

**Mathematics**

Advanced GCE

Unit **4724**: Core Mathematics 4

**Mark Scheme for January 2013**

---

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

© OCR 2013

## Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given

**Subject-specific Marking Instructions for GCE Mathematics Pure strand**

- a. Annotations should be used whenever appropriate during your marking.

**The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks.** It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

- b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c. The following types of marks are available.

**M**

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

**A**

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

**B**

Mark for a correct result or statement independent of Method marks.

**E**

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep \*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

- g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

- h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question	Answer	Marks	Guidance	
1	$u = x$ and $dv = \cos 3x$  $x \times \frac{1}{3} \sin 3x - \int \frac{1}{3} \sin 3x dx$  $\frac{x}{3} \sin 3x + \frac{1}{9} \cos 3x [+c]$ cao www ISW	M1  A2  A1  <b>[4]</b>	integration by parts as far as $f(x) \pm \int g(x) dx$  <b>A1</b> for $x \times k \sin 3x - \int k \sin 3x dx$ ; $k \neq \frac{1}{3}$ or 0  Not ..... $\frac{1}{3} \left( \frac{1}{3} \cos 3x \right)$ or ..... $-\frac{1}{9} \cos 3x$	Check if labelled $v, du$  $k$ may be negative
2	<u>The first 3 marks refer to the expansion...</u>  First 2 terms = $1 - \frac{8}{3}x$  $3^{\text{rd}}$ term = $\frac{\frac{3}{2} \cdot \frac{1}{2}}{1.2} \left( -\frac{16x}{9} \right)^2$  $= \frac{32}{27}x^2$  Complete expansion $\approx 27 - 72x + 32x^2$  valid for $\frac{-9}{16} < x < \frac{9}{16}$ or $ x  < \frac{9}{16}$	.....  B1  M1  A1  A1  B1  <b>[5]</b>	<u>of <math>\left(1 - \frac{16x}{9}\right)^{\frac{3}{2}}</math> and to no other expansion</u>  Allow any equiv fraction for the $-\frac{8}{3}$ and ISW  Allow clear evidence of intention, e.g. $\frac{\frac{3}{2} \cdot \frac{1}{2} - 16x^2}{1.2 \cdot 9}$  Allow any equiv fraction for the $\frac{32}{27}$ and ISW  cao No equivalents. Ignore any further terms  oe Beware, e.g. $x < \left  \frac{9}{16} \right $	$\frac{3}{2} \cdot -\frac{16}{9}$ is not an equiv fraction       If expansion $(a+b)^n$ used, award B1, B1, B1 for $27, -72x, 32x^2$  condone $\leq$ instead of $<$

Question	Answer	Marks	Guidance	
3	For attempt at product rule on $xy^2$ $\frac{d}{dx}(y^2) = 2y \frac{dy}{dx}$ $\frac{dy}{dx} = \frac{2x - y^2}{2xy} \text{ or } \frac{1 - x^{-2}}{2y}$ Stationary point $\rightarrow$ (their) $\frac{dy}{dx} = 0$ soi  $x^2 = 1$ or $y^2 = 2$ or $y^4 = 4$  $(1, \sqrt{2}), (1, -\sqrt{2})$	M1 B1  A1  M1  A1  A1,A1  <b>[7]</b>	or changing equation to $y^2 = x + x^{-1}$ soi in the differentiating process  Award <u>B1</u> for $(\pm)\frac{1}{2}(x + x^{-1})^{-\frac{1}{2}}(1 - x^{-2})$  Ignore any other values  Accept 1.41 or $4^{\frac{1}{4}}$ for $\sqrt{2}$	SR. Award <b>A1</b> only if extra co-ordinates presented with both correct answers
4 (i)	Produce (at least 2) relevant equations Eliminate either $\lambda$ or $\mu$ from 2 of them and solve for the other ( $\mu$ or $\lambda$ ) $\lambda = 2$ and $\mu = -1$ cao Check that $(\lambda, \mu) = (2, -1)$ satisfies all eqns  P is (5, 4, 6) cao www	M1 M1  A1  B1  A1  <b>[5]</b>	e.g. $1 + 2\lambda = 6 + \mu, 2 + \lambda = 8 + 4\mu, 3\lambda = 1 - 5\mu$ soi by correct $(\lambda, \mu)$  or e.g. $\lambda = 2$ from 2 different pairs <u>This must be convincing.</u> Check unusual arguments  Allow any reasonable vector notation	Dep previous M1M1A1 earned
4 (ii)	Using $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 4 \\ -5 \end{pmatrix}$  Using $\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{ \mathbf{a}   \mathbf{b} }$ giving value $\frac{n}{\sqrt{a}\sqrt{b}}$ 68.2°... (not 111.8...)	M1  M1  A1  <b>[3]</b>	i.e. correct parts for direction vectors   for any 2 meaningful vectors in this question using meaningful scalar product & modulus or 1.19 (radians)	Expect $\frac{-9}{\sqrt{14}\sqrt{42}}$



Question		Answer	Marks	Guidance
5	(i)	<p>their <math>\frac{dy}{d\theta} / \frac{dx}{d\theta}</math></p> <p><math>\frac{dy}{dx} = \frac{2 \sin \theta}{3 \cos \theta}</math></p> <p>their <math>\frac{dy}{dx} = \frac{1}{2}</math></p> <p><math>\tan \theta = \frac{3}{4}</math></p> <p><math>(3.8, -0.6)</math> or <math>\left(\frac{19}{5}, -\frac{3}{5}\right)</math> or <math>x = 3.8, y = -0.6</math></p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[5]</p>	<p>If <math>\tan \theta = \frac{3}{4}</math> not seen, award this A1 only if coords are correct</p>
5	(ii)	<p>Manipulating equations into form <math>\sin \theta = f(x)</math> and <math>\cos \theta = g(y)</math> and then using <math>\sin^2 \theta + \cos^2 \theta = 1</math></p> <p><math>\frac{(x-2)^2}{9} + \frac{(1-y)^2}{4} = 1</math> oe www ISW</p> <p>Accept e.g. <math>\left(\frac{x-2}{3}\right)^2</math></p> <p><math>4x^2 + 9y^2 - 16x - 18y - 11 = 0</math></p>	<p>M1</p> <p>A1</p> <p>[2]</p>	<p>If part (ii) is attempted first, and then part (i), allow</p> <p>B1 for obtaining <math>\frac{dy}{dx} = \frac{4(x-2)}{9(y-1)}</math></p> <p>M1 for equating their <math>\frac{dy}{dx}</math> to <math>\frac{1}{2}</math></p> <p>A1 for obtaining <math>9y - 8x = -7</math></p> <p>M1 for eliminating <math>x</math> or <math>y</math> from above eqn...</p> <p>A1 for <math>(3.8, -0.6)</math></p> <p>the following marks in part (i):-</p> <p>....and their Cartesian equation</p>

Question	Answer	Marks	Guidance
6	Attempt diff to connect $du$ & $dx$ Correct result e.g. $\frac{du}{dx} = 2$ or $du = 2 dx$  Indef integ in terms of $u = \frac{1}{2} \int \frac{2u-3}{u^5} (du)$  Integrate to $\frac{u^{-3}}{-3} - \frac{3u^{-4}}{-8}$ oe  Use correct variable & correct values for limits $= \frac{-23}{384}$ oe ( $-0.059895 \dots$ )  [ISW, e.g. changing to $\frac{23}{384}$ ]	M1 A1  A1  A1A1  M1 A1  <b>[7]</b>	or find $\frac{du}{dx}$ or $\frac{dx}{du}$  Must be completely in terms of $u$ . or (using 'by parts') $\frac{(2u-3)u^{-4}}{-8} - \frac{u^{-3}}{12}$  Provided minimal attempt at $\int f(u)du$ made Accept decimal answer only if minimum of first 3 marks scored

Award B1,B1 for  $\frac{4u^{-3}}{-3} - \frac{3u^{-4}}{-2}$   
 or for  $\frac{2u^{-3}}{-3} - \frac{3u^{-4}}{-4}$   
 or for  $\frac{(2u-3)u^{-4}}{-2} - \frac{u^{-3}}{3}$   
 or for  $\frac{(2u-3)u^{-4}}{-4} - \frac{u^{-3}}{6}$

Question		Answer	Marks	Guidance	
7	(i)	<p><b>I</b></p> $\frac{\cos x}{1 + \sin x} - \frac{-\sin x}{\cos x} \text{ or } \frac{\cos x}{1 + \sin x} + \frac{\sin x}{\cos x}$ $\frac{+/- \cos^2 x + +/- \sin x(1 + \sin x)}{(1 + \sin x)\cos x}$ $\frac{1 + \sin x}{\cos x(1 + \sin x)} = \frac{1}{\cos x} \quad \underline{\text{www}} \quad \text{AG}$ <p><b>II</b></p> <p>Change to <math>\ln\left(\frac{1 + \sin x}{\cos x}\right)</math></p> <p>Change to <math>\ln(\sec x + \tan x)</math></p> <p>Diff as <math>\frac{\text{attempt at } \frac{d}{dx}(\sec x + \tan x)}{\sec x + \tan x}</math></p> <p>Reduce to <math>\sec x = \frac{1}{\cos x}</math></p> <p><b>III</b></p> <p>Change to <math>\ln\left(\frac{1 + \sin x}{\cos x}\right)</math></p> <p>Diff as <math>\frac{\text{attempt at quotient differentiation}}{\frac{1 + \sin x}{\cos x}}</math></p> <p>Fully correct differentiation</p> <p>Correct reduction to <math>\frac{1}{\cos x}</math></p>	<p>B2</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[4]</p>	<p>Each half (including 'middle' sign) scores B1</p> <p>Combine, <u>provided</u> derivative was of form <math>f'(x)/f(x)</math></p> <p><math>\cos^2 x + \sin^2 x = 1</math> in intermediate step required</p> <p><u>Not</u> <math>\ln\left(\frac{1}{\cos x} + \tan x\right)</math></p>	<p>Allow only variations num signs</p>
7	(ii)	<p>Indef integral = <math>\ln(1 + \sin x) - \ln(\cos x)</math></p> <p>[Method I]</p> <p>Substitute limits &amp; use log manipulation</p> <p>Answer = <math>\ln(2 + \sqrt{3})</math></p>	<p>B1</p> <p>M1</p> <p>B1</p> <p>[3]</p>	<p>or <math>\ln(\sec x + \tan x)</math> [Method II]</p> <p>Use of <math>\ln A - \ln B = \ln \frac{A}{B}</math> anywhere in question</p> <p>Accept <math>\ln 3.73</math> or <math>\ln \frac{2 + \sqrt{3}}{1}</math> .....but not <math>\ln \frac{1 + \sqrt{3}/2}{1/2}</math></p> <p>Answer has <u>not</u> been given</p>	

Question		Answer	Marks	Guidance	
8	(i)	$AB = \sqrt{(+/-2)^2 + (+/-2^2 + (+/-4)^2)}$ $AD = \sqrt{(+/-2)^2 + (+/-4)^2 + (+/-2)^2}$	B1 B1 <b>[2]</b>	oe oe	If $AB^2 = AD^2 = 24$ , then SR B1 $AB = AD$ to be stated for 2 <sup>nd</sup> B1
8	(ii)	midpoint is (3, 5, 0)  Clear method for finding direction vector  $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(3\mathbf{j} - \mathbf{k})$ oe or e.g. $\mathbf{r} = 3\mathbf{i} + 5\mathbf{j} + \mu(-3\mathbf{j} + \mathbf{k})$ cao	B1 M1 A1 <b>[3]</b>	Accept any reasonable vector notation.  Expect $3\mathbf{j} - \mathbf{k}$ or $-3\mathbf{j} + \mathbf{k}$  “ $\mathbf{r} =$ ” is essential. No f.t. for wrong mid-point.	
8	(iii)	substitution of $\lambda = +/-5$ or $\mu = +/-4$	M1  <b>[1]</b>	Based on correct answer to (ii)	
8	(iv)	Kite	B1  <b>[1]</b>		

Question		Answer	Marks	Guidance	
9	(i)	Separating variables $\int \frac{1}{\theta+20} d\theta = \int -k dt$ $\ln(\theta+20) = -kt (+c)$ or equivalent $\theta = Ae^{-kt} - 20$ oe (i.e. $\theta = e^{-kt+c} - 20$ )	M1 A1 A1 <b>[3]</b>	or invert each side: $\frac{dt}{d\theta} = -\frac{1}{k(\theta+20)}$ “Eqn A” “Eqn B”	Must see $\frac{1}{\theta+20}$ ; ignore posn ‘k’
9	(ii)	$(- )3 = -k(40+20)$ $k = \frac{1}{20}$ oe Subst $t = 0, \theta = 40$ & their $k$ (where necessary) into their Eqn A or their Eqn B and solve for the arbitrary constant Subst $\theta = 0$ & their values of $k$ and the arbitrary constant into their Eqn A or their Eqn B $t = 21.9722 = 22$ minutes cao www	M1 *A1 M1 M1 dep*A1 <b>[5]</b>	Using $t = 0, \theta = 40, \frac{d\theta}{dt} = (-)3$ in <u>given</u> equation Not $k = -\frac{1}{20}$	
9	(iii)	$k$ is larger	B1 <b>[1]</b>		

Question	Answer	Marks	Guidance
10 (i)	Clear start to algebraic division (Quotient) = $x - 1$ (Remainder) = $x + 7$  Final answer: $x - 1 + \frac{x + 7}{x^2 - x - 6}$	M1 A1 A1  A1   <b>[4]</b>	at least as far as $x$ term in quot & subseq mult back & attempt at subtraction   final answer in correct form This must be shown in part (i) or, if not, then implied in part (ii)  If no long division shown but only comparison of coefficients or otherwise, SR M0 B1 B1 B1  Accept $A = 1, B = -1, C = 1, D = 7$
10 (ii)	Convert their $\frac{Cx + D}{x^2 - x - 6}$ to Partial Fractions  $\frac{x + 7}{x^2 - x - 6} = \frac{2}{x - 3} - \frac{1}{x + 2}$ <u>Their....</u> $\int Ax + B \, dx = \frac{1}{2} Ax^2 + Bx$ or $\frac{(Ax + B)^2}{2A}$ $\int \frac{E}{x - 3} + \frac{F}{x + 2} \, dx = E \ln(x - 3) + F \ln(x + 2)$  Using limits in a correct manner  $8 + \ln \frac{27}{4} \left( 8 + \ln \frac{54}{8} \right)$ isw	M1  A1A1  B1 ft B1 ft  M1  A1  <b>[7]</b>	<u>Correct</u> fraction converted to <u>correct</u> PFs          Tolerate some wrong signs provided intention clear  Answer required in the form $a + \ln b$ , so giving <u>only</u> a decimalised form is awarded A0

**OCR (Oxford Cambridge and RSA Examinations)**  
1 Hills Road  
Cambridge  
CB1 2EU

**OCR Customer Contact Centre**

**Education and Learning**

Telephone: 01223 553998

Facsimile: 01223 552627

Email: [general.qualifications@ocr.org.uk](mailto:general.qualifications@ocr.org.uk)

**[www.ocr.org.uk](http://www.ocr.org.uk)**

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

**Oxford Cambridge and RSA Examinations**  
is a Company Limited by Guarantee  
Registered in England  
Registered Office; 1 Hills Road, Cambridge, CB1 2EU  
Registered Company Number: 3484466  
OCR is an exempt Charity

**OCR (Oxford Cambridge and RSA Examinations)**  
Head office  
Telephone: 01223 552552  
Facsimile: 01223 552553

© OCR 2013

