

**GCE**

**Mathematics A**

Unit **H240/02**: Pure Mathematics and Statistics

Advanced GCE

**Mark Scheme for June 2018**

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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## Annotations and abbreviations

<b>Annotation in scoris</b>	<b>Meaning</b>
✓ and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
<b>Other abbreviations in mark scheme</b>	<b>Meaning</b>
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

**Subject-specific Marking Instructions for A Level Mathematics A**

- a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.
- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.  
If you are in any doubt whatsoever you should contact your Team Leader.
- c The following types of marks are available.

**M**

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

**A**

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

**B**

Mark for a correct result or statement independent of Method marks.

**E**

Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation *isw*. However, this would not apply to a case where a candidate

- passes through the correct answer as part of a wrong argument.
- d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep\*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.
- f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for g. E marks will be lost except when results agree to the accuracy required in the question.
- g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.
- h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. ‘Fresh starts’ will not affect an earlier decision about a misread. Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.
- i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j If in any case the scheme operates with considerable unfairness, consult your Team Leader.

Final Version

Question		Answer	Mks	AO	Guidance
1	(i)	$2(x^2 - 6x + 11.5)$ $2((x - 3)^2 + 11.5 - 9)$  $2(x - 3)^2 + 5$	<b>B1</b> <b>B1</b> <b>M1</b> <b>A1</b> <b>[4]</b>	<b>1.1a</b> <b>1.1</b> <b>1.1</b> <b>1.1</b>	or $a = 2$ or $b = -3$ $23 - 2(\text{their } b)^2$ or $c = 5$
1	(ii)	$2(x + 3)^2 + 5$ is always +ve or $2(x + 3)^2 + 5 > 0$ or $2(x + 3)^2 + 5 \geq 5$ Hence no real roots	<b>B1f</b> <b>[1]</b>	<b>1.1</b>	or $2(x + 3)^2 = -5$ , which is impossible or "+ve quadratic" and min on $y = 5$ or "+ve" quadratic; TP at (3, 5). Both Hence no real roots Must use (i), not use D or $12^2 - 8k = 0$
1	(iii)	$2(x - 3)^2 = 2(x^2 - 6x + 9)$ $k = 18$	<b>M1</b> <b>A1</b> <b>[2]</b>	<b>1.1a</b> <b>2.2a</b>	
2	(i)	$(1 - (-3))^2 + (-2 - (-1))^2 + (5 - 2)^2$ (= 26) Length = $\sqrt{26}$ or 5.10 or 5.1 (2 sf)	<b>M1</b> <b>A1</b> <b>[2]</b>	<b>1.1a</b> <b>1.1</b>	Attempt. Allow with one sign error  $\sqrt{\quad}$ not nec'y
2	(ii)	$\begin{pmatrix} -1 \\ -1.5 \\ 3.5 \end{pmatrix}$	<b>B1</b>  <b>[1]</b>	<b>1.1</b>	
2	(iii)	$\vec{BA} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$  $\vec{PQ} = \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$  $BA = PQ$ and $BA \parallel PQ$  and hence $ABPQ$ is a parallelogram <b>(AG)</b>	<b>M1</b>  <b>M1</b>  <b>A1</b> <b>[3]</b>	<b>2.1</b>  <b>1.1</b>  <b>2.2a</b>	or quote result for $\vec{BA}$ from (ii) or (i)(a)  or similar methods with $AQ$ & $BP$ or $AB$ and $QP$ etc Allow find eg $AB$ and $PQ$ or $\vec{BA} = \vec{PQ}$ with arrows or $ BA  =  PQ $ & $ BP  =  AQ $ shown & stated or $BA \parallel PQ$ & $BP \parallel AQ$ shown & stated Both statements needed, dep M1M1

Question		Answer	Mks	AO	Guidance	
3	(i)	7 hours	<b>B1</b> <b>[1]</b>	<b>2.2b</b>	Allow between 6 and 8 hours.	
3	(ii)	Dave will gain no marks even if he does no revision	<b>B1</b> <b>[1]</b>	<b>3.5a</b>	oe	
3	(iii)	(Bob believes) too much revision leads to tiredness (or less sleep or stress or forgetting) and hence a lower mark. (Ayesha does not.)	<b>B1</b> <b>[1]</b>	<b>3.5a</b>	oe Must suggest reason for drop in marks	
3	(iv)	(She believes that) however much or little revision she does it will make no difference to the mark she obtains	<b>B1</b> <b>[1]</b>	<b>2.2b</b>	or (She thinks) revision will not increase her mark or revision is unnec'y to obtain high mark	revision is unhelpful oe
4		<u>Summary of method</u> Use of $\cos(A+B)$ or $\sin(A+B)$ or $\cos 2\theta$ formula Correct result  Use of one of the above or $\sin 2\theta$ formula Correctly obtain result  <u>Example of method</u> $\sin^2(\theta+45) - \cos^2(\theta+45) \equiv -\cos 2(\theta+45)$ $\equiv -\cos(2\theta+90)$ $\equiv -[\cos 2\theta \cos 90 - \sin 2\theta \sin 90] \equiv \sin 2\theta$ <b>AG</b>	<b>M1</b> <b>A1</b>  <b>M1</b> <b>A1</b>  <b>M1</b> <b>A1</b> <b>A1</b> <b>[4]</b>	<b>3.1a</b> <b>2.1</b>  <b>1.1</b> <b>1.1</b>	Correct formula  Correct formula  <u>Use of correct <math>\cos 2\theta</math> formula</u> Correct result <u>Use of correct <math>\cos(A+B)</math> formula</u> Must see this step and final answer	
5	(i)	eg $1+3=4$ or $4+5=9$ or $9+7=16$	<b>B1</b> <b>[1]</b>	<b>1.1</b>	or $25+11=36$ or any correct example	
5	(ii)	If $m-n=1$ (or $-1$ ) then $(m-n)(m+n)$ could be prime	<b>E1</b> <b>[1]</b>	<b>2.3</b>	or One of the factors of $p$ could be 1	(or if $m+n=1$ )
5	(iii)	Let $S=n^2$ $\Rightarrow$ Other square number is $(n+1)^2$ $\Rightarrow 853=(n+1)^2-n^2=2n+1$ $\Rightarrow n=426$	<b>M1</b> <b>M1</b> <b>A1</b>	<b>3.1a</b> <b>2.2a</b> <b>1.1</b>	or Other square number is $(\sqrt{S}+1)^2$ $\Rightarrow 853=(\sqrt{S}+1)^2-S=2\sqrt{S}+1$ $\Rightarrow \sqrt{S}=426$	$853=m^2-n^2$ & $m-n=1$ $\Rightarrow 853=m+n$ $\Rightarrow 853=2n+1$ $\Rightarrow n=426$

Question		Answer	Mks	AO	Guidance	
		$\Rightarrow S = 181476$	<b>A1</b>	<b>3.2a</b>	$\Rightarrow S = 181476$ $m - n = 1, m + n = 853$ M1 $2m = 854$ M1 $m = 427 \quad n = 426$ A1 $n^2 = 181476$ A1	$\Rightarrow S = 181476$ T & I: 426 seen M1M1A1 S = 181476 A1
<b>6</b>		<b>DR</b>				
<b>6</b>	<b>(i)</b>	$\frac{\ln x}{x} = 0$ $\Rightarrow \ln x = 0$ or $\frac{\ln 1}{1} = 0$ $\Rightarrow x = 1$	<b>M1</b> <b>A1</b> <b>[2]</b>	<b>1.1a</b> <b>1.1</b>	May not be seen May be implied	
<b>6</b>	<b>(ii)</b>	y-coordinates are $\frac{\ln 2}{2}$ and $\frac{\ln 4}{4}$ $\frac{\ln 4}{4} = \frac{2 \ln 2}{4} = \frac{\ln 2}{2}$ oe $\Rightarrow AB$ is // to x-axis <b>AG</b>	<b>B1*</b> <b>B1dep*</b> <b>[2]</b>	<b>1.1</b> <b>3.1a</b>	Allow $\frac{\ln 4}{4} = \ln 4^{\frac{1}{4}} = \ln \sqrt{2} = \frac{\ln 2}{2}$ Show that $\frac{\ln 4}{4} = \frac{2 \ln 2}{4}$ and conclusion	Both = 0.346... B0B0 use of $\frac{\ln 4}{4} - \frac{\ln 2}{2} = 0$ unjustified B0B0
<b>6</b>	<b>(iii)</b>	$\frac{dy}{dx} = \frac{x \times \frac{1}{x} - 1 \times \ln x}{x^2}$ or $\frac{1}{x} \times \frac{1}{x} + \ln x \times (-\frac{1}{x^2})$ oe $\frac{1}{x^2} - \frac{\ln x}{x^2} = 0$ or $\frac{1 - \ln x}{x^2} = 0$ $1 - \ln x = 0$ oe $x = e$ or 2.72 or 2.7 (2 sf) Coordinates are $(e, \frac{1}{e})$	<b>M1</b> <b>M1</b> <b>A1</b> <b>A1</b> <b>[4]</b>	<b>3.1a</b> <b>1.1</b> <b>1.1</b> <b>1.1</b>	Attempt diff, $\geq$ one term correct oe, their $\frac{dy}{dx} = 0$ Allow $(e, 0.368)$ or $(e, 0.37)$	or $(2.7, 0.37)$ (2 sf)



Question	Answer	Mks	AO	Guidance	
<p>6 (iv)</p>	<p>Attempt <math>\frac{d^2y}{dx^2}</math></p> <p><math>= \frac{x^2(-\frac{1}{x}) - 2x(1-\ln x)}{x^4}</math> or <math>\frac{-3+2\ln x}{x^3}</math> oe</p> <p>Substitute <math>x = e</math> (or 2.72) into <math>\frac{d^2y}{dx^2}</math></p> <p><math>\frac{d^2y}{dx^2} = -\frac{1}{e^3}</math> oe or -0.0498</p> <p><math>\frac{d^2y}{dx^2} &lt; 0</math>, hence maximum</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>B1f</p> <p>[5]</p>	<p>2.1</p> <p>1.2</p> <p>1.1</p> <p>1.1</p> <p>3.2a</p>	<p>Attempt diff their <math>\frac{dy}{dx}</math></p> <p>All correct, not necessarily simplified cao</p> <p>Sub their <math>x</math> from (iii) into their <math>\frac{d^2y}{dx^2}</math></p> <p>cao Allow or - 0.0497 or -0.05</p> <p>ft their result of sub their <math>x</math> into their <math>\frac{d^2y}{dx^2}</math></p> <p>dep see result</p>	<p>Example of grad method</p> <p>Sub 2.7 &amp; 2.8 in <math>\frac{dy}{dx}</math> M1</p> <p>0.00093, -0.0038 A1A1</p> <p>State grad +ve &amp; -ve or show on diag dep A1A1 M1</p> <p>Hence max B1f dep M1A1A1</p> <p>No proof, no marks</p>
<p>7</p>	<p><b>Summary of marks:</b></p> <p>Attempt find <math>x</math> at intersection of curves <math>x = 1</math></p> <p>Correct integral, any limits</p> <p>Correct numerical result</p> <p>Attempt area of part or all of <math>2 \times 2</math> square</p> <p>Wholly correct method</p> <p><math>\frac{44}{3}</math></p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[7]</p>	<p>3.1a</p> <p>1.1</p> <p>3.1a</p> <p>1.1</p> <p>1.1</p> <p>2.1</p> <p>1.1</p>	<p>Can be implied</p> <p>from correct limits</p>	

Question	Answer	Mks	AO	Guidance	
	<p><b>Examples of methods:</b></p> <p><u>Method 1</u>  <math>3 - 2x^2 = x</math>  <math>2x^2 + x - 3 = 0</math>  <math>x = 1</math>  <math>\int_0^1 (3 - 2x^2) dx</math>                      or <math>\int_{-1}^1 (3 - 2x^2) dx</math>  <math>= \left[ 3x - \frac{2x^3}{3} \right]_0^1</math>                      or <math>\left[ 3x - \frac{2x^3}{3} \right]_{-1}^1</math>  <math>= \frac{7}{3}</math>    or <math>\frac{14}{3}</math>  <math>"\frac{7}{3}" - 1 (= \frac{4}{3})</math>                              or <math>"\frac{14}{3}" - 2 (= \frac{8}{3})</math>  <math>8 \times "\frac{4}{3}" + 4</math>                              or <math>4 \times "\frac{8}{3}" + 4</math>  <math>= \frac{44}{3}</math></p> <p><u>Method 2</u>  <math>3 - 2x^2 = x</math>  <math>x = 1</math>  <math>\int_1^3 \left(\frac{y-3}{2}\right)^{\frac{1}{2}} dy</math>    <math>= \dots\dots</math>  <math>= \frac{4}{3}</math>  <math>\frac{4}{3} + \frac{1}{2} (= \frac{11}{6})</math>  <math>8 \times \frac{11}{6}</math>  <math>= \frac{44}{3}</math></p>	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>		<p>Ignore other root</p> <p>Correct integrand with any limits</p> <p>Attempt area above <math>y = 1</math> or above <math>y = x</math></p> <p>Complete correct method</p> <p><u>Method 3</u>  <math>3 - 2x^2 = x</math>  <math>x = 1</math>  <math>\int_0^1 (3 - 2x^2 - 1) dx</math>  <math>= \left[ 2x - \frac{2x^3}{3} \right]_0^1</math>  <math>= \frac{4}{3}</math>  <math>\frac{4}{3} + \frac{1}{2} (= \frac{11}{6})</math>  <math>8 \times \frac{11}{6}</math>  <math>= \frac{44}{3}</math></p>	<p>or <math>3 - 2x^2 = -x</math>  <math>2x^2 - x - 3 = 0</math>  <math>x = -1</math></p> <p>or <math>"\frac{14}{3}" - 1 (= \frac{11}{3})</math></p> <p><math>4 \times \frac{11}{3}</math></p> <p>Other correct methods may be seen</p>

Question			Answer	Mks	AO	Guidance
8	(i)	(a)	0.0478 or 0.048 (2 sf)	B1 [1]	1.1	BC
	(i)	(b)	22.5 or 23 (2 sf)	B1 [1]	1.1	BC
	(i)	(c)	$P(X < 20 + b) = 0.75$ or $P(X > 20 + b) = 0.25$ $20 + b = 22.02..$ or 22.0 or 22 $b = 2.02$ or 2.0 (2 sf) Allow $b = 2$	M1 A1 A1 [3]	1.1a 1.1 1.1	$P(X < 20 - b) = 0.25$ $20 - b = 17.98$ or 18  $b = 22(.02)$ M1A1A0  T & I method: Try 2 values, one $\approx 2$ M1 Correct probs for two values in [2, 2.1] A1 Correct probs for two values in [2, 2.05] A1 & ans 2.0 or 2 A1
8	(ii)		$\frac{1.5\mu - \mu}{\mu/3}$  $= \frac{3}{2}$  $P(X > 1.5\mu) = 0.0668$ or 0.67 (2 sf)	M1 A1 A1 [3]	1.1a 1.1 1.1	SC (eg) Let $\mu = 1$ ; $N(1, \frac{1}{9})$ M1 $X = \frac{3}{2}$ A0 $P(X > \frac{3}{2}) = 0.067$ A1
9			$H_0: p = \frac{1}{6}$ $H_0: p > \frac{1}{6}$ where $p = P(2 \text{ on one throw})$ $B(35, \frac{1}{6})$ $P(X \geq 10) = 1 - P(X \leq 9)$ or $P(X \geq 11) = 1 - P(X \leq 10)$ $P(X \geq 10) = 0.055$ $P(X \geq 11) = 0.023$ (0.04 lies between these hence) rejection region is $X \geq 11$ Allow eg $a \geq 11$ <u>Special case, using N~Bin; Method A</u> $H_0: \mu = \frac{35}{6}$	B1 B1 M1 M1 A1 A1 A1 B1	1.1 2.5 3.3 1.1a 2.1 3.4 2.2a 1.1	B1B0 one error eg undefined $p$ or two-tail stated or implied unless clearly using $N(\ )$  $\geq 1$ of these probabilities stated  BC BC  dep $\geq$ one of above probs seen & correct
						or $P(X \leq 9)$ , $P(X \leq 10)$  $P(X \leq 9) = 0.945$ $P(X \leq 10) = 0.977$ (0.96 between these) rej'n region is $X \geq 11$

Question		Answer	Mks	AO	Guidance	
		$H_0: \mu > \frac{35}{6}$ where $\mu$ = pop mean no. of 2's $N(\frac{35}{6}, \frac{175}{36})$ or $N(5.833, 4.861)$ soi $P(X \geq 10) = 1 - P(X < 9.5)$ or $P(X \geq 11) = 1 - P(X < 10.5)$ $P(X \geq 10) = 0.048$ $P(X \geq 11) = 0.017$ (0.04 lies between these hence) rejection region is $X \geq 11$ <u>Special case, using <math>N \sim \text{Bin}</math>; Method B</u> $H_0: \mu = \frac{35}{6}$ $H_0: \mu > \frac{35}{6}$ where $\mu$ = pop mean no. of 2's $N(\frac{35}{6}, \frac{175}{36})$ or $N(5.833, 4.861)$ soi $P(X > a) = 0.04$ soi $\frac{35}{6} + 1.751 \times \sqrt{\frac{175}{36}}$ $= 9.69$ or $9.7$ rejection region is $X \geq 11$	<b>B1</b> <b>M1</b> <b>M1</b> <b>A0</b> <b>A1</b>  <b>A1</b>  <b>B1</b>  <b>B1</b>  <b>M1</b>  <b>M1</b>  <b>A1</b>  <b>A0</b> <b>A1</b> <b>[7]</b>	<b>2.5</b> <b>3.3</b> <b>1.1a</b> <b>2.1</b> <b>3.4</b>  <b>2.2a</b>  <b>1.1</b>  <b>2.5</b>  <b>3.3</b>  <b>1.1a</b>  <b>2.1</b>  <b>3.4</b> <b>2.2a</b>	<b>B1B0</b> one error eg undefined $\mu$ or two-tail Allow incorrect variance $\geq 1$ of these probabilities attempted <b>BC</b> <b>BC</b> dep $\geq$ one of above probs seen & correct  <b>B1B0</b> one error eg undefined $\mu$ or two-tail Allow incorrect variance $z = \phi^{-1}(0.96)$ (= 1.751) dep $\phi^{-1}(0.96)$ attempt. May be implied <b>BC</b>	$P(X < 9.5)$ or $P(X < 10.5)$ $P(X < 9.5) = 0.952$ $P(X < 10.5) = 0.983$ (0.96 between these) rej'n region is $X \geq 11$
<b>10</b>	<b>(i)</b>	Only 784 trees and $810 > 784$	<b>E1</b> <b>[1]</b>	<b>2.4</b>	or other similar	
<b>10</b>	<b>(ii)</b>	eg Each no. not independent of previous no. Each no. is related to the next	<b>E1</b>  <b>[1]</b>	<b>2.3</b>	Allow 2nd digit of each no. is 1st of next Consecutive nos share two digits  Ignore all else	or similar correct Digits are re-used

Question		Answer	Mks	AO	Guidance		
10	(iii)	<p><math>H_0: \mu = 4.2</math>  <math>H_1: \mu &lt; 4.2</math> where <math>\mu</math> is mean height of trees (in the wood)  <math>\bar{X} \sim N(4.2, \frac{0.8^2}{50})</math> and <math>\bar{X} &lt; 4.0</math> or <math>\bar{X} \leq 4.0</math>  <math>P(\bar{X} &lt; 4.0) = 0.038549\dots</math> or 0.039                      Compare 0.02                      Do not reject <math>H_0</math>                      There is insufficient evidence that mean height of these trees in the wood is less than 4.2m.</p>	<p><b>B1</b>  <b>B1</b>  <b>M1</b>  <b>A1</b>  <b>A1</b>  <b>M1</b>  <b>A1f</b>                      [7]</p>	<p><b>1.1</b>  <b>2.5</b>  <b>3.3</b>  <b>3.4</b>  <b>1.1</b>  <b>2.2b</b>  <b>3.5a</b></p>	<p>Allow other letters except <math>X</math> or <math>\bar{X}</math>                      One error, eg undefined <math>\mu</math> or 2-tail: B0B1                      Stated or implied                      Allow <math>\bar{X} &gt; 4.0</math> or <math>\bar{X} = 4.0</math>  <b>BC</b> Allow 0.038                      NB 0.038... implies M1A1                      dep <math>P(\bar{X} &lt; 4.0)</math> attempted                      Allow Accept <math>H_0</math>                      dep <math>P(\bar{X} &lt; 4.0)</math> attempted                      In context, not definite; eg "Mean height not less than 4.2m": A0</p>	<p><math>\Phi^{-1}(0.98)</math> (= 2.054)  <math>4.2 - 2.054 \times \frac{0.8}{\sqrt{50}}</math>                      (= 3.968)                      comp their 3.968 with 4.0                      Can be implied by conclusion</p>	
11	(i)	(a)	<p>Both the number of employees using public transport and the number of employees using private vehicles depend on the LA population.                      [1]</p>	<b>E1</b>	<p>2.1                      or similar, but must be in context.                      Ignore all else</p>	<p>NOT No. using pt is prop to no. using pv</p>	
11	(i)	(b)	<p>Negative                      If a large prop use public transport then a smaller prop drive (and vice versa)                      [2]</p>	<p><b>E1ind</b>  <b>E1ind</b>                      [2]</p>	<p>2.2b                      2.4</p>	<p>Ignore "strong" or "slight" etc                      or similar in context</p>	<p>NOT Inverse prop'n                      NOT "as <math>a</math> increases <math>b</math> decreases" unless in context</p>
11	(ii)	(a)	<p>Decrease the size of <math>r</math> or Make <math>r</math> less negative                      [1]</p>	<b>E1</b>	<p>2.2b                      Make (value of ) <math>r</math> increase  <math>r</math> closer to 0                      Ignore eg "greatly"                      Ignore all else</p>	<p>NOT Make <math>r</math> decrease                      NOT Weaken the corr'n                      NOT Make corr'n less</p>	
11	(ii)	(b)	<p>Little effect (because the population of the LA is small compared with the whole population)                      [1]</p>	<b>E1</b>	<p>2.2b                      or No effect or similar                      Ignore all else</p>		
11	(ii)	(c)	<p>Ignore all reference to public transport  <u>Type 1 answers</u>                      People don't travel far to work                      Jobs are close                      High proportion walk (or cycle)                      [1]</p>	<b>E1</b>	<p>2.4  <u>Type 2 answers</u>                      Any suggested <u>reason</u> why few drive                      eg Few garages; Parking expensive                      or similar in context</p>	<p>NOT just Few drive</p>	

Question		Answer	Mks	AO	Guidance
12	(i)	$a(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}) = 1$ soi $a = \frac{16}{31}$	M1 A1 [2]	3.1a 1.1	or $\frac{16}{31}(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}) = 1$ oe seen correctly obtained
12	(ii)	$P(X = 1, 3 \text{ or } 5) = \frac{21}{31}$ Or 0.677 or 0.68 (2 sf)	B1 [1]	1.1a	
12	(iii)	$P(\text{sum odd}) = P(\text{OE}) + P(\text{EO})$ $= 2 \times \frac{21}{31} \times (1 - \frac{21}{31})$ $= \frac{420}{961}$ or 0.437 or 0.44 (2 sf)	M1 A1 [2]	2.1 1.1	or correct "long" method  Allow without "2 ×"
12	(iv)	$P(\text{Sum} > 8 \text{ \& odd}) = P(\text{Sum} = 9)$ $= P(4, 5) + P(5, 4)$ $= \frac{2}{31} \times \frac{1}{31} + \frac{1}{31} \times \frac{2}{31}$ $(= \frac{4}{961})$ $\frac{P(\text{Sum} > 8 \text{ \& odd})}{P(\text{Sum odd})}$ $= \frac{4}{961} \div \frac{420}{961}$ $= \frac{1}{105}$ or 0.00952 or 0.0095 (2 sf)	M1 M1 A1 [3]	1.1a 2.4 1.1	or $P(> 8) \times P(O   > 8)$ $= \frac{5}{961} \times \frac{4}{5}$ Attempt ft their (iii) and their $P(\text{Sum} > 8 \text{ \& odd})$ cao <b>NB</b> $\frac{2}{961} \div \frac{210}{961} = \frac{1}{105}$ M0M1A0
12	(v)	$S_{\infty} = \frac{p}{1-0.5} = 1$ $P(X = 1) = 0.5$	M1 A1 [2]	3.4 3.4	Correct ans, no working M1A1
12	(vi)	Eg Y. (Y takes all values, but) X cannot be > 5 Eg X because > 5 is very unlikely	B1 [1]	3.5b	oe, eg Y. It may take more than 5 attempts or "limited no." oe instead of 5

Question		Answer	Mks	AO	Guidance	
13		<b>DR</b>				
13	(i)	$N(450 \times 0.15, 450 \times 0.15 \times 0.85)$ or $N(67.5, 57.375)$ oe $P(Y > \mu + \sigma) \approx \frac{1}{6}$ or $\Phi^{-1}(\frac{1}{6}) = 0.9674$ $'67.5' + \sqrt{57.375}$ or $'67.5' + 0.9674 \times \sqrt{57.375}$ $= 74$ or $75$ or $76$	<b>M1</b>  <b>M1</b> <b>A1</b>	<b>3.1b</b>  <b>1.2</b> <b>1.1</b>	seen or implied $P(Y < a) = \frac{5}{6}$ or 74.83 seen; ft their $\mu$ & $\sigma$ for M1 only Integer. No ft Dep M1M1 Correct ans, inadequate wking: M0M0A0 <b>NB 450/6 = 75</b> <b>M0M0A0</b>	$B(450, 0.15)$ with T & I method using $\geq$ one of 74, 75, 76, 61, 60, 59 $P(X > 74) = 0.177$ $P(X > 75) = 0.145$ both $a = 74$ or $75$ or $76$
13	(ii)	$\frac{\frac{50!}{r!(50-r)!} \times 0.15^r \times 0.85^{50-r}}{\frac{50!}{(r+1)!(50-(r+1))!} \times 0.15^{r+1} \times 0.85^{50-(r+1)}} \quad \text{oe}$ eg $\frac{\frac{1}{50-r} \times 0.85}{\frac{1}{r+1} \times 0.15}$ or $\frac{0.85}{50-r} \times \frac{r+1}{0.15}$ oe $= \frac{17(r+1)}{3(50-r)}$ <b>AG</b>	<b>M1</b>  <b>A1</b>  <b>A1</b>	<b>1.1a</b>  <b>2.1</b>  <b>1.1</b>	$\frac{{}^{50}C_r \times 0.15^r \times 0.85^{50-r}}{{}^{50}C_{r+1} \times 0.15^{r+1} \times 0.85^{50-(r+1)}}$ Any correct simplification without factorials <b>OR</b> without indices Any correct simplification without factorials <b>AND</b> without indices and correctly obtain result	Fully correct  or $\frac{17}{20} \times \frac{20}{3} \times \frac{r+1}{50-r}$
13	(iii) (a)	$\frac{17(r+1)}{3(50-r)} \leq 1 \quad \text{oe}$ $17r + 17 \leq 150 - 3r$ $20r \leq 133 \quad \text{oe}$ $r \leq 6.65$ $r$ is an integer so $r \leq 6$	<b>M1</b>  <b>A1</b> <b>A1</b> <b>A1</b>	<b>3.1b</b>  <b>1.1</b>  <b>1.1</b> <b>1.1</b>	$\frac{1}{50-r} \times 0.85 \leq \frac{1}{r+1} \times 0.15 \quad \text{oe}$ $0.85(r+1) \leq 0.15(50-r)$ $r \leq 50 \times 0.15 - 0.85$  SC: $P(X=6)=0.142, P(X=7)=0.157, P(X=8)=0.149$ B1 (must be these three) hence $r \leq 6$ B1dep	M1 No factorials or indices A1 Correct, in form $ar \leq b$ or $r <$ correct expr'n  No wking B0B0

Question			Answer	Mks	AO	Guidance	
13	(iii)	(b)	$P(X = r) \leq P(X = r + 1)$ for $r \leq 6$ Hence most likely value is $r$ is 6 or 7 $\frac{P(X=6)}{P(X=7)} = \frac{17(6+1)}{3(50-6)} = 0.902 < 1$ Most likely value is 7	<b>B1</b>  <b>B1</b> [2]	<b>2.1</b>  <b>3.2a</b>	or $P(X = 6) = 0.142$ & $P(X = 7) = 0.157$  indep, but dep on some reasonable explanation	NOT 6.65 rounds to 7 B0B0  No expl'n: B0B0
				<b>100</b>			



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