8 - Algebraic Manipulation

BIDMAS - Brackets, Indices, Division, Multiplication, Addition, Subtraction

Negatives and Indices:

- \(-x^2 = -x^2\)
- Even Index: \((-x)^2 = +x^2\)
- \((-x)^3 = -x^3\)
- Odd Index: \((-x)^3 = -x^3\)

Expanding Brackets:

\((x + 2)^2 = (x +2)(x +2)\)
e.g. \((3-x)(3-x) = 9 - 6x + x^2\)
when in brackets, — belongs to x, so \((-x)^2\)

Expanding Brackets:

Claw Method =

\((x-9)(x+6)\)

\(x^2 + 6x - 9x - 54\)

Factorising:

When coefficient of x = 1:
e.g. \(x^2 -7x + 10\)

\(-7 = \text{sum} \quad 10 = \text{product}\)

\(-5 + -2 = -7\)
\(-5 \times -2 = 10\)

Factorising difference of two squares \(x^2 - y^2\):
e.g. \(x^2-100 = (x+10)(x-10)\)

must have \(\sqrt{a}\) and \(\pm \sqrt{b}\)

\(x^2 - 64\)

\(\uparrow \uparrow\)

\(a \quad b\)
Factorising completing the square \( x^2 + y^2 + 2yx \):

e.g. \( x^2 + 14x + 49 \)

\[
\begin{align*}
\uparrow & \quad \uparrow & \quad \uparrow \\
x^2 + 2yx + y^2
\end{align*}
\]

1. Divide second term by \( 2x \) to get \( y \) —— \( y = 14x \div 2x = 7 \)
2. Square why to check \( y^2 \) —— \( 7^2 = 49 = y^2 \)
3. Put into brackets = \( (x + 7)^2 \)

Algebraic Manipulation

- An equation is anything with an equal sign i.e. it is true for only certain value/s of \( x \)
e.g. \( x^2 - 16 = 0 \) —— \( x \) has a particular value
- An identity is true for all values of \( x \) i.e. like a rule or formula
e.g. \( (x+1)^2 = x^2 + 2x + 1 \) —— \( x \) can be input as any value
To prove it is an identity, the formula must be able to work when the value is not only an integer

Expansion of 2+ Binomials

1. Expand 1st two brackets + simplify
e.g. \( (x+3)(x-4)(x-10) \)
2. Put answer from expansion into brackets
3. Expand 3rd bracket with new bracket
4. Simplify - highest power first
e.g. \( (x+3)(x-4) = x^2 - x - 12 \)
2. \( (x^2 - x - 12) \)
3. \( (x-10)(x^2 - x - 12) = x^3 - x^2 - 12x - 10x^2 + 10x + 120 \)
4. \( (x^3 - 11x^2 - 2x + 120) \)