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Pure Core 4

Report on the Examination

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General

A full range of marks was seen on this paper. It was clear that some students were very poorly prepared for the examination, showing little knowledge of the specification. In contrast, there were some outstanding performances in which students showed an excellent grasp of the mathematics across the specification. Most students were able to demonstrate some knowledge of the specification with questions **1** and **2** generally being done well. Most students attempted all the questions, although the latter parts of questions **6** and **7** were sometimes omitted or treated superficially. These question parts, together with question **4(a)(ii)** presented the greatest difficulty, with few gaining full marks. Presentation was generally good, with working set out clearly, although there were some students whose work was difficult to follow through poor setting out or illegible hand-writing.

Question 1

This question provided a good start for the students with about 75% of the entry gaining 5 or more of the 7 marks available. However, many did not use the most efficient algebraic methods and will have paid a time penalty later in the paper.

Part **(a)** Most students gave a correct answer showing clearly their use of the Remainder Theorem. Those who used long division gained no credit.

Part **(b)(i)** A full justification was required to gain the mark, but some students just wrote $\mathrm{d} = 3$ and the given answer. An uncommon error was to find d as -4.

Part **(b)(ii)** Students were expected to find the value of a by inspection, although some used long division or equating coefficients. Some students did not respond to the request to write $\mathrm{g}(x)$ as a product of three linear factors until it was required in part **(iii)**.

Part **(b)(iii)** Many students took the expected route of fully factorising the denominator and cancelling down the common factors with those of $\mathrm{g}(x)$ and simplifying the result. Other methods, such as multiplying by the denominator and equating coefficients or using long division, were usually successful but unnecessarily time consuming.

Question 2

Overall, this question was answered well and was a good source of marks for most students. Over half the entry scored 10 or more of the 11 marks available.

Part **(a)** The vast majority of students answered correctly, showing confidence in finding partial fractions. The most common method was to substitute $x = 3$ and $x = -\frac{1}{3}$, although some students used simultaneous equations and others hybrid methods. Few mistakes were seen, although some students did manage to swap over the given coefficients A and B.

Part **(b)(i)** Many students approached this by writing out the full expression using their values of A and B and using the binomial expansions of both terms clearly and correctly to arrive at the correct answer. The common errors were not to take the 3 out of $(3-x)$ correctly, if at all, and/or to get confused with the index -1, this sometimes being combined with the 2 (value of A) to become $\frac{3}{2}$, 6 or $\frac{1}{6}$. The other common error was to drop the minus sign in the expansion of $\left(1-\frac{x}{3}\right)^{-1}$. Although most students made a clear attempt to combine the two expansions, some made sign errors. Very few students attempted to use the binomial theorem result from the formula book, or did not use the partial fractions and multiplied their expansions together with $(7x-1)$, although a few correct solutions were seen using this method.

Part **(b)(ii)** Most students were aware that this question involved the range of validity of the expansions, with many finding the range correctly but not relating this to the given value of 0.4; this didn't answer the question. Other students seemed to just make a guess, such as '*not enough terms have been considered*' or even '*because 0.4 is a decimal number'*.

Question 3

The majority of students did well on part **(a)** of this question and well over half of the cohort scored 6 marks or more. Most students were less successful on part **(b)**, although many started by writing down several relevant looking identities.

Part **(a)(i)** The vast majority of students answered correctly, with relatively few finding $\alpha = 56.3°$ rather than the expected $\alpha = 33.7°$.

Part **(a)(ii)** Although most students scored the 3 marks some failed to state the minimum value where an explicit statement was required. Similarly, some students just wrote down the value of x at which the minimum occurred without justifying it. Less common errors were to use $\cos^{-1}\left(-\frac{1}{R}\right)$ or to assume the minimum was at $x = 90°$.

Part **(b)(i)** Many students wrote the given expression in terms of $\sin x$ and $\cos x$, but then went on to treat the identity as if it were an equation, often equating to zero. Those who used a common denominator or took out a factor of $\frac{1}{\sin x}$ usually completed successfully.

Part **(b)(ii)** Although most students recognised that they had to use the result from part **(i)**, many failed to consider both $\cot x = 0$ and $\cos 2x = 0$ and so gained no credit. Of those who did consider both these equations many students thought $\cot x = 0$ had no solutions, or they changed it to $\tan x = 0$. Most students did solve $\cos 2x = 0$ correctly, although not always giving both solutions within the given range.

Question 4

This question was not done well, with fewer than 10% of the entry gaining full marks, although over half did gain 4 or more of the 8 marks available.

Part **(a)(i)** Most students were successful here; using implicit differentiation as expected and clearly showing the derivatives of the three terms. Those few who solved y in terms of x before differentiating were rarely successful, particularly those who thought that $y = x - \sqrt{8}$. Some students left their derivative in terms of x and y rather than the requested p and q.

Part **(a)(ii)** Many students gained the first 2 marks for a correct expression for the tangents, especially those who used $y - y_1 = m\left(x - x_1\right)$. The relatively few who tried to use $y = mx + c$ often made an error in finding c in terms of p and q. The common mistake was a sign error in the gradient at $\left(p, -q\right)$, reducing the attempt at solving the two tangent equations simultaneously to nonsense. The simplest way to establish the requested result is to add the two tangent equations, but few students did this. Most equated the two expressions for y and attempted to find x, often making an algebraic error. This is a valid method if the expression for x is substituted back in a tangent equation and $y = 0$ is shown, but few attempted to do this. Other students assumed $y = 0$ and attempted to find an expression for x but most were unsure how to proceed from there. Other students wrote their tangent equations in the form $ax + by + c = 0$ and equated them both to zero. Again it is possible to solve the problem this way, although most students could not complete successfully.

Part **(b)** A variety of attempts at this question were seen, including those who thought they had been asked to find the derivative using the parametric equations, which were irrelevant. Some students seemed not to understand the question at all. Those who squared the expressions for x and y and subtracted were usually successful, although many made an error in the squaring by missing a term or making a sign error. Very few factorised $x^2 - y^2$ and then used the parametric equations to eliminate t, but some students did establish the given result by other algebraically valid eliminations of t.

Question 5

A full spread of marks was seen on this question, with about 20% of the entry gaining full marks, and about another 20% only scoring 2 marks or fewer.

Part **(a)** Many students attempted this integral by using parts and produced some complicated looking expressions using incorrect integrals. This integral cannot be done by parts; some students did realise this and swapped to the required substitution method. Those who did use an appropriate substitution were usually successful, with an occasional error being made in the coefficient. Few students could do the integral by inspection.

Part **(b)** Most students knew they had to separate the variables and integrate both sides, with the vast majority using their result from part **(a)**. Most used correct notation in the separation, but some students read e^{2y} as $e^{2}y$ and some did not multiply across correctly by e^{2y}. Of those who did, most solved the integral of e^{2y} correctly. Those who also solved the integral in x correctly usually went on to complete successfully, although the common error was to think $\ln(a+b)=\ln a+\ln b$. Students who did used their result from part **(a)**, whatever it was, could score method marks for using $(1,0)$ to find a value for their constant, and to use logarithms to express their final answer as $y=\mathrm{f}(x)$.
Most students did include a constant, and did attempt to take logs to obtain a final solution.

Question 6

There was a wide range of responses to this question and relatively few students gained full marks, although about 25% of the entry did score 11 or more of the 15 marks available.

Part **(a)(i)** Most students answered this correctly, others making the occasional sign error.

Part **(a)(ii)** Most students found either the vector $\overrightarrow{BC}$ or $\overrightarrow{CB}$ correctly, but some students mixed the two up and others made sign errors. Most students did work with the vectors $\pm\overrightarrow{AC}$, $\pm\overrightarrow{BC}$ so focussing on the required angle, although some found the obtuse rather than the acute angle; they could gain full marks if they explained why their result was negative. Few students tried to work with the wrong vectors, $\overrightarrow{OA}$ and $\overrightarrow{OB}$ being the usual choice of those who did. The scalar product based formula for $\cos\theta$ is well known, but some students did approach the problem using the cosine rule with most being successful by this method. Some students only scored 3 of the marks as they gave insufficient justification as to how their expression became that given in the question.

Part **(b)** Most students knew what was required here although many scored only 1 mark due to poor notation. $\overrightarrow{AC}=\mathbf{a}+\lambda\mathbf{d}$ or line $AC=\mathbf{a}+\lambda\mathbf{d}$ or similar were common rather than $\mathbf{r}=\mathbf{a}+\lambda\mathbf{d}$

Part **(c)(i)** Many students showed they understood the question and found a vector equation for line BD and solved simultaneously with that of line AC to find values for the parameters. The fairly common error was to use the vector $\overrightarrow{BD}$ itself as the equation of the line, which resulted in no credit. Some students omitted their parameter in the third equation and so found $p=-6$ without attempting to find the value of their parameter. Many did find the value of p correctly, with relatively few making mistakes in the algebra although some had difficulty in solving $\frac{1}{2}p=-6$. Those few who chose point D as a point on line BD made the algebra unnecessarily complex and often solved incorrectly.

Part **(b)(ii)** The expected approach to showing $ABCD$ is a rhombus was to find the lengths of the four sides. Although some students started to do this either by finding the vectors first, or going directly to their moduli, few attempted all four and so could gain no credit. If they had made an earlier mistake 2 marks were still available, but many students seem to give up if they weren't getting equal lengths. Those who just stated all lengths are 7 (from part **(a)** and $\left|\overrightarrow{BC}\right|$) or showed no detail of where 7 came from for each side, gained no credit. Those relatively few students who found the scalar product $\overrightarrow{AC}\bullet\overrightarrow{BD}=0$ and claimed the diagonals are perpendicular hence the shape is a rhombus, scored 2 of the 4 marks as they didn't consider that the diagonals also needed to bisect each other. Few students drew a diagram for this part of the question, which might have aided their solution.

Question 7

The range of marks seen in this question typically varied between up to 4, scored by about 85% of the students, to up to 7 scored by about 25%. Very few completed the whole question successfully, with many making only superficial attempts at part **(b)**.

Part **(a)(i)** and **(ii)** Most students answered correctly with only the occasional arithmetic error.

Part **(a)(iii)** Many students scored 2 out of the 3 marks available; although they understood what to do and followed correct manipulation, taking logarithms correctly, they didn't respond to the request for integers in the final expression. Those students who didn't simplify the expressions in their working often made an error.

Part **(b)(i)** Students were expected to attempt the required differentiation using the quotient or chain rule. Those that did this often made a sign error or had a wrong coefficient, sometimes including a t in their expression. Although some excellent algebra was seen in deriving the given result using the initial expression for N, many were unsuccessful in eliminating the $9e^{-\frac{t}{8}}$ term, and some gave up without trying. However, many students tried to rearrange the given equation for N to make $e^{-\frac{t}{8}}$ or t the subject, and then differentiate. It is difficult to see why they considered this approach would be helpful, but many made an algebraic error and little progress. Some tried to solve the differential equation and made little or no progress.

Part **(b)(ii)** Most students who attempted this understood the question to mean finding the maximum value of N rather than that of the growth rate $\frac{dN}{dt}$, but most had to abandon their attempt. Of those who did differentiate the growth rate expression, most wrote $\frac{d^2N}{dt^2}$, which was condoned; if their attempt led to the maximum growth rate occurring when $N = 250$ they usually completed successfully; those who had worked with inequalities were condoned. Some students thought $T = 250$ was the required answer. Few students saw that the maximum value of the growth rate was at $N = 250$ by inspection, (due to the symmetries of the quadratic function involved) but some concise and corrects solutions to the problem were seen.

Mark Ranges and Award of Grades

Grade boundaries and cumulative percentage grades are available on the Results Statistics page of the AQA Website: http://www.aqa.org.uk/over/stat.html

Converting Marks into UMS marks

Convert raw marks into Uniform Mark Scale (UMS) marks by using the link below.

UMS conversion calculator www.aqa.org.uk/umsconversion