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Mark Scheme (Results)
January 2009

GCE

GCE Mathematics (6666/01)

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January 2009
6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme		Marks
1 (a)	$C\colon\ y^2 - 3y = x^3 + 8$		
	$\left\{ \cancel{\frac{\mathrm{d}y}{\mathrm{d}x}} \cancel{=} \right\}\quad 2y\frac{\mathrm{d}y}{\mathrm{d}x} - 3\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2$	Differentiates implicitly to include either $\pm ky\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\pm 3\frac{\mathrm{d}y}{\mathrm{d}x}$. (Ignore $\left(\frac{\mathrm{d}y}{\mathrm{d}x} = \right)$.)	M1
		Correct equation.	A1
	$(2y-3)\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2$	A correct (condoning sign error) attempt to combine or factorise their '$2y\frac{\mathrm{d}y}{\mathrm{d}x} - 3\frac{\mathrm{d}y}{\mathrm{d}x}$'. Can be implied.	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x^2}{2y-3}$	$\underline{\frac{3x^2}{2y-3}}$	A1 oe
			(4)
(b)	$y = 3 \Rightarrow 9 - 3(3) = x^3 + 8$	Substitutes $y = 3$ into C.	M1
	$x^3 = -8 \Rightarrow \underline{x = -2}$	Only $\underline{x = -2}$	A1
	$(-2, 3) \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3(4)}{6-3} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 4$	$\frac{\mathrm{d}y}{\mathrm{d}x} = 4$ from correct working. Also can be ft using their 'x' value and $y = 3$ in the correct part (a) of $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x^2}{2y-3}$	A1 $\sqrt{}$
			(3)
	1(b) final A1 $\sqrt{}$. Note if the candidate inserts their x value and $y = 3$ into $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x^2}{2y-3}$, then an answer of $\frac{\mathrm{d}y}{\mathrm{d}x} =$ their x^2, ***may*** indicate a correct follow through.		
			[7]

Question Number	Scheme		Marks	
2 (a)	$\text{Area}(R) = \int_0^2 \frac{3}{\sqrt{(1+4x)}}\,dx = \int_0^2 3(1+4x)^{-\frac{1}{2}}\,dx$			
	$= \left[\frac{3(1+4x)^{\frac{1}{2}}}{\frac{1}{2}.4}\right]_0^2$	***Integrating*** $3(1+4x)^{-\frac{1}{2}}$ to give $\pm k(1+4x)^{\frac{1}{2}}$.	M1	
		Correct integration. Ignore limits.	A1	
	$= \left[\frac{3}{2}(1+4x)^{\frac{1}{2}}\right]_0^2$			
	$= \left(\frac{3}{2}\sqrt{9}\right) - \left(\frac{3}{2}(1)\right)$	Substitutes limits of 2 and 0 into a changed function and subtracts the correct way round.	M1	
	$= \frac{9}{2} - \frac{3}{2} = \underline{3}\,(\text{units})^2$	$\underline{3}$	A1	
				(4)
	(Answer of 3 with no working scores M0A0M0A0.)			
(b)	$\text{Volume} = \pi\int_0^2 \left(\frac{3}{\sqrt{(1+4x)}}\right)^2 dx$	Use of $V = \pi\int y^2\,dx$. Can be implied. Ignore limits and dx.	B1	
	$= (\pi)\int_0^2 \frac{9}{1+4x}\,dx$			
	$= (\pi)\left[\frac{9}{4}\ln\lvert 1+4x\rvert\right]_0^2$	$\pm k\ln\lvert 1+4x\rvert$	M1	
		$\frac{9}{4}\ln\lvert 1+4x\rvert$	A1	
	$= (\pi)\left[\left(\frac{9}{4}\ln 9\right) - \left(\frac{9}{4}\ln 1\right)\right]$ Note that $\ln 1$ can be implied as equal to 0.	Substitutes limits of 2 and 0 and subtracts the correct way round.	dM1	
	So Volume $= \underline{\frac{9}{4}\pi\ln 9}$ Note the answer must be a one term exact value. Note, also you can ignore subsequent working here.	$\underline{\frac{9}{4}\pi\ln 9}$ or $\underline{\frac{9}{2}\pi\ln 3}$ or $\underline{\frac{18}{4}\pi\ln 3}$ Note that $= \frac{9}{4}\pi\ln 9 + c$ (oe.) would be awarded the final A0.	A1 oe isw	(5)
				[9]

Question Number	Scheme		Marks
3 (a)	$27x^2 + 32x + 16 \equiv A(3x+2)(1-x) + B(1-x) + C(3x+2)^2$	Forming this identity	M1
	$x = -\frac{2}{3}$, $12 - \frac{64}{3} + 16 = \left(\frac{5}{3}\right)B \Rightarrow \frac{20}{3} = \left(\frac{5}{3}\right)B \Rightarrow B = 4$ $x = 1$, $27 + 32 + 16 = 25C \Rightarrow 75 = 25C \Rightarrow C = 3$	Substitutes either $x = -\frac{2}{3}$ or $x = 1$ into their identity or equates 3 terms or substitutes in values to write down three simultaneous equations.	M1
		Both $B = 4$ and $C = 3$ **(Note the A1 is dependent on both method marks in this part.)**	A1
	Equate x^2: $27 = -3A + 9C \Rightarrow 27 = -3A + 27 \Rightarrow 0 = -3A$ $\Rightarrow A = 0$ $x = 0$, $16 = 2A + B + 4C$ $\Rightarrow 16 = 2A + 4 + 12 \Rightarrow 0 = 2A \Rightarrow A = 0$	Compares coefficients or substitutes in a third x-value or uses simultaneous equations to show $A = 0$.	B1
			(4)
(b)	$f(x) = \frac{4}{(3x+2)^2} + \frac{3}{(1-x)}$		
	$= 4(3x+2)^{-2} + 3(1-x)^{-1}$	Moving powers to top on any one of the two expressions	M1
	$= 4\left[2\left(1+\frac{3}{2}x\right)^{-2}\right] + 3(1-x)^{-1}$		
	$= 1\left(1+\frac{3}{2}x\right)^{-2} + 3(1-x)^{-1}$		
	$= 1\left\{\underline{1 + (-2)(\tfrac{3x}{2}); + \frac{(-2)(-3)}{2!}(\tfrac{3x}{2})^2 + \ldots}\right\}$	Either $1 \pm (-2)(\frac{3x}{2})$ or $1 \pm (-1)(-x)$ from either first or second expansions respectively	dM1;
	$+ 3\left\{\underline{1 + (-1)(-x); + \frac{(-1)(-2)}{2!}(-x)^2 + \ldots}\right\}$	Ignoring 1 and 3, any one correct $\{\underline{\ldots\ldots\ldots}\}$ expansion.	A1
		Both $\{\underline{\ldots\ldots\ldots}\}$ correct.	A1
	$= \left\{1 - 3x + \frac{27}{4}x^2 + \ldots\right\} + 3\left\{1 + x + x^2 + \ldots\right\}$		
	$= 4 + 0x ; + \frac{39}{4}x^2$	$4 + (0x)$; $\frac{39}{4}x^2$	A1; A1
			(6)

Question Number	Scheme		Marks
(c)	Actual $= \mathrm{f}(0.2) = \dfrac{1.08+6.4+16}{(6.76)(0.8)}$ $= \dfrac{23.48}{5.408} = 4.341715976... = \dfrac{2935}{676}$	Attempt to find the actual value of f(0.2) or seeing awrt 4.3 and believing it is candidate's actual f(0.2).	
	Or Actual $= \mathrm{f}(0.2) = \dfrac{4}{(3(0.2)+2)^2} + \dfrac{3}{(1-0.2)}$ $= \dfrac{4}{6.76} + 3.75 = 4.341715976... = \dfrac{2935}{676}$	Candidates can also attempt to find the actual value by using $\dfrac{A}{(3x+2)} + \dfrac{B}{(3x+2)^2} + \dfrac{C}{(1-x)}$ with their A, B and C.	M1
	Estimate $= \mathrm{f}(0.2) = 4 + \frac{39}{4}(0.2)^2$ $= 4 + 0.39 = 4.39$	Attempt to find an estimate for f(0.2) using their answer to (b)	M1 $\sqrt{}$
	%age error $= \dfrac{\lvert 4.39 - 4.341715976...\rvert}{4.341715976...} \times 100$	$\left\lvert \dfrac{\text{their estimate - actual}}{\text{actual}} \right\rvert \times 100$	M1
	$= 1.112095408... = 1.1\%(2\text{sf})$	1.1%	A1 **cao** (4)
			[14]

Question Number	Scheme		Marks
4 (a)	$\mathbf{d}_1 = -2\mathbf{i} + \mathbf{j} - 4\mathbf{k}$, $\mathbf{d}_2 = q\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$		
	As $\left\{ \mathbf{d}_1 \bullet \mathbf{d}_2 = \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} \bullet \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix} \right\} = \underline{(-2\times q) + (1\times 2) + (-4\times 2)}$	Apply dot product calculation between two direction vectors, ie. $\underline{(-2\times q) + (1\times 2) + (-4\times 2)}$	M1
	$\mathbf{d}_1 \bullet \mathbf{d}_2 = 0 \Rightarrow -2q + 2 - 8 = 0$ $-2q = 6 \Rightarrow \underline{q = -3}$ AG	Sets $\mathbf{d}_1 \bullet \mathbf{d}_2 = 0$ and solves to find $\underline{q = -3}$	A1 cso
			(2)
(b)	Lines meet where:		
	$\begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} = \begin{pmatrix} -5 \\ 11 \\ p \end{pmatrix} + \mu \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix}$		
	First two of $\mathbf{i}: 11 - 2\lambda = -5 + q\mu$ (1) $\mathbf{j}: 2 + \lambda = 11 + 2\mu$ (2) $\mathbf{k}: 17 - 4\lambda = p + 2\mu$ (3)	Need to see equations (1) and (2). Condone one slip. (Note that $q = -3$.)	M1
	(1) + 2**(2)** gives: $15 = 17 + \mu \Rightarrow \mu = -2$	Attempts to solve (1) and (2) to find one of either λ or μ	dM1
		Any one of $\underline{\lambda = 5}$ or $\underline{\mu = -2}$	A1
	(2) gives: $2 + \lambda = 11 - 4 \Rightarrow \lambda = 5$	Both $\underline{\lambda = 5}$ and $\underline{\mu = -2}$	A1
	(3) $\Rightarrow 17 - 4(5) = p + 2(-2)$	Attempt to substitute their λ and μ into their **k** component to give an equation in p alone.	ddM1
	$\Rightarrow p = 17 - 20 + 4 \Rightarrow \underline{p = 1}$	$\underline{p = 1}$	A1 cso
			(6)
(c)	$\mathbf{r} = \begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + 5 \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} -5 \\ 11 \\ 1 \end{pmatrix} - 2 \begin{pmatrix} -3 \\ 2 \\ 2 \end{pmatrix}$	Substitutes their value of λ or μ into the correct line l_1 or l_2 .	M1
	Intersect at $\mathbf{r} = \underline{\begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix}}$ or $\underline{(1, 7, -3)}$	$\underline{\begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix}}$ or $\underline{(1, 7, -3)}$	A1
			(2)

Question Number	Scheme		Marks
(d)	Let $\overrightarrow{OX} = \mathbf{i} + 7\mathbf{j} - 3\mathbf{k}$ be point of intersection		
	$\overrightarrow{AX} = \overrightarrow{OX} - \overrightarrow{OA} = \begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix} - \begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} = \begin{pmatrix} -8 \\ 4 \\ -16 \end{pmatrix}$	Finding vector $\overrightarrow{AX}$ by finding the difference between $\overrightarrow{OX}$ and $\overrightarrow{OA}$. Can be ft using candidate's $\overrightarrow{OX}$.	M1 $\sqrt{\ }$ $\pm$
	$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OA} + 2\overrightarrow{AX}$		
	$\overrightarrow{OB} = \begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} + 2\begin{pmatrix} -8 \\ 4 \\ -16 \end{pmatrix}$	$\begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} + 2\begin{pmatrix} \text{their } \overrightarrow{AX} \end{pmatrix}$	dM1 $\sqrt{\ }$
	Hence, $\overrightarrow{OB} = \underline{\begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}}$ or $\overrightarrow{OB} = \underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$	$\underline{\begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}}$ or $\underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$ or $\underline{(-7, 11, -19)}$	A1
			(3)
			[13]

Question Number	Scheme		Marks
5 (a)	Similar triangles $\Rightarrow \dfrac{r}{h}=\dfrac{16}{24} \Rightarrow r=\dfrac{2h}{3}$	Uses similar triangles, ratios or trigonometry to find either one of these two expressions oe.	M1
	$V=\dfrac{1}{3}\pi r^2 h = \dfrac{1}{3}\pi\left(\dfrac{2h}{3}\right)^2 h = \dfrac{4\pi h^3}{27}$ **AG**	Substitutes $r=\frac{2h}{3}$ into the formula for the volume of water V.	A1
			(2)
(b)	From the question, $\dfrac{dV}{dt}=8$	$\dfrac{dV}{dt}=8$	B1
	$\dfrac{dV}{dh}=\dfrac{12\pi h^2}{27}=\dfrac{4\pi h^2}{9}$	$\dfrac{dV}{dh}=\dfrac{12\pi h^2}{27}$ or $\dfrac{4\pi h^2}{9}$	B1
	$\dfrac{dh}{dt}=\dfrac{dV}{dt}\div\dfrac{dV}{dh} = 8\times\dfrac{9}{4\pi h^2} = \dfrac{18}{\pi h^2}$	Candidate's $\dfrac{dV}{dt}\div\dfrac{dV}{dh}$;	M1;
		$8\div\left(\dfrac{12\pi h^2}{27}\right)$ or $8\times\dfrac{9}{4\pi h^2}$ or $\dfrac{18}{\pi h^2}$ oe	A1
	When $h=12$, $\dfrac{dh}{dt}=\dfrac{18}{144\pi}=\dfrac{1}{8\pi}$	$\dfrac{18}{144\pi}$ or $\dfrac{1}{8\pi}$	A1 oe isw
	Note the answer must be a one term exact value. Note, also you can ignore subsequent working after $\dfrac{18}{144\pi}$.		(5)
			[7]

Question Number	Scheme		Marks
6 (a)	$\int \tan^2 x \, dx$		
	$\left[NB: \underline{\sec^2 A = 1 + \tan^2 A} \text{ gives } \underline{\tan^2 A = \sec^2 A - 1}\right]$	The correct underlined identity.	M1 oe
	$= \int \sec^2 x - 1 \, dx$		
	$= \underline{\tan x - x} \, (+\,c)$	Correct integration with/without + c	A1
			(2)
(b)	$\int \frac{1}{x^3} \ln x \, dx$		
	$\left\{\begin{array}{l} u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = x^{-3} \Rightarrow v = \frac{x^{-2}}{-2} = \frac{-1}{2x^2} \end{array}\right\}$		
	$= -\frac{1}{2x^2} \ln x - \int -\frac{1}{2x^2} \cdot \frac{1}{x} \, dx$	Use of 'integration by parts' formula in the correct direction. Correct direction means that $u = \ln x$.	M1
		Correct expression.	A1
	$= -\frac{1}{2x^2} \ln x + \frac{1}{2} \int \frac{1}{x^3} \, dx$	An attempt to multiply through $\frac{k}{x^n}$, $n \in$ □ , n …2 by $\frac{1}{x}$ and an attempt to ...	
	$= \underline{-\frac{1}{2x^2} \ln x + \frac{1}{2}\left(-\frac{1}{2x^2}\right)} \, (+\,c)$	… "integrate"(process the result);	M1
		correct solution with/without + c	A1 oe
			(4)

Question Number	Scheme		Marks
(c)	$\int \frac{e^{3x}}{1+e^x}\,dx$		
	$\left\{ u = 1+e^x \Rightarrow \underline{\frac{du}{dx} = e^x}, \ \underline{\frac{dx}{du} = \frac{1}{e^x}}, \ \underline{\frac{dx}{du} = \frac{1}{u-1}} \right\}$	Differentiating to find any one of the <u>three underlined</u>	<u>B1</u>
	$= \int \frac{e^{2x}.e^x}{1+e^x}\,dx = \int \frac{(u-1)^2.e^x}{u}.\frac{1}{e^x}\,du$ or $= \int \frac{(u-1)^3}{u}.\frac{1}{(u-1)}\,du$	Attempt to substitute for $e^{2x} = f(u)$, their $\frac{dx}{du} = \frac{1}{e^x}$ and $u = 1+e^x$ or $e^{3x} = f(u)$, their $\frac{dx}{du} = \frac{1}{u-1}$ and $u = 1+e^x$.	M1*
	$= \int \frac{(u-1)^2}{u}\,du$	$\underline{\int \frac{(u-1)^2}{u}\,du}$	A1
	$= \int \frac{u^2-2u+1}{u}\,du$ $= \int u - 2 + \frac{1}{u}\,du$	An attempt to multiply out their numerator to give at least three terms and divide through each term by u	dM1*
	$= \frac{u^2}{2} - 2u + \ln u \ (+c)$	Correct integration with/without +c	A1
	$= \frac{(1+e^x)^2}{2} - 2(1+e^x) + \ln(1+e^x) + c$	Substitutes $u = 1+e^x$ back into their integrated expression with at least two terms.	dM1*
	$= \frac{1}{2} + e^x + \frac{1}{2}e^{2x} - 2 - 2e^x + \ln(1+e^x) + c$		
	$= \frac{1}{2} + e^x + \frac{1}{2}e^{2x} - 2 - 2e^x + \ln(1+e^x) + c$		
	$= \frac{1}{2}e^{2x} - e^x + \ln(1+e^x) - \frac{3}{2} + c$		
	$= \frac{1}{2}e^{2x} - e^x + \ln(1+e^x) + k$ **AG**	$\underline{\frac{1}{2}e^{2x} - e^x + \ln(1+e^x) + k}$ must use a $+c$ and " $-\frac{3}{2}$ " combined.	A1 **cso**
			(7)
			[13]

Question Number	Scheme		Marks
7 (a)	At A, $x=-1+8=7$ & $y=(-1)^2=1 \Rightarrow A(7,1)$	$A(7,1)$	B1 (1)
(b)	$x=t^3-8t$, $y=t^2$,		
	$\frac{dx}{dt}=3t^2-8$, $\frac{dy}{dt}=2t$		
	$\therefore \frac{dy}{dx}=\frac{2t}{3t^2-8}$	Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ Correct $\frac{dy}{dx}$	M1 A1
	At A, $m(\mathbf{T})=\underline{\frac{2(-1)}{3(-1)^2-8}}=\underline{\frac{-2}{3-8}}=\underline{\frac{-2}{-5}}=\underline{\frac{2}{5}}$	Substitutes for t to give any of the four underlined oe:	
	$\mathbf{T}$: $y-(\text{their } 1)=m_T(x-(\text{their } 7))$ or $1=\frac{2}{5}(7)+c \Rightarrow c=1-\frac{14}{5}=-\frac{9}{5}$ Hence $\mathbf{T}$: $y=\frac{2}{5}x-\frac{9}{5}$	Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y=(\text{their gradient})x+"c"$.	dM1
	gives $\mathbf{T}$: $\underline{2x-5y-9=0}$ **AG**	$\underline{2x-5y-9=0}$	A1 **cso** (5)
(c)	$2(t^3-8t)-5t^2-9=0$	Substitution of both $x=t^3-8t$ and $y=t^2$ into $\mathbf{T}$	M1
	$2t^3-5t^2-16t-9=0$		
	$(t+1)\{(2t^2-7t-9)=0\}$ $(t+1)\{(t+1)(2t-9)=0\}$	A realisation that $(t+1)$ is a factor.	dM1
	$\{t=-1 \text{ (at } A)\}$ $t=\frac{9}{2}$ at B	$t=\frac{9}{2}$	A1
	$x=\left(\frac{9}{2}\right)^2-8\left(\frac{9}{2}\right)=\frac{729}{8}-36=\frac{441}{8}=55.125$ or awrt 55.1	Candidate uses their value of t to find either the x or y coordinate	ddM1
	$y=\left(\frac{9}{2}\right)^2=\frac{81}{4}=20.25$ or awrt 20.3	One of either x or y correct. Both x and y correct.	A1 A1
	Hence $B\left(\frac{441}{8},\frac{81}{4}\right)$	awrt	(6)
			[12]