16 - Quadratic Equations

Quadratics can be solved by:
- factorisation
- using the formula
- completing the square
- graphically (using x intercept/s - only when line touches x axis)

Solving
(x-5)(x-5) = 0
- One of the brackets has to = 0 in order to multiply to get 0
- so (x-5) = 0
- so x = 5
- X is inverse of what is in the bracket

Always show that brackets = 0 because it is an equation not an expression

Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$ax^2 + bx + c = 0$

Completing the square
a/co-efficient of x^2 must be 1

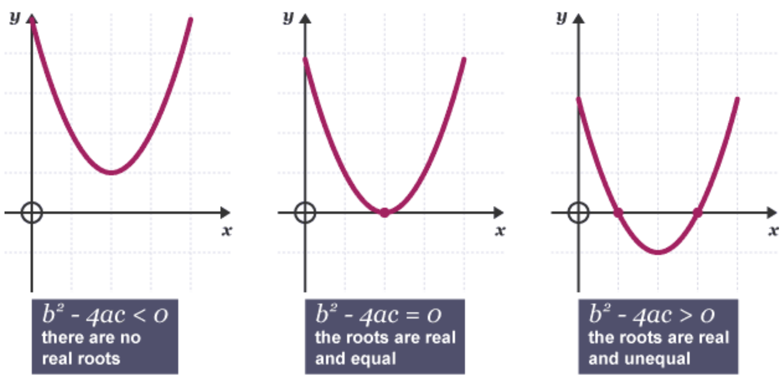
$x^2 + 5x + 4 = 0$
1. Divide b by 2
2. Put $x^2 + b/2x$ into bracket
3. Expand to find $(b/2)^2$
4. Repeat step 2, but $-(b/2)^2$ and +c

$5 \div 2 = 5/2$
$(x^2 + 5/2x)$
$x^2 + 5x + 25/4 — 5/2x = 25/4$
$(x^2 + 5/2x) -25/4 + 4$
$(x^2 + 5/2x) - 2.25 = 0$

Solving
e.g. $(x - 2)^2 - 5 = 0$
$(x - 2)^2 = 5$
$(x - 2) = \sqrt{5}$
$x = \sqrt{5} + 2$
or $x = - \sqrt{5} + 2$
(quadratics often have 2 solutions)

The discriminant $\sqrt{b^2 - 4ac}$

Positive = $b^2 - 4ac > 0$ — 2 solutions
Neutral = $b^2 - 4ac = 0$ — 1 solution
Negative = $b^2 - 4ac < 0$ — no solution
Math error = no solution a.k.a on graph the parabola does not cut the x axis



The Turning Point
Vertex of the parabola - halfway between the roots
If the turning point is the highest point = maximum point

If the turning point is the lowest point = minimum point

By completing the square of a quadratic, you can work out the turning point

$y = a(x+p)^2 + q$
a > 0 — minimum point (below x axis)
a < 0 — maximum point (above x axis)

turning point = (p,q)

$a(x+p)^2 + q$
p = inverse of p — you must change the sign
q = q

Completing the square when a >1
$y = 2x^2 + 12x + 1$

1. Divide everything by co-efficient of x to put it outside the bracket — $2(x^2 + 6x + ½)$
2. Complete the square within the bracket — $2[(x + 3)^2 -9 + ½]$, $2[(x + 3)^2 -8.5]$
3. Expand (by co-efficient) to eliminate outside bracket — $2(x + 3)^2 -17$
4. Then locate p and x — p = -3, q = -17 — turning point (-3,-17); a = 2 > 0 — minimum point

Linear and Non-linear simultaneously

$y = x^2$ — these = each other because they both = y
$y = 3 - 2x$

$x^2 = 3-2x$ — if we move these to one side, we get a quadratic
$x^2 + 2x -3 = 0$

$(x+3)(x-1) = 0$ — Factorise/use equation to workout x values
x = -3, x = +1

$y = x^2$ — Input to find x
—> $y = -3^2 = 9$
—> $y = 1^2 = 1$

(-3,9) (1,1) — Co-ordinates

Quadratic inequalities

1. Solve the inequality as an equation
2. Check if graph is min/max point (co-efficient of x > or < 0)

$x^2 - b > 0$ — x > highest, x < lowest
$x^2 - b < 0$ — lowest < x < highest