revisely

4 - Number and Sequences

Sequences

- **Linear/Arithmetic Sequence** ⟶ a number pattern that increases/decreases by the same amount between terms ($dn+e$)
- Common Difference ⟶ the difference between 2 terms of a linear/arithmetic sequence (when the difference is the same

e.g. Linear →	1st		2nd		3rd		4th	→terms/n
	14		20		26		32	→sequence
		+6		+6		+6		→common difference

6 = common difference ↗
nth term = 6n + 8
d in front of n = common difference
n = term number
$\pm e$ = extra difference = 0th term

- **Quadratic Sequences** ⟶ $xn^2 \pm e$

Sequences that involve n^2
We also know that a sequence is quadratic if the 2nd difference is the same.

Sample Sequence =	1st		2nd		3rd		4th		5th	→terms/n
	-1		5		15		29		47	→sequence
		+6		+10		+14		+18		→1st difference
			+4		+4		+4			→2nd difference(constant)

① To find what value goes in front of n^2, we divide the 2nd difference by 2.
$xn^2+e \longrightarrow x$ = 2nd difference ÷ 2

In the sample sequence, 4 = 2nd difference
so $x = 4 \div 2 = 2 = 2n^2$

② To find the rest of the nth term (e), we use a comparison method/table ⟶

1st row = Terms
2nd row = xn^2, the value of x changes depending on 2nd difference ÷ 2

3rd row = original quadratic sequence
4th row = the difference between xn^2 to the original quadratic sequence.

Term (n)	1	2	3	4	5
$2n^2$	2	8	18	32	50
Sequence	-1	5	15	29	47
Subtract (d) /difference	**-3**	**-3**	**-3**	**-3**	**-3**

Now we know that our nth Term rule = $2n^2 - 3$

- **Complex Quadratic ⟶**

However, A quadratic sequence can also appear as → $xn^2 + bn + e$

x = multiplier of n^2 e.g. $2n^2/3n^2$

b = multiplier of n

x/b are variables, n^2/e are constant

e = extra value

To work out a $xn^2 + bn + d$ sequence
e.g. 5 15 31 53

① Make the table after working out 1st/2nd difference
2nd difference = 6 → so $x = 3$ (6/2)

Term (n)	1	2	3	4
Sequence	5	15	31	53
$3n^2$	3	12	27	48
Subtract/Difference (e)	2	3	4	5

Here we see that the difference between the original and $3n^2$ is not constant.
Now we need to create a new table, a different table to go back to the 0th term.

Term (n)	0	1	2	3	4
Original sequence (e)	1	5	15	31	53
1st Diff ($x+b$)	+4	+10	+16	+22	
2nd Diff ($2x$)	+6	+6	+6		

Using the formulas table, what we already know and the 0th term ⟶

$2x = 6 \Longrightarrow$

$x = 3 \Longrightarrow$

$x + b = 4 \Longrightarrow$

$b = (4 - 3) = 1 \Longrightarrow$

$e = 1$

$xn^2 + bn + e$

$3n^2 + 1n + 1$

The pattern we see is now $3n^2 + n + 1$ = nth term rule

- **Special Sequences ⟶**

2^n	2,4,3,16,32,64
10^n	10,100,1000,10000
Prime Numbers	2,3,5,7,11,13,17,19
n^2	1,4,9,16,25,36,47,64
n^3	11,8,27,64,125,216
Triangular numbers = $½n(n+1)$	1,3,6,10,15,21

- **Fibonacci Sequence ⟶** a b a+b

Next term = 2 previous terms added

e.g. 1 1 2 3 5 8

a,b, a+b, a+2b, 2a+3b, 3a+5b, 5a+8b, 8a+13b, 13a+21b

the 3rd term always = 1st + 2nd term

- **Geometric ⟶**

Geometric involves multiplication of terms e.g.

7	21	63	189	(×3)
a_1	a_2	a_3	a_4	

For geometric sequences we need to find the common ratio = 2nd term ÷ 1st term = a_2/a_1
e.g. 21/7 = 3

then we use the formula $a_1 \times r^{n-1}$

e.g. $5 \times 2^{n-1}$

1st term =
① 1-1 = 0 ($^{n-1}$)
② $2^0 = 1$ (r^{n-1})
③ 5 × 1 = 5 ($a_1 \times r^{n-1}$)
④ 5 = a_1 in sequence

2nd term =
① 2-1 = 1
② $2^1 = 2$
③ 5 × 2 = 10 ($a_1 \times r^{n-1}$)
④ 10 = a_2

a_1 varies depending on 1st term of the geometric sequence

r, varies depending on the common ratio

n varies depending on which term you are looking for

Sequence =	5	10	20	40
	a_1	a_2	a_3	a_4

Sequence Recap

Geometric sequences =	$a_1 \times (r)^{n-1}$
Arithmetic/Linear =	$dn + e$
Quadratic =	$xn^2 + e$
Fibonacci =	$a, b, (a + b)$

a = **1st** term of geo sequence
r = common ratio of geo sequence
e = extra value
d = common difference
$2d$ = 2nd difference
n = Term

To workout e (extra value), we work out the difference original sequence and the current.
e = the difference between them
e.g.

$2n^2$ ⟶	2	8	18	32	⟹	e = 2 here
$2n^2 + e$ ⟶	4	10	20	34	⟹	

* $a_1 \times (r)^{n-1}$ not $(a_1 \times r)^{n-1}$