a term such as $0.6\times\frac{x}{50}$ or $\frac{3}{5} \times p$, using the same letter for both unknowns. Some realised that the second unknown was not the same as the first and wrote, for example, $0.4\times\frac{x}{50} + 0.6\times\frac{y}{50} = 0.54$. However, few realised that there was a second simultaneous equation, namely $x + y = 50$. The better scoring candidates wrote an equation such as $0.4 \times\frac{x}{50} + 0.6 \times\frac{50-x}{50} = 0.54$. A few candidates muddled red and blue, writing a correct equation such as $0.6\times\frac{x}{50} + 0.4 \times \frac{50-x}{50} = 0.54$ and correctly finding $x = 35$, but then gave their answer as 35 red discs, rather than 15. A few candidates used a trial and improvement method, some with success. Several gave an incorrect answer of 16 red discs, being deceived by the fact that this value does give a probability of 0.54, although only when rounded to 2 significant figures.

**9)** In all parts of this question some candidates appeared not to recognise that there is a geometric distribution involved and used either a binomial distribution or some other incorrect method. Very few were able to distinguish between the two variables "*X*" and "The number of steps".
A few candidates interchanged 0.2 and 0.8. If they did so consistently in parts (i)(a) and (i)(b) they were able to score some marks.

**(i)(a)** The usual errors were seen, such as $0.8^{10}$ alone, $0.8^9 \times 0.2$ and $0.8^{11} \times 0.2$. Also binomial methods such as ${}^{11}C_{10} \times 0.8^{10} \times 0.2$ were seen.

**(i)(b)** Common errors were $0.8^{10}$ (alone), $1 - 0.8^9$, $1 - 0.8^{11}$ and $1 - 0.8^{10} \times 0.2$. Also as usual, some candidates attempted the "long" method, ie
$0.2 + 0.8 \times 0.2 + \ldots + 0.8^9 \times 0.2$. But the last or first term was often omitted.

**(ii)** Few correct answers were seen. Some candidates tried to use $\Sigma xp$ but usually failed. Some found $\frac{1}{0.8}$, although they had not interchanged 0.2 and 0.8 in the previous parts. Others found $0.8 \times 0.2$.

**(iii)** Many gave the same answer as for part (ii). Others who had answered part (ii) correctly ($\frac{1}{0.2} = 5$), found $\frac{1}{0.8} = 1.25$ in this part.

# 4733 Probability & Statistics 2

### General Comments

This paper placed a premium on understanding. Candidates were able to carry out standard calculations accurately; on questions that required carrying out of straightforward hypothesis tests, such as question 6, there were many excellent answers seen, even though the process is quite intricate. However, weaknesses identified in previous papers were still very much in evidence here. These included:

- Misunderstanding of the meaning of probability density functions
- Misunderstanding of the conditions for a Poisson distribution to be a good model
- Misunderstanding of the Central Limit Theorem
- Confusion between the roles of sample mean and population mean in hypothesis tests.

This year the question on continuous distributions (question 5) was particularly poorly answered.
Many candidates could respond to questions requiring verbal answers only by quoting phrases learnt from textbooks or from past mark schemes. Examinations and mark schemes are designed to reward candidates who understand the concepts, at the expense of those who can merely quote phrases. Candidates need to be able to go beyond familiar words, and show that they have understood the concepts by applying them to the specific requirements of the question. Too many candidates seem to think that learning standard phrases is the same as learning the ideas.

### Comments on Individual Questions

**1)(i)** Most got this right, though some failed to add 1 to the numbers (answering "88, 89, 90, 90, 91") and some rushed to the conclusion that the numbers always went up by 1.

**(ii)** Quite a few candidates showed that they had misunderstood the words "which candidates *would be picked*", by answering that the same pupils *had been picked* twice and that repeats should be ignored. All that was needed was the comment that not all pupils were equally likely to be selected, and that the random numbers should be multiplied by 1000, rejecting numbers greater than 853.

**2)** Some candidates failed to multiply the mean and variance by 40. The continuity correction caused problems; even though it affects only the fourth significant figure, it needs to be considered. The best way of handing it was to write "$1.999\times10^6 - ½$". Many used, for example, $1.9985\times10^6$, which gives a very wrong answer. A few divided throughout by $10^6$, forgetting that the denominator involves a square root and so this is incorrect.

**3)(i)** Candidates would certainly have been expecting a reverse-normal distribution question. This was slightly non-standard as it involved the sample mean; quite a number of candidates failed to include the $\sqrt{80}$ factor. Some correctly found 1.282 but then found $\Phi(0.7)$ rather than $\Phi^{-1}(0.7)$ (tables gave 1.282 directly but not 0.524). There were also inaccuracies in finding 0.524, with some candidates using 0.521. Some candidates clearly expected the two standardised equations to have different signs. In giving the final answer it is not acceptable to state the standard deviation as an exact surd such as $40\sqrt{5}$, as the $z$ values are not exact. Some candidates got the right value of $\sigma$ and then multiplied or divided it by $\sqrt{80}$. However, a pleasingly large number of correct answers were seen.

**(ii)** Many candidates thought that the Central Limit Theorem was used in moving from $\sigma$ to $\sigma/\sqrt{n}$. This is not so. The Central Limit Theorem states that the *distribution* of the mean of a sufficiently large sample is approximately normal – it is a statement about the *shape* of the distribution, *not* about the parameters. The statement that the mean of a random sample of size $n$ has standard error $\sigma/\sqrt{n}$ is *exactly* true for all $n$, however small – a different result entirely.

In parts (b) and (c), some candidates could not distinguish between necessary and sufficient conditions, as in several previous examinations.

**4)** This standard question on a Poisson hypothesis test was answered well by many, but a large number of candidates failed to consider the distribution for 10 years, Po(3.2). It was particularly dispiriting to see candidates trying to find the probability of 0.6 floods from Po(0.32). However, the number of candidates who wrongly considered P(> 6) or P(≤ 6) or P(= 6), instead of the correct P(≥ 6), seemed, pleasingly, to be smaller than in the past.

Candidates who gave the over-assertive conclusion "There is significant evidence that there has not been an increase in the mean number of floods" lost the final mark. Hypothesis tests cannot prove $H_0$ correct; a correct conclusion might have been "There is insufficient evidence that there has been an increase in the mean number of floods".

**5)(i)** As usual, there were many who wrongly drew the graphs beyond the limits of [0, *a*]. Many could not put the parabola in the right place.

**(ii)** It was surprising to find so many S2 candidates who could not multiply out $(x - a)^2$. Commonly seen were $x^2 - 2a + a^2$ and $x^2 - 2ax + a$. Most at least knew that to integrate $x(x - a)^2$ they had to multiply out (or use parts). There were other frequent algebraic errors, too.

**(iii)** This question brought out all too clearly how few candidates understand pdfs. A correct answer is that $T$ is equally likely to take any value in the range, whereas $S$ is more likely to take values towards 0, so $T$ has the greater variance.

The most common wrong answer was that "$T$ is constant so has small variance" (few committed themselves to saying "zero variance"). There is a recurring misconception here. The *input* values ($x$) for the pdf represent the possible values taken by the random variable; the *output* values ($y$, or f($x$)) indicate the likelihood that the random variable takes those values. The default reaction of the lower scoring candidates is that the output values are the possible values taken by the random variable. Some make explicit their misconception that $x$ represents some sort of input parameter, which apparently determines whether or not $S$ or $T$ "occurs" – as if $S$ or $T$ were "events" and not variables representing random numbers. Alternatively, some candidates probably think that "variance" measures variability in probability and not in the values of the random variable.

**6)** This question requiring a hypothesis test for the mean $\mu$ of a normal distribution, based on an unbiased estimate of the population variance, was probably the best done on the entire paper – and of course it was the most similar to questions that have been asked before. Many scored most or all of the marks. However, one error that led to substantial loss of marks was omission of the √50 in standardising.

Previous reports have drawn attention to the serious misunderstanding represented by statements such as $\bar{X} \sim \text{N}(36.68, 56.25 / 50)$. 36.68 is the sample mean and not the population mean; the underlying basis of the test is the assumption that $\mu$ is the hypothesised value (38.4) and not the sample mean. Likewise it is important to get the standardisation the right way round: $\frac{36.68 - 38.4}{\sqrt{56.25 / 50}}$ and not $\frac{38.4 - 36.68}{\sqrt{56.25 / 50}}$.

**7)** This hypothesis test involving a normal approximation to binomial was generally well done, apart from those who used the sample proportion 5/12 instead of the hypothesised proportion 0.35. The plan is to convert from B(120, 0.35) to N(42, 27.3) and either to find a critical value or to find P(≥ 50). Common causes of loss of marks were:

- Omission of the continuity correction (42.5 for the CV, 49.5 for the probability)
- Failure to justify the approximation fully (examiners needed to see $nq$ = 78 if the condition $nq$ > 5 was used, while "$npq$ > 5" is wrong, as is "$n$ large and $np$ > 5")

- Stating the hypotheses in terms of $\mu$ rather than the original parameter $p$
- Attempts to use √120 or √50 in the standardisation.

**8)(i)** This question was a good discriminator. Good candidates knew that they had to start by finding the critical region using B(14, 0.25), and then find the probability of being outside the critical region using B(14, 0.4). Quite a lot of candidates attempted to use a left-hand tail rather than a right-hand tail; the alternative hypothesis $H_1$: $p > 0.25$ made the correct choice unambiguous.

**(ii)** This was quite well done, though some answers were poorly explained. For (b), sensible reasons were that "increasing P(Type I error) leads to decreasing P(Type II error)", or "increasing $\alpha$ increases the critical region" [or "decreases the acceptance region"]. Answers that merely expanded what a Type II error was, such as "the probability of wrongly accepting $H_0$ decreases", did not gain full marks.

**9)(i)** It was a pity that so many candidates did not attempt to see what was wrong with the given statement but merely wrote out learnt phrases. Centres and Candidates are again reminded that if they write out things like "breakdowns must occur randomly in space or time, singly, independently and at a constant rate" they will not gain much, if any, credit. That list, adapted from a standard textbook, is both unhelpful and actively misleading.

The phrase "constant rate" is the essence of this question, and it needs amendment (to "constant *average* rate", or the equivalent) simply because its absence implies that events occur at exactly equal and predictable intervals. Some candidates actually indicated that they thought this exact predictability was required for the Poisson distribution; of course exact predictability negates the whole point of dealing with uncertainly by using probability theory.

**(ii)** Many candidates were able to answer that a Poisson distribution was probably not applicable here because the rate within one day would vary – there are more breakdowns at some times of the day (say, rush hours) than at others. It should be noted that the question talked about "breakdowns *per day*", and so answers that referred to differences between one whole day and another (for instance, weekdays/weekends, or summer/winter) were wrong.

**(iii)** A fair number of candidates stated the correct answers, $B_0 = 13$ and 0.0739. Inequalities were not permitted in the answers.

**(iv)** All but a few candidates were able to start with $e^{-\lambda}\dfrac{\lambda^2}{2} = 0.0072$. Some could not then see how to turn this into $\lambda = 0.12e^{\lambda/2}$, often trying to use logarithms; mistakes were often made in taking the square root of $e^{\lambda}$. Some candidates did not read the question correctly, failing to give answers to 4 decimal places or working out

$0.12e^{k\lambda} - \lambda$ rather than $0.12e^{k\lambda}$ as required. In the final conclusion, "$\lambda$ is between 8.41 and 8.84" is too weak a deduction; "$\lambda$ is nearer 8.5 than 8.6" is too strong.

Some candidates who could not see what to do attempted to solve $8.5 = 0.12e^{8.5k}$ and $8.6 = 0.12e^{8.6k}$, getting $k = 0.5012$ and 0.4967. Some then guessed that $k$ was 0.5, but this was not an acceptable method.

# 4734 Probability & Statistics 3

**General Comments**

There were 391 candidates, the highest in recent years. Overall, the candidates found the paper rather easy. In the hypothesis test questions, there were few over-assertive answers.
There was no evidence of candidates running out of time.

**Comments on Individual Questions**

**1)** Most candidates scored full marks on this question. There were two common errors. Firstly, candidates said that $\sigma = 38 \times 0.9 = 34.2$, instead of $\sigma^2 = 38 \times 0.9^2$. These candidates usually scored three marks. Secondly, some candidates used $n = 37$ or 39 instead of the correct 38. If they made no further error they could score four marks, which most of them did.

**2)(i)** Most candidates scored both marks. Some lost one mark for failing to give their answer as an integer. A significant minority had no idea, $0.096 \times 500 = 48$ and $500 + 250 = 750$ being common incorrect methods.

**(ii)** Most candidates scored full marks. A significant minority scored two, because they used the value of $n$ found in (i) instead of 250. Nearly all candidates used the correct $z$-value, 1.96.

**(iii)** Most candidates scored full marks. Many who had scored two marks in (ii), scored the first method mark for 500/their answers to (ii). As in (i), some candidates lost a mark for failing to give their answer as an integer. Those with no idea in (i) usually failed to score here.

**3)(i)** Most candidates scored three marks for reaching $\frac{a}{2}+\frac{b}{3}=1$. Better candidates realised that $a = b$ and went on to score full marks. Some candidates wasted time by trying to use $E(X) = 1$, which is wrong anyway. Almost all candidates had some idea of how to answer this question.

**(ii)** Most candidates who scored full marks in (i) went on to score full marks here. Many candidates with incorrect $a$ and $b$, or no $a$ and $b$, scored the first two marks. Some gained the first method mark and then made an error. A very small number of candidates scored zero.

**4)(i)** Almost all candidates gained full marks. Those who did not used an incorrect z-value or $\sigma = \frac{1.58}{500}$ or similar.

**(ii)** Most candidates gained the first two marks for $z \times \frac{1.58}{\sqrt{n}} < 0.05$. Sometimes the LHS was multiplied by 2. Those who used $z = 1.96$ went on score full marks, unless the answer was left as an inequality or the final answer was not given as an integer.

**(iii)** Almost all candidates gained this mark for mentioning CLT.

**5)(i)** Most candidates scored three marks for knowing that they needed to show $\int_1^e \ln y \, dy = 1$ and using integration by parts to do so. Most overlooked that they also needed to state that f is non-negative over [1, e]. A few candidates realised that the non-negative property was required, but made no attempt at the integral.

**(ii)** The candidates who gained at least three marks in (i) usually scored all three marks here. Some candidates did not score in this part, even though they knew integration by parts was required again. They either used no limits and no '+ $c$' or incorrect limits.

**(iii)** Most candidates gained both marks by showing F(2.45) < 0.75 < F(2.46). Some candidates did not realise that this is all that was required. Many of these candidates scored a method mark for $Q_3\ln Q_3 - Q_3 + 1 = 0.75$ and then tried to rearrange this. Many gave up, some realised that they needed to use a method similar to the standard solution and a very small minority produced the correct answer 2.455(11..) by using their GC, iteration or Newton-Raphson.

**(iv)** Most candidates gained at least three marks. Many of these lost the first method mark for eg P($\ln y < x$) etc, but they knew that $\mathrm{F}(\mathrm{e}^x)$ was needed. Some did not produce $0 \le x \le 1$.

**6)(i)** Almost all candidates gained full marks.

**(ii)** Almost all candidates scored the first mark for 4.2 < 5, or equivalent. Most candidates did not gain the second mark, choosing to combine Chemistry and Art. The logical choice is to combine the Science subjects. Logic of the context should be preferred to combining the small class with the next smallest.

**(iii)** Most candidates who chose to combine Chemistry and Biology went on to score full marks in this part. Those who chose to combine Chemistry and Art were allowed a maximum of six marks out of eight in this part and most achieved this. Some candidates using both combinations lost a mark for inadequate hypotheses. A small minority did not use Yates correctly. An even smaller minority treated the six cells as separate and combined only eg Male/Chemistry with Male/Art, losing many marks.

**7)(i)** Many candidates produced fully correct solutions. Almost all knew the assumption of equal variances. Some gave inadequate hypotheses. A significant minority did not pool variances and were allowed a maximum of eight marks out of twelve. As before, most achieved this or lost a mark for inadequate hypotheses or not knowing the assumption. There were a few numerical errors, but generally this question was answered well by most candidates.

**(ii)** Many candidates gained full marks, but some candidates who correctly used a pooled variance in (i) decided to use $\frac{2.12}{10} + \frac{2.89}{8}$ here. They lost the method mark, and of course, the final accuracy mark as well. Another error was to use the same CV as in (i).

# 4735 Probability & Statistics 4

**General Comments**

There were 81 candidates, approximately twice as many as usual. In general, the candidates found this paper very easy.
Many candidates lost marks for inadequate hypotheses.
There were few over-assertive responses.
There was no evidence of candidates running out of time.

**Comments on Individual Questions**

**1)(i)** Approximately two-thirds of candidates scored both marks. The others scored zero, usually for $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

**(ii)** Almost all the candidates scored full marks. The main error was to consider $E(S + F)$ instead of $E(SF)$.

**2** Just over half the candidates gained full marks. Of those who did not, most lost a mark for hypotheses which did not refer to the population or parameter.

**3)(i)** Almost all the candidates knew that a single exponential term was required and attempted integration by parts. Some candidates lost marks for sign errors and missing denominators. Most candidates gave a satisfactory reason for the condition $t < ½$, though it should be noted that the validity of the series expansion of $(1 – 2t)^{-2}$ is less relevant than the convergence of the integral.

**(ii)** Almost all the candidates gained full marks. Those who did lose marks made errors in differentiation.

**4)(i)** Only one-third of the candidates gained this mark. Most candidates wrote 'groups independent', or equivalent.

**(ii)** Most candidates scored at least seven out of nine. Marks were lost for inadequate hypotheses, use of $\frac{1}{2}$ instead of $\frac{1}{12}$ in the formula for variance and use of 0.05 instead of 0.025.

**5)(i)** Almost all the candidates gained at least two marks. The candidates who lost a mark did so for not factorising the power series.

**(ii)** Almost all the candidates gained full marks.

**(iii)** Almost all the candidates knew that the distribution was binomial with $n = 4$, but $p$ was sometimes wrong.

**(iv)** Most candidates did not answer this question well. Use of even powers of $t$ was a common mistake.

**6)(i)** Almost all the candidates scored full marks.

**(ii)** Almost all the candidates scored full marks.

**(iii)** Almost all candidates gained marks on this question, but there were many errors in the coefficients of $\sigma^2$.

**(iv)** Approximately half the candidates gained full marks. A few made errors in using $E(X^2) = Var(X) + [E(X)]^2$, but most did not realise this formula was necessary.

**7(i)** Almost all the candidates scored full marks.

**(ii)** Almost all the candidates scored full marks.

**(iii)** Over half the candidates gained full marks, but there were some very confused attempts, muddling the various methods given on the mark scheme. Many obtained an incorrect value of $p$, using the answer to (ii) instead of (i).

# Overview – Decision Mathematics

Two common themes emerge from OCR's Decision Maths papers 4736 and 4737. They apply also across MEI's suite, 4771, 4772 and 4773.

The first is that most candidates are well prepared for the fundamental requirements of the papers. The techniques of Decision Maths are algorithms, and it is clear that candidates are practised in dealing with the relevant algorithms for each paper. However, the rationale for Decision Maths is not the memorising and application of algorithms, it is rather to develop skills in mathematical modelling.
Arguably two of the most important underlying skills of the mathematical modeller are the abilities both to reason and to communicate that reasoning. This communication is important for its own sake, for unless one can communicate clearly, even to oneself, then one cannot be reasoning clearly. Clear communication and clear thinking are symbiotic.

But the second common theme, seen across all units of both specifications, is that of candidates showing poor communication skills, for example "*Many candidates did not clearly show which values had been compared and whether or not they had been swapped.*" (PE report on 4736) and "*Candidates usually had the right idea about why the flow in FC could not exceed 15 litres per second, but did not give a tight enough explanation, often assuming that because they could see why the result was true from the diagram they did not need to explain it.*" (PE report on 4737). If candidates are going to succeed at more than a very basic level in Decision Maths, then they must be able to communicate well. Indeed, if they are to gain benefit from studying Decision Maths, then an improvement in their communication skills must surely be one of the fundamental observable outputs.
It can be more difficult for candidates working in isolation to develop such skills. If candidates can work in pairs or groups, writing explanations, criticising descriptions, etc, then progress is much easier to achieve.

It might be argued that to put so much stress on communication is not commensurate with mark weightings, but the argument is that by stressing and practising these skills, mathematical performance will increase across all aspects of the papers.

# 4736 Decision Mathematics 1

**General Comments**

Most candidates were able to attempt every question and they were generally able to fit their answers into the spaces provided in the answer book.
Those candidates who used extension sheets usually labelled their answers to show which part of which question the work referred to. Some candidates used additional sheets for rough working, it would have been helpful if they had labelled this work with the question and part number and with the word ‘rough’ or ‘working’ or ‘scrap’. Similarly, when candidates make more than one attempt at an answer, they should indicate which one they want to have marked. Where there is more than one answer, examiners are instructed to mark the last complete attempt.

On some questions candidates lost marks for not showing their working and on others marks were lost through not answering all of the requests.

Centres should make sure that candidates know that work that has been overwritten is rarely able to be read properly. The scanning is set at a high level for Maths papers, so even when work has been done in pencil and then erased it can still show through. Highlighter pen does not usually scan, although sometimes it can show as a grey block that obscures whatever was written beneath it. Also, apart from graph paper, the scanning is done in black and white, so when candidates use colour the examiners cannot tell what they have done.

**Comments on Individual Questions**

**1)(i)** Most candidates were able to carry out bubble sort, although some sorted into decreasing order. Many candidates did not clearly show which values had been compared and whether or not they had been swapped.

Some students wrote out a separate list for each comparison and underlined the values that had been compared, others gave just the list that resulted at the end of the pass but wrote down which values had been compared (in order) and whether or not they had been swapped. Using crosses on a single list to show swaps did not usually convey the passage of 57 to the end of the list so it was ambiguous exactly which elements had been compared.

There were the usual issues with candidates misreading, miscopying or omitting values from the list.

**(ii)** This part was generally answered well, apart from when candidates were sorting into decreasing order or when they did not understand what constitutes a pass in bubble sort.

**(iii)** There were many fully correct responses to this part. The question had told candidates that there were five passes and had asked for the number of swaps in each, so candidates did not get any credit if they only gave four numbers, only gave the number of swaps for the final two passes or only gave the total number of swaps.

Several candidates thought that they had spotted a pattern and claimed either 5,4,3,2,1 or 4,3,2,1,0. In fact the final pass involved 1 swap and the required numbers were 4,3,2,1,1

**2)(i)(a)** Many correct graphs were seen. Most candidates gave a connected graph with four vertices and five arcs but some had graphs with two odd vertices.

**(i)(b)** Some candidates gave concise and convincing explanations, showing why a connected Eulerian graph with four vertices and five arcs cannot be simple. Others explained why a simply connected graph with four vertices and five arcs cannot be Eulerian or why a simply connected Eulerian graph with four vertices cannot have five arcs.

Several candidates assumed that they should start from a simply connected Eulerian graph with four vertices and four arcs and explain why a fifth arc could not be added, without considering the possibility of having a simply connected semi-Eulerian graph with four vertices and four arcs and explaining why adding a fifth arc to make it Eulerian will also make it non-simple. Many of the explanations dealt only with one specific case and were not general arguments.

Several candidates copied out the definition of simply connected and stated that an Eulerian graph has all even order vertices, but did no more than this. Some candidates claimed that it was impossible to have an Eulerian graph because there was an odd number of arcs or that it was impossible to have a simply connected graph because there were more arcs than vertices. Neither of these reasons are true in general.

**(ii)(a)** The relationship between the number of arcs and the sum of the vertex orders was lost on many candidates, so although they were able to offer $10 \times 2 = 20$ here they were not able to use this in parts (ii)(b) and (c)

**(ii)(b)** Most candidates who had understood what was being asked were able to state 2 as the minimum vertex order. One or two candidates claimed 1 or 0. Fewer were able to identify the maximum vertex order as 6, those who did correctly identify 2 and 6 were usually able to draw a correct graph.

Incorrect graphs usually had too many arcs, had some odd vertices or were not simple. It was evident in this part that some candidates had started by trying to draw the graph without thinking about the possible vertex orders.

The only way to make a total of 20 from eight positive even numbers is to have seven 2's and a 6 or six 2's and two 4's.

Some answers were so messy that it was impossible to deduce which arcs candidates intended to have included and which they had erased. There was enough space for candidates to draw a tidy graph alongside their first attempt, although they would have needed to label it as being their answer.

**(ii)(c)** Many good responses, although again some candidates drew graphs that had odd vertices, were not simple or had too many vertices of order 4. Some candidates seemed to be trying to draw a simply connected graph with eight vertices each of order 4.

**3)(i)** Most candidates were able to gain some marks on this question, with many fully correct responses. Some candidates assumed that whenever two values were subtracted they should always take the smaller from the larger, or made arithmetic slips. Quite a few candidates did not record their output values for $F$ and $M$. The mark scheme accommodated individual errors with only minor penalties.

Some candidates wasted time by writing the value of every variable in each row of the table, this incurred no loss of marks provided the values were correct. At the other extreme, a few candidates tried to show all their working in a single row in the table, or showed multiple steps in a single row in the table, in these cases it was usually not possible to identify which sets of values went together and some marks were lost.

Many candidates used both the table in the answer book and the spare table, without indicating which one they wanted marked as their final answer.

**(ii)** Most candidates were able to complete the first three steps in the table to reach $D = 30$ and $E = 18$. Although some candidates realised that from this point on the algorithm was essentially the same as in part (i), apart from the calculation of $G$ at the end, a more common response was to claim, incorrectly, that the algorithm would not terminate.

**(iii)** A number of correct responses were seen. Some candidates assumed that the output would be the same as in part (i) and a few possibly realised that $F$ was a common factor of $A$ and $B$ but gave $F = 2$, $M = 48$ instead of $F = 4$, $M = 24$.

**4)(i)** Candidates were required to apply Kruskal's algorithm to find a minimum spanning tree, they had been provided with a list of arcs sorted by increasing arc weight which they could use to show which arcs they had chosen and which they had rejected.

Most candidates found the correct tree and gave its weight. Some candidates then added extra arcs to their tree, in finding the route of weight 82 km, and in doing so prevented their tree from being seen.

A number of candidates did not list a suitable route, either because their route was not of the required length or because they gave it in two parts (such as 'tree + *GFBA*') instead of writing out the complete route.

**(ii)** Some candidates misunderstood what was being asked for here, or had not read the question carefully enough. The first part asked for a minimum spanning tree on a reduced network and then the second part asked candidates to use this answer to find a route of length 81 km that passed through each of the seven farmhouses once.

Some candidates gave the weight of the minimum spanning tree from part (i) instead of the minimum spanning tree on the reduced network, and others gave the weight of the minimum spanning tree on the reduced network but then wrote down a cycle on the reduced network as well.

A few candidates thought that this part was asking them to find a lower bound for tsp, but in this case it went further than that and asked them to find an achievable travelling salesperson route.

**(iii)** Many fully correct responses. Some candidates did not answer all three requests. The question asked candidates to show that the nearest neighbour method could not be used to create a cycle through all the vertices when started from $A$, but that it succeeded when started from $B$, candidates were then asked to adapt this cycle to create a cycle through all the vertices of length less than 70 km.

Some candidates seemed to think that at each vertex they should choose the nearest vertex, with the possible exception of the one they had just come from, whereas nearest neighbour says to choose the nearest vertex that has not yet been visited. When starting from $A$ nearest neighbour forms a route through every vertex but there is no way to close the cycle without repeating vertices, when started from $B$ the route ends at $G$ and the arc $GB$ can be used to close the cycle without revisiting any vertices. Swapping $F$ and $G$ then gave a cycle through the vertices that had weight 69 km.

Some candidates did not make it apparent which of the requests they were answering, others helpfully wrote, for example, 'From $A$', 'From $B$' and 'Improved route'.

**5)(i)** Most candidates are able to apply Dijkstra's algorithm, apart from the odd arithmetic slip. Some candidates overlooked the arc $AF$ and some omitted to write down the shortest route. A few candidates seemed to have found the shortest route 'by inspection' and then attempted to use it to construct the labels, and some candidates only gave the values of the permanent labels. A significant minority gave all the temporary values, instead of only recording them when they are an improvement on the value that is currently recorded. Candidates do not need to make a note of how temporary labels were reached, the route should be found by tracing back through the permanent labels at the end (although candidates would not normally be expected to show any working for this).

**(ii)** The calculation of the run time for a quadratic order algorithm had the usual misunderstandings, either squaring the time or squaring the measure of the size of the problem for only one of the problems.

The vast majority of the candidates were able to calculate the approximate run time as being 90000 seconds, although some left it at that and some thought that dividing this by 60 was enough to convert it to hours.

One candidate commented that a computer would not take 25 hours to solve a problem, this is the whole point about why we need to use efficient algorithms with large scale problems.

**(iii)** A number of correct responses were seen, but too many answers stopped short of finding how much shorter the path needed to be, without specifying that what they had calculated was (presumably) the new length of *CE*. An answer of 5 on its own was not enough, but *CE* = 5 gained partial credit. Candidates need to give some indication of what their calculated values represent.

Many candidates assumed that the new path needed to be a unique shortest path ('less than' the current shortest length, rather than 'less than or equal to'), this combined with an assumption that all values had to be integers led to many candidates claiming an answer of 4 (either as the new length of *CE* or as the amount by which it needed to be reduced).

**(iv)** Most candidates understood what was required here, with a number of fully correct responses. Some candidates did not allow for the blocked arcs, and so thought that *B*, *C*, *E*, *F* were the odd nodes, rather than *A*, *B*, *C* and *E*. Other candidates claimed that there were only two odd nodes (or sometimes three!)

Most candidates who identified the odd nodes correctly were able to calculate the shortest weight for at least one pairing, although some candidates used a blocked arc or made arithmetic errors.

Quite a few candidates forgot to reduce the total weight to remove the blocked arcs, ending up with 224 + 32 = 256 instead of 205 + 32 = 237. Some did not use the total given in the question, usually resulting in a wrong value for the total weight. A few candidates did not use the route inspection method and just wrote down what they believed to be the shortest route, with no evidence that it could not be improved.

**6)(i)** Some candidates could not draw the lines correctly, and several did not get all three lines, even with a generous tolerance on plotting. The intersections with the axes were sometimes dramatically wrong and some candidates assumed that all three lines would have a positive gradient. The lines should be drawn using a sharp pencil and a ruler.

Even when candidates had the three lines correct, they often assumed that the feasible region would be one of the triangular regions.

Some candidates used the spare copy of the graph, but did not indicate which attempt was the one they wanted marked, and some crossed out their work with a big cross that ended up looking like extra lines. A few candidates drew profit lines on their graphs, if these are used they should be labelled and drawn well away from the feasible region.

Candidates should be aware that if they draw lines and then erase them the original lines may still show through on the scanned answer. Any lines that are not to be marked should be indicated as such.

**(ii)** Candidates who had identified the feasible region correctly usually answered this part well. Other candidates were able to write down one or more of the coordinates of the region they had identified, although often this resulted in too few 'non-trivial' profit calculations (in the sense of having non-zero values for both $x$ and $y$). Some candidates managed to claim coordinates that were well off the graph (often by making errors with simultaneous equations), or had at least one of $x$ and $y$ negative.

Some candidates went back to the equations and solved them simultaneously to find the three points where pairs of lines intersected. This usually led to them then considering the feasible region to be the triangle with these three points as its vertices, resulting in them claiming that (8, 6) was the optimal point.

**(iii)** Some candidates assumed that they had to hunt for integer solutions near their optimal vertex of (4, 4.5), or if they had given (8, 6) claimed that they had already answered this part. Many candidates listed all the integer-valued coordinates in the feasible region, instead of giving the coordinates where $y$ had its maximum integer value for each $x$, or only considered some of the feasible values for $x$.

**(iv)** There was little understanding of what the problem was with the third constraint, and some candidates tried to explain why they thought the new version of the third constraint was wrong. Many responses were ambiguous as to whether they were talking about constraints, the objective, the coefficients or the values on the RHS. The word 'positive' was often used when 'non-negative' was meant, this was particularly an issue when candidates were talking about the values that could appear on the RHS of the inequalities.

Some candidates explained that the inequality sign for the third constraint needed to become 'less than or equal to' or talked about needing to be able to add (non-negative) slack. A worrying number of answers appeared to be saying that the coefficient of $x$ should not be negative.

**(v)** Irrespective of what they had written in part (iv), most candidates were able to give a correct initial tableau. Only a small minority omitted the columns corresponding to the slack variables, and most candidates had the signs of the coefficients in the objective row correct (ie had the objective row corresponding to the equation $P - 5x - 8y = 0$).

**(vi)** Most candidates achieved some marks on this final part. Some did not indicate their pivot operations or did not record the interim solution. A few candidates chose a pivot from the wrong column, which gained partial credit provided what they were doing was valid, others tried to pivot on a negative value.

Where candidates attempted the second iteration, fully correct responses were often seen. A few candidates made small numerical errors, these did not penalise the candidate unless they resulted in the final values (including slack variables) being wrong. There was some evidence of candidates realising that the solution would be the same as their answer to part (ii) and trying to produce a tableau to give this, however these attempts rarely showed that there was a slack variable with value 9 and the candidates often ended up with gaps in their tableau.

# 4737 Decision Mathematics 2

**General Comments**

Most candidates were able to attempt every question and they were generally able to fit their answers into the spaces provided in the answer book.
Those candidates who used extension sheets usually labelled their answers to show which part of which question the work referred to. Some candidates used additional sheets for rough working, it would have been helpful if they had labelled this work with the question and part number and with the word 'rough' or 'working' or 'scrap'. Similarly, when candidates make more than one attempt at an answer, they should indicate which one they want to have marked.
The quality of written answers was usually good, although many candidates were not specific enough in their responses to questions where they were asked for explanations.

Centres should make sure that candidates know that work that has been overwritten is rarely able to be read properly. The scanning is set at a high level for Maths papers, so even when work has been done in pencil and then erased it can still show through. Highlighter pen does not usually scan, although sometimes it can show as a grey block that obscures whatever was written beneath it. Also, apart from graph paper, the scanning is done in black and white, so when candidates use colour the examiners cannot tell what they have done.

**Comments on Individual Questions**

**1)(i)** The bipartite graph was usually correct. A few candidates gave an arc joining *F* to 2, 5 or 6.

**(ii)** Most candidates drew their incomplete matching correctly, although some then drew over it with their alternating path and some candidates wasted time redrawing the original bipartite graph first (which often meant that their incomplete matching could not be seen, for example if they had used coloured pens). A few candidates did not show the arcs from *A*, *B* and *C*.

Nearly all candidates identified that Fred was not happy with this arrangement.

**(iii)** Many candidates found the alternating path and gave the corresponding complete matching. Some candidates who had made earlier slips were able to recover marks here.

The alternating path and the complete matching needed to be written down, not drawn on a diagram.

**2) (i)** Most candidates knew that they needed to subtract every value from a constant (10 was the usual choice) to convert to a minimisation problem. Some candidates said 'subtract by 10', but then gave an example to show what they meant.

**(ii)** Inevitably there were some numerical slips. Because the final reduced cost matrix had been given in the question, candidates needed to show how they had reduced the rows and columns, not just say that they had. In general, this is a good thing to do anyway, for example by writing the amounts by which each row or column is to be reduced on the matrix preceding the one where the reduction is made (or on the matrix where the reduction is made, but not a mixture of the two methods).

**(iii)** Several candidates used five lines to cross through the 0's, it could be done with four lines. Apart from that, the augmentations were done well with most candidates knowing which values needed to be decreased and which needed to be increased, and by how much. There were some arithmetic slips, or a value or two that were not changed when they should have been.

A number of candidates assumed that only one augmentation was necessary and then changed one of the entries to make part (iv) work, only a few candidates did more than two augmentations.

Candidates sometimes did the augmentation but did not write down on which day Fred cooks.

**(iv)** Some candidates did not read the stem and so did not have Alice cooking on Sunday, and some had a correct matching but did not include Alice and/or Fred in their final list. The candidates who had stopped after one augmentation in part (iii) did not seem to realise that as they could not find a complete matching their job in part (iii) was only half done.

**3)(i)** Many good diagrams were seen, although sometimes candidates did not direct their arcs and several used more dummy activities than they needed.

**(ii)** Most candidates were able to make the forwards and backwards passes to find the early and late event times. Those who had problems had usually failed to take account of the dummy activities. It was not unusual to have the late event time for the event at the start of activities *D* and *G* given as 25 instead of 30.

**(iii)** Candidates usually had the correct minimum completion time. Some candidates claimed that *A* and/or *G* were also critical activities, usually when they had given the late event time at the start of *D* and *G* as 25.

Some candidates answered this part with parts (i) and (ii) and then had their answers in the wrong spaces for the next few parts. If this happens candidates should change the numbering in the margin or write a short message saying ‘this is the answer to (iii)’, for example.

**(iv)** Mostly done well, apart from candidates who had Molly doing *A* first. Some candidates coloured in the blocks, as if they were drawing a resource histogram, and then the activities could not be determined. The easiest way to show a schedule is to write the activity letter in each cell where it is happening, although the use of blocks (with bold outlines and the activity letter contained within the box) is also acceptable.

Some candidates had more than one attempt at this part, in this case they needed to indicate which attempt was the one they wanted to have marked.

**(v)** Also done well, although quite a few candidates gave *A* and/or *F* to the friend, often because they had assumed that Molly would have to complete all the critical activities herself. Some candidates split activities between Molly and her friend, this cannot be done unless the question specifically says that an activity can be shared.

**4)(i)** Almost all the candidates were able to say how they knew that *F* is the source and were able to state that *J* is the sink.

**(ii)** Although most candidates gave the value as 30, there were quite a few candidates who gave the value of 8 + 10 + 12 as 20. This seemed to mostly be due to an arithmetic error, although some candidates had changed a correct answer of 30 into 20 possibly as a response to what happens later in the question.

**(iii)** Most candidates realised that *BA* could not carry its full capacity of 6 litres per second because the most that could flow through *A* was 5 litres per second (*AD* being the only possible route for flow from *BA* and having a capacity of 5 litres per second).

Candidates usually had the right idea about why the flow in *FC* could not exceed 15 litres per second, but did not give a tight enough explanation, often assuming that because they could see why the result was true from the diagram they did not need to explain it. A complete answer should make sense even if the diagram is removed. Usually candidates were able to explain why the maximum flow out of *B* was 15 litres per second, but they did not explain that flow through *FC* had to flow along *CB* (because there were no other arcs into or out of *C*) and hence that the maximum flow in *FC* was also 15 litres per second.

**(iv)** Many correct solutions.

Candidates usually showed a flow in which there was no flow along *ED*, *FE* and *IH*, but some tried to also have each of *FC*, *IL* and *KJ* at their upper capacities rather than having the maximum possible feasible flow through them, despite explaining in part (iii) that the flow in *FC* could not exceed 15 litres per second.

Some candidates just wrote the upper capacities, which did not make a flow, feasible or otherwise. Other candidates wrote values that would have been a flow with the required conditions if it had been feasible.

Some candidates gave a feasible flow but it only had 12 litres per second flowing through *KJ* although 13 litres per second was possible.

**(v)** Several candidates had the excess capacities and potential backflows reversed, on some or all the arcs. Excess capacities should show how much more could flow along the arc (and are labelled in the direction of the flow), potential backflows show how much less could flow along the arc (and are labelled in the direction opposing the flow). When the arcs have upper capacities only, the potential backflow shows the actual flow but in the wrong direction.

Many candidates assumed that the required cut would be that from part (ii), sometimes changing their answer to part (ii) to fit the maximum flow of 20 litres per second that they had found here.

**5)(i)** Candidates needed to say both what was being maximised and what was being minimised, as well as making it evident that they were talking about a route and not a single arc weight.

First all routes from the start to the finish are considered and the minimum arc weight for each route is recorded, then the route for which its minimum value is greatest is chosen. The dynamic programming method enables the maximin route to be found without explicitly carrying out these two stages.

**(ii)** Inevitably it was the action column that caused the most problems. The action value should correspond to the state label for the state that is being moved into (in the next stage, ie from the rows further up the table). So, starting from (4;0), the first stage that needs to be recorded is stage 3 with states 0, 1 and 2, each with action 0 (showing that the transition is ‘into (4; 0)’. Similarly, stage 2 will have states 0, 1 and 2; with state 0 having actions 0 and 1 - transitions from (2; 0) to (3; 0) and from (2; 0) to (3; 1); state 1 having actions 0 and 2 – transitions from (2; 1) to (3; 0) and from (2; 1) to (3; 2); and state 2 having actions 1 and 2. Stage 1 will have states 0, 1 and 2; with state 0 having actions 0, 1 and 2; state 1 having actions 0 and 2; and state 2 having actions 1 and 2. Finally, stage 0 has state 0 which has actions 0, 1 and 2.

A few candidates solved the maximum weight route problem instead of the maximin problem.

Some candidates did not give the weight that could be carried or list both routes that could be used.

**(iii)** Most candidates realised that the maximum weight would now be 18 tonnes, although a few thought that it was still 17 tonnes or that it would increase to 20 tonnes.

The reasons given were often confused or incomplete. The change could only affect routes that included the arc connecting (2; 0) to (3; 1), so explanations did not need to include consideration of routes through (3; 0) or (3; 2). This could be done by considering the two routes that use (2; 0) to (3; 1) by looking at what happens between (0; 0) and (2; 0), or by looking at what happens between (3; 1) and (4; 0), or by recognising that 18 is the maximum weight on any arc from stage 1.

**6)(i)** Some candidates did not read the question carefully enough and assumed that the table already represented a zero-sum game, resulting in an answer of –2.

**(ii)** Mostly done correctly, some candidates felt the need to use additional sheets for the interim step in the calculation.

**(iii)** Candidates were asked to show their working, which required showing the row minima and the column maxima and then identifying the strategy that resulted in the largest row minimum value (the row maximin) and the strategy that resulted in the smallest column minimum value (the column minimax). Some candidates showed their working but wrote the values of the play-safe strategies instead of the strategies themselves. Several candidates did not answer all the requests in this part, often omitting to explain how they knew that the game was not stable.

**(iv)** Most candidates showed that they had compared appropriate values and several stated that the outcome for Greg was better than the outcome for Iona in each column. A few candidates just said that Greg's row dominated Iona's without showing why and some tried to involve Hakkim as well.

**(v)** This required candidates to use the original table, with the row for Iona removed. A few of the candidates who had used the zero-sum table realised why they had a different expression for Jeff to the one given and were able to revert their expressions for Kathy and Leo by reversing the coding.

**(vi)** Some candidates did not use the scale given on the axes, the scale had been given to enable the relevant intersections to be able to be read from the graph. Some candidates assumed that the scale on the horizontal axis should be 1 square = 1 unit, but this would have allowed probability values from 0 to 5. Most candidates whose expressions were correct were able to use either the graph or simultaneous equations to find the optimal value of $p$, they usually also recorded the expected number of legs won.

**(vii)** Several candidates were able to give a specific example in which one team won at most 3 rounds but at least 18 legs.

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