1. The points A, B and C have position vectors $3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$, $-\mathbf{i} + 6\mathbf{k}$ and $7\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}$ respectively. M is the midpoint of BC.

(a) Show that the magnitude of $\overrightarrow{OM}$ is equal to $\sqrt{17}$. [2]

(b) Point D is such that $\overrightarrow{BC} = \overrightarrow{AD}$. Show that position vector of the point D is $11\mathbf{i} - 8\mathbf{j} - 6\mathbf{k}$. [3]

2. The equations of two lines are

$$\mathbf{r} = \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} \text{ and } \mathbf{r} = \begin{pmatrix} -1 \\ 8 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 1 \\ -5 \end{pmatrix}.$$

Find the coordinates of the point where these lines intersect. [4]

3. (i) Write down a vector equation of the line through the points A (5, 1, 9) and B (8, 7, 15). [1]

P is the point (11, –2, 15).

(ii) Show that triangle APB is isosceles and find angle PAB. [4]

The point D lies on the line through A and B. Angle PAD = angle PDA.

(iii) Find the coordinates of D. [4]

4. Points A, B and C have position vectors $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} -4 \\ 0 \\ 3 \end{pmatrix}$ respectively.

(a) Find the exact distance between the midpoint of AB and the midpoint of BC. [4]

Point D has position vector $\begin{pmatrix} x \\ -6 \\ z \end{pmatrix}$ and the line CD is parallel to the line AB.

(b) Find all the possible pairs of x and z. [4]

5. The points A and B have position vectors $\begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix}$ respectively.

(a) Find the exact length of AB. [2]

(b) Find the position vector of the midpoint of AB. [1]

The points P and Q have position vectors $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix}$ respectively.

(c) Show that $ABPQ$ is a parallelogram. [3]

6. The points A, B and C have position vectors **a**, **b** and **c**, relative to an origin O, in three dimensions. The figure $OAPBSCTU$ is a cuboid, with vertices labelled as in the following diagram. M is the midpoint of AU.

Prove that the lines OM and AS intersect, and find the position vector of the point of intersection. [9]

7. Points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$. Point C lies on AB such that $AC : CB = p : 1$.

(a) Show that the position vector of C is $\dfrac{1}{p+1}(\mathbf{a} + p\mathbf{b})$. [3]

It is now given that $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ and $\mathbf{b} = -6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}$, and that C lies on the y-axis.

(b) Find the value of p. [4]

(c) Write down the position vector of C. [1]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Guidance
1		a	$\overrightarrow{OM}=\frac{1}{2}\left(\overrightarrow{OC}+\overrightarrow{OB}\right)=3\mathbf{i}-2\mathbf{j}+2\mathbf{k}$ $\left\lvert\overrightarrow{OM}\right\rvert=\sqrt{3^2+(-2)^2+2^2}=\sqrt{9+4+4}=\sqrt{17}$	M1(AO1.1) E1(AO2.1) [2]	Attempt to find $\overrightarrow{OM}$ AG
		b	$\overrightarrow{BC}=8\mathbf{i}-4\mathbf{j}-8\mathbf{k}$ $\overrightarrow{OD}=\overrightarrow{OA}+\overrightarrow{AD}=\overrightarrow{OA}+\overrightarrow{BC}$ $\overrightarrow{OD}=3\mathbf{i}-4\mathbf{j}+2\mathbf{k}+8\mathbf{i}-4\mathbf{j}-8\mathbf{k}$ $=11\mathbf{i}-8\mathbf{j}-6\mathbf{k}$	M1(AO1.1) E1(AO2.4) E1(AO2.1) [3]	Express $\overrightarrow{OD}$ in terms of known vectors AG An intermediate step must be seen
			Total	5	
2			Any two from $3+\lambda=-1-3\mu$ $\lambda=8+\mu$ $2+3\lambda=2-5\mu$ solve simultaneously to obtain a value of λ or μ $\lambda=5$ or $\mu=-3$ (8, 5, 17) isw	B1 M1 A1 A1	may be in vector form

				[4]	allow vector form **Examiner's Comments** Most candidates scored full marks on this question. Very few were unable to make some progress, and those that went wrong usually did so through careless slips.
			Total	4	
3		i	$r = \begin{pmatrix} 5 \\ 1 \\ 9 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 6 \\ 6 \end{pmatrix}$ oe isw	B1 [1]	**NB** eg $r = \begin{pmatrix} 8 \\ 7 \\ 15 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ — **B0** for just the RHS, must see "$r =$" oe **Examiner's Comments** Most knew what to do here. A few candidates wrote an expression and not an equation, or missed out the parameter, thus losing an easy mark.
		ii	$6\times3-3\times6+6\times6=\sqrt{6^2+(-3)^2+6^2}\times\sqrt{3^2+6^2+6^2}\cos A$ $36 = 81\cos A$ or $-36 = 81\cos A$ or better $A = 63.6°$ or 1.11 rad	M1 A1 A1	allow sign errors and 1 algebraic slip eg omission of power — $\cos A = \dfrac{9^2+9^2-(their\sqrt{90})^2}{2\times9\times9}$ $PB = 3\sqrt{10}$

<table>
<tr><td></td><td></td><td></td><td>eg $AB = \sqrt{3^2 + 6^2 + 6^2}$ and $AP = \sqrt{6^2 + (-3)^2 + 6^2}$
[so isosceles]</td><td>B1

[4]</td><td><table><tr><td>if obtuse angle found, clear explanation needed if acute angle stated as answer

NB AB = 9 and AP = 9 stated is sufficient
B0 if answer spoiled</td><td>A = 63.6° or 1.11 rad

NB 58.2° or $\cos\theta = \frac{\sqrt{10}}{6}$</td></tr></table>
Examiner's Comments

This was done well by most candidates. A few made errors in calculating the lengths, and some calculated either the external angle of the triangle or angle APB or ABP.</td></tr>
<tr><td></td><td></td><td>iii</td><td>$\overrightarrow{PD} = \begin{pmatrix} 5+3\lambda \\ 1+6\lambda \\ 9+6\lambda \end{pmatrix} - \begin{pmatrix} 11 \\ -2 \\ 15 \end{pmatrix}$ oe

$(3\lambda - 6)^2 + (3 + 6\lambda)^2 + (6\lambda - 6)^2 = 9^2$ oe

$\lambda = \frac{8}{9}$[or 0]

$\left(\frac{23}{3}, \frac{19}{3}, \frac{43}{3}\right)$</td><td>M1

M1

A1

A1</td><td><table><tr><td>allow one algebraic slip eg omission of one power

NB

$\lambda = \frac{8}{3}$ if direction vector is</td><td>NB $\overrightarrow{PD} = \begin{pmatrix} 3\lambda - 6 \\ 3 + 6\lambda \\ 6\lambda - 6 \end{pmatrix}$</td></tr></table></td></tr>
</table>

				[4]	$\begin{pmatrix}1\\2\\2\end{pmatrix}$	
			Alternatively	M1		
			$AD^2 = 9^2 + 9^2 - 2 \times 9 \times 9 \times \cos(180 - 2 \times 63.6)$			
			$(3\lambda)^2 + (6\lambda)^2 + (6\lambda)^2 =$ their AD^2 oe	M1* dep	NB $AD = 8$	
			$\lambda = \frac{8}{9}$	A1	$\lambda = \frac{8}{3}$ if direction vector is $\begin{pmatrix}1\\2\\2\end{pmatrix}$	
			$\left(\frac{23}{3}, \frac{19}{3}, \frac{43}{3}\right)$	A1		
				[4]		
			Alternatively	M1		
			$\overrightarrow{PE} = \begin{pmatrix}5+3\lambda-11\\1+6\lambda--2\\9+6\lambda-15\end{pmatrix}$	M1	*E* is the foot of the	

			Answer	Marks	Guidance	
			$\overrightarrow{PE} \cdot \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = 0$	A1	perpendicular from P to AB	
			$\lambda = \frac{4}{9}$	A1		
				[4]		
			$\left(\frac{23}{3}, \frac{19}{3}, \frac{43}{3}\right)$		from $\overrightarrow{AD} = 2\overrightarrow{AE}$	
				M1		
				M1		
			Alternatively			
			$\overrightarrow{PD}$ found as above	A1		
			$\overrightarrow{AP} \cdot \overrightarrow{AD} = \overrightarrow{DA} \cdot \overrightarrow{DP}$ oe	A1	eg $\begin{pmatrix} 6 \\ -3 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 3\lambda \\ 6\lambda \\ 6\lambda \end{pmatrix} = \begin{pmatrix} -3\lambda \\ -6\lambda \\ -6\lambda \end{pmatrix} \cdot \begin{pmatrix} 6-3\lambda \\ -3-6\lambda \\ 6-6\lambda \end{pmatrix}$	or $\begin{pmatrix} -2 \\ -6 \\ -6 \end{pmatrix} \cdot \begin{pmatrix} 6-3\lambda \\ -3-6\lambda \\ 6-6\lambda \end{pmatrix} = 36$ or $\begin{pmatrix} 6 \\ -3 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 3\lambda \\ 6\lambda \\ 6\lambda \end{pmatrix} = 32$
			$\lambda = \frac{8}{9}$	[4]		
			$\left(\frac{23}{3}, \frac{19}{3}, \frac{43}{3}\right)$			
					Examiner's Comments	

					Many candidates did not know where to begin and often failed to score. However, a variety of successful approaches were taken and some excellent work demonstrating thorough understanding was seen.
			Total	9	
4		a	Position vectors of midpoints *AB* & *BC* are $\begin{pmatrix}1.5\\0.5\\4\end{pmatrix}\begin{pmatrix}-1\\-0.5\\4\end{pmatrix}$ $2.5^2 + 1^2 (+ 0^2)$ $\text{Distance} = \dfrac{\sqrt{29}}{2}$	M1(AO 1.1a) A1(AO 1.1) M1(AO 1.1) A1(AO 1.1) [4]	Correct method for one midpoint Both midpoints correct ft their midpoints; √ not necessary for M1
		b	$\overrightarrow{AB} = \begin{pmatrix}1\\-3\\2\end{pmatrix} \quad \overrightarrow{CD} = \begin{pmatrix}-4-x\\6\\3-z\end{pmatrix}$ $\overrightarrow{CD} = -2\overrightarrow{AB}$ $-x - 4 = -2 \Rightarrow x = -2$ $3 - z = -4 \Rightarrow z = 7$	M1(AO 3.1a) M1(AO 1.2) A1(AO 1.1) A1(AO 1.1) [4]	For scale factor –2
			Total	8	

5		a	$(1-(-3))^2+(-2-(-1))^2+(5-2)^2\ (=26)$ Length = $\sqrt{26}$ or 5.10 or 5.1 (2 sf)	M1 (AO 1.1a) A1 (AO 1.1) [2]	Attempt. Allow with one sign error — $\sqrt{}$ not nec'y Examiner's Comments This question was very well answered. A few candidates made sign errors.
		b	$\begin{pmatrix}-1\\-1.5\\3.5\end{pmatrix}$	B1 (AO 1.1) [1]	Examiner's Comments A surprisingly large number of candidates simply halved their answer to part (a). Perhaps they did not understand the concept of a "position vector".
		c	$\overrightarrow{BA}=\begin{pmatrix}4\\-1\\3\end{pmatrix}$ $\overrightarrow{PQ}=\begin{pmatrix}5\\1\\3\end{pmatrix}-\begin{pmatrix}1\\2\\0\end{pmatrix}\quad \left(=\begin{pmatrix}4\\-1\\3\end{pmatrix}\right)$ $BA = PQ$ and $BA\ //\ PQ$	M1 (AO 2.1) M1 (AO 1.1)	or quote result for $\overrightarrow{BA}$ from (b) or (a)(i) or similar methods with AQ & BP or AB and QP etc Allow find eg AB and PQ or $\overrightarrow{BA}=\overrightarrow{PQ}$ with arrows or $\|BA\| = \|PQ\|$ & $\|BP\| = \|AQ\|$ shown & stated or $BA\ //\ PQ$ & $BP\ //\ AQ$ shown & SC Incorrect, but equal, vectors BA & PQ with correct conclusion SC B1 Allow without method SC Lengths only seen: M1M0 Just $\|BA\| = \|PQ\|$ A0

			and hence *ABPQ* is a parallelogram (AG)	A1 (AO 2.2a) [3]	stated Both statements needed, dep M1M1 Examiner's Comments Only a minority of candidates answered this question in the most efficient way, using the vector forms of one pair of sides. Many found the vector forms of both pairs of sides. Then they commented either that both pairs consisted of two parallel lines, or they found the lengths of all four sides and commented that both pairs consisted of lines of equal length. Some candidates found the vector form for two opposite sides and then stated that because these two sides are parallel, *ABPQ* is a parallelogram. A few candidates made sign errors while finding vectors. Some did not use correct vector notation. Some candidates (quite reasonably) replaced the column vector notation by the **i**, **j**, **k** notation. Some candidates discussed the "gradients" of opposite sides.
			Total	6	
6			$AU = OS = \mathbf{b} + \mathbf{c}$ $OM = OA + 0.5\,AU = \mathbf{a} + 0.5(\mathbf{b} + \mathbf{c})$ $AS = \mathbf{b} + \mathbf{c} - \mathbf{a}$ Let X lie on OM such that $OX = \mu OM$ Let Y lie on AS such that $AY = \lambda AS$ $OX = \mu(\mathbf{a} + 0.5\mathbf{b} + 0.5\mathbf{c})$ $OY = \mathbf{a} + \lambda(\mathbf{b} + \mathbf{c} - \mathbf{a})$ Let $OX = OY$ $\mu(\mathbf{a} + 0.5\mathbf{b} + 0.5\mathbf{c}) = \mathbf{a} + \lambda(\mathbf{b} + \mathbf{c} - \mathbf{a})$	B1 (AO3.1a) B1 (AO1.1a) M1 (AO2.1) A1 (AO1.1) M1FT (AO3.1a)	One of these stated or implied One correct

			a: $\mu = 1 - \lambda$ b: $0.5\mu = \lambda$ c: $0.5\mu = \lambda$ $\lambda = \frac{1}{3}$ and $\mu = \frac{2}{3}$ Satisfy all three equations hence *OM* and *AS* intersect Point of intersection has position vector $\frac{2}{3}(\mathbf{a} + 0.5\mathbf{b} + 0.5\mathbf{c})$ $= \frac{1}{3}(2\mathbf{a} + \mathbf{b} + \mathbf{c})$ oe	A1 (AO1.1) A1 (AO2.2a) E1 (AO2.4) A1 (AO3.2a) [9]	their *OX* = their *OY*, each in terms of a parameter All three Statement needed or $\mathbf{a} + \frac{1}{3}(\mathbf{b} + \mathbf{c} - \mathbf{a})$
			Total	9	
7		a	$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ $\overrightarrow{AC} = \frac{p}{p+1}(\mathbf{b} - \mathbf{a})$ $\mathbf{c} = \boldsymbol{a} + \frac{p}{p+1}(\mathbf{b} - \mathbf{a})$	M1 (AO 2.1) M1 (AO 1.1) A1 (AO 1.1) [3]	$\overrightarrow{AC}$ attempted, i.t.o. **a**, **b** and *p*. $\mathbf{a} + \overrightarrow{AC}$ attempted

			$= \dfrac{\mathbf{a}(p+1) + p(\mathbf{b}-\mathbf{a})}{p+1}$ $\quad (= \dfrac{1}{p+1}(\mathbf{a} + p\mathbf{b})$ **AG**)		Any correct intermediate form with denominator $p + 1$, and final answer
		b	$\mathbf{c} = \dfrac{2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} + p(-6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k})}{p+1}$ $= \dfrac{(2-6p)\mathbf{i} + (3+4p)\mathbf{j} + (-4+12p)\mathbf{k}}{p+1}$ $2 - 6p = 0$ $\Rightarrow p = \dfrac{1}{3}$ $-4 + 12p = 0$ also satisfied by $p = \dfrac{1}{3}$	M1 (AO 1.1) M1 (AO 1.1) A1 (AO 1.1) B1 (AO 2.1) [4]	
		c	$\mathbf{c} = \dfrac{13}{4}\mathbf{j}$	B1FT (AO 1.1) [1]	ft their p; answer must be of form $k\mathbf{j}$
			Total	8	