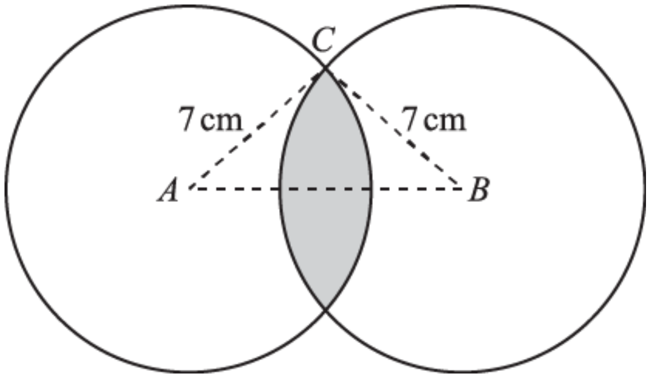
1.



The diagram shows two circles of radius 7 cm with centres *A* and *B*. The distance *AB* is 12 cm and the point *C* lies on both circles. The region common to both circles is shaded.

i. Show that angle *CAB* is 0.5411 radians, correct to 4 significant figures.

[2]

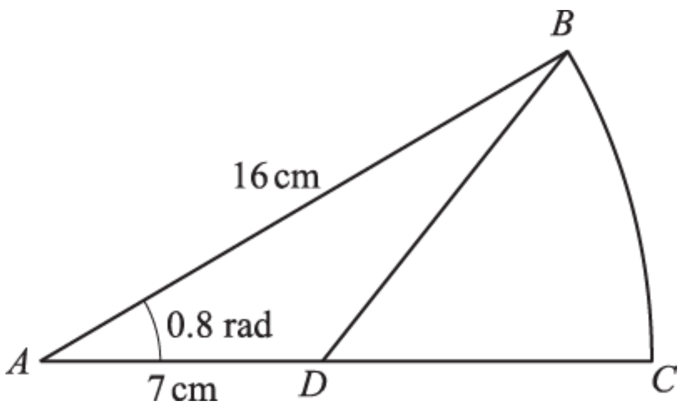
ii. Find the perimeter of the shaded region.

[2]

iii. Find the area of the shaded region.

[5]

2.



The diagram shows a sector *BAC* of a circle with centre *A* and radius 16 cm. The angle *BAC* is 0.8 radians. The length *AD* is 7 cm.

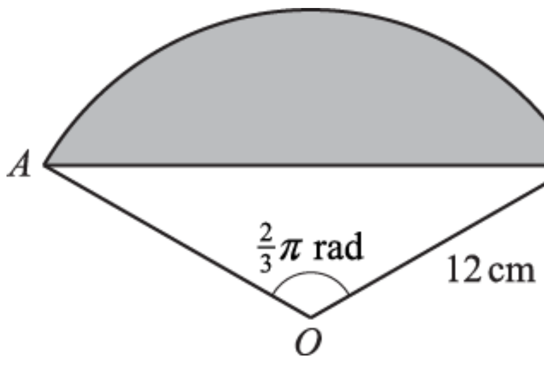
i. Find the area of the region *BDC*.

[4]

ii. Find the perimeter of the region *BDC*.

[4]

3.



The diagram shows a sector *OAB* of a circle, centre *O* and radius 12 cm. The angle *AOB* is $\frac{2}{3}\pi$ radians.

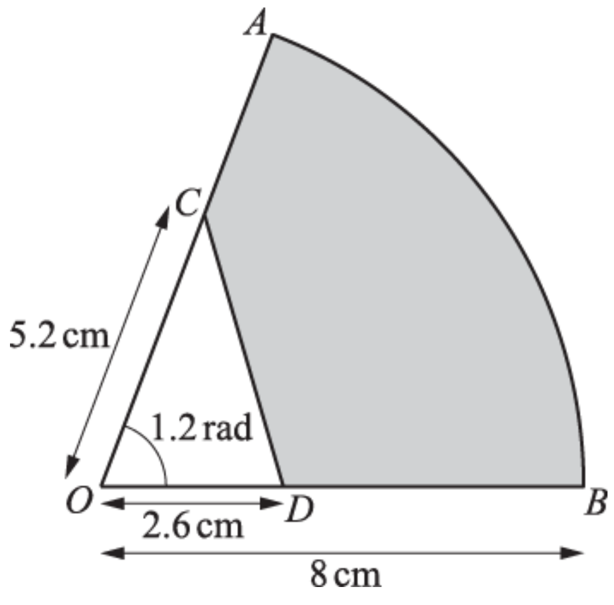
i. Find the exact length of the arc *AB.*

[2]

ii. Find the exact area of the shaded segment enclosed by the arc *AB* and the chord *AB*.

[5]

4.



The diagram shows a sector *AOB* of a circle with centre *O* and radius 8 cm. The angle *AOB* is 1.2 radians. The points *C* and *D* lie on *OA* and *OB* respectively such that *OC* = 5.2 cm and *OD* = 2.6 cm. *CD* is a straight line.

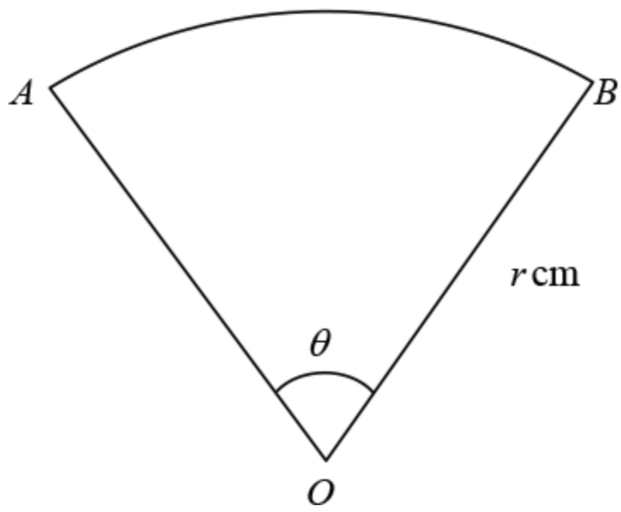
i. Find the area of the shaded region *ACDB*.

[4]

ii. Find the perimeter of the shaded region *ACDB*.

[5]

5.

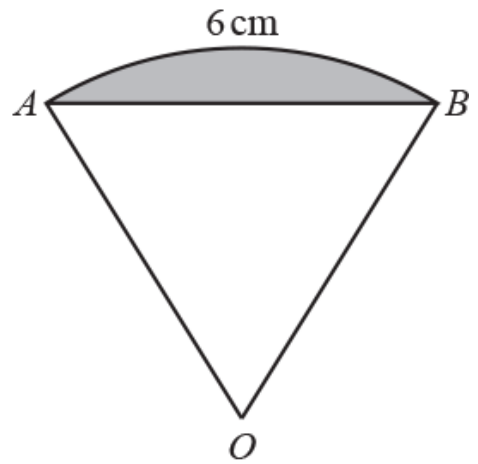


The diagram shows a sector *AOB* of a circle with centre *O* and radius *r* cm. The angle *AOB* is θ radians. The arc length *AB* is 15 cm and the area of the sector is 45 cm^2.

(a) Find the values of *r* and θ. [4]

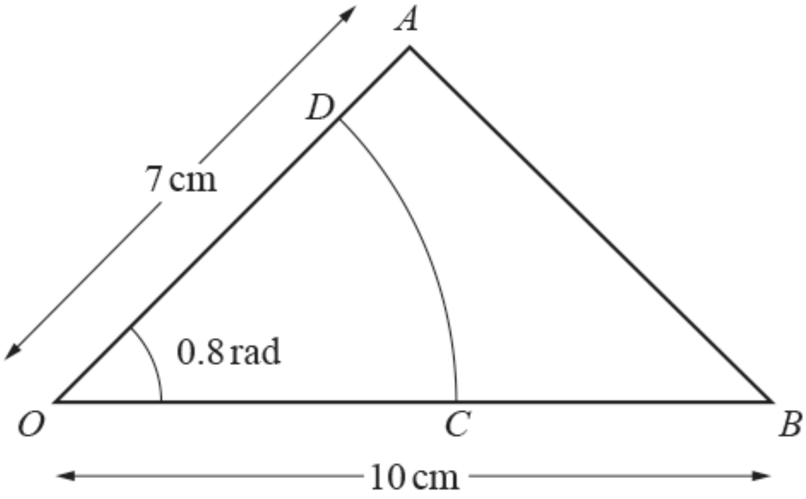
(b) Find the area of the segment bounded by the arc *AB* and the chord *AB*. [3]

6.



The diagram shows a sector *AOB* of a circle with centre *O*. The length of the arc *AB* is 6 cm and the area of the sector *AOB* is 24 cm^2. Find the area of the shaded segment enclosed by the arc *AB* and the chord *AB*, giving your answer correct to 3 significant figures. [6]

7.



The diagram shows the triangle *AOB*, in which angle *AOB* = 0.8 radians, *OA* = 7 cm and *OB* = 10 cm. *CD* is the arc of a circle with centre *O* and radius *OC*. The area of the triangle *AOB* is twice the area of the sector *COD*.

(a) Find the length *OC*. [3]

(b) Find the perimeter of the region *ABCD*. [4]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	$\cos^{-1} {}^{6}/_{7} = 0.5411$ AG	M1	Attempt correct method to find angle *CAB*	Either use cosine rule or right-angled trigonometry Allow M1 for $\cos A = {}^{6}/_{7}$ or equiv from cosine rule If first finding another angle, they must get as far as attempting angle *CAB* for the M1 Allow in degrees or radians
		i		A1	Obtain 0.5411	Must be given to exactly 4sf, as per question If angle found as 31° then conversion to radians must be shown explicitly
		i				<u>Examiner's Comments</u> The majority of candidates were able to find the angle correctly, providing enough detail of the method used to be convincing. Using the cosine rule was a more popular approach than using a right-angled triangle. A surprising number of candidates first gave the required angle in degrees and then showed their conversion from degrees to radians, rather than simply setting their calculator into the appropriate mode.
		ii	arc length = 7 × (2 × 0.5411) = 7.575 perimeter = 15.2	M1	Attempt arc length using 7θ	Must be using $r = 7$ Allow if using $\theta = 0.5411$ not 1.0822 If no method shown then award M1 for value seen in the range [7.56, 7.58] M0 if using angle other than 0.5411 or 1.0822 (inc M0 for 1.0822π) but allow M1 if required angle is intended eg 0.54 or a slip when doubling 0.5411 Allow valid method with degrees, but M0 for 7θ with θ in degrees

		ii		A1	Obtain perimeter as 15.2, or better	Allow equivalent method using fractions of the circle Allow 15.15, or anything that rounds to this with no errors seen
		ii				**Examiner's Comments** Nearly all of the candidates could attempt to find the length of an arc, but not enough thought was given to deciding which angle to use. Most candidates did gain full marks, either by first doubling the given angle to get 1.0822 radians and then using this in the appropriate formula, or by using the original angle of 0.5411 radians and then multiplying the associated arc length by four.
		iii	sector area = ½ × 7^2 x (2 × 0.5411) = 26.51 triangle area = ½ × 7^2 x sin 1.082 = 21.63 area of segment = 4.88 shaded area = 9.76 cm^2	M1*	Attempt area of one sector using (½) × 7^2 x θ, or equiv	Allow if using θ = 0.5411 not 1.0822 Allow M1 if 13.3 or 26.5 seen with no method M0 if using angle other than 0.5411 or 1.0822 (inc M0 for 1.0822π) unless clearly intended as correct angle Allow equivalent method using fractions of the circle Allow valid method with degrees, but M0 for (½)$r^2\theta$ with θ in degrees Condone omission of ½, but no other error May be seen explicitly or implied in method eg as part of ½$r^2(\theta - \sin\theta)$
		iii		M1*	Attempt area of relevant triangle or area of rhombus	Condone omission of ½ from ½$ab\sin\theta$ Allow if using θ = 0.5411 not 1.0822 in (½) × 7^2 x sinθ Allow if attempting area of triangle *ABC* Could be using radians or degrees Allow even if evaluated in incorrect mode If using a right-angled triangle, it must be ½bh, and valid use of trig to find b or h

		iii		A1	Obtain 4.88, or better, either as final answer or soi in method	Could come from finding area of segment but omitting to double it Allow inaccuracy – values in range [4.85, 4.9] Allow even if value not seen explicitly – could be implied by part of a calculation or even by final answer
		iii		M1d*	Attempt correct method to find required area	Must be full and valid method – including attempted use of correct angle and subtraction in correct order Could find area of one segment and double it Other methods are possible eg 2 × sector minus rhombus
		iii		A1	Obtain 9.76, or better	Allow answer rounding to 9.76, no errors seen
		iii				<u>Examiner's Comments</u> Most candidates were able to gain the first two marks on this question, one for attempting the area of a sector and one for attempting the area of a triangle, though this was not always relevant. Many candidates then made no further progress, usually because they had used the angle of 0.5411 incorrectly in $\frac{1}{2}r^2(\theta - \sin\theta)$, or an equivalent method. Some candidates gave a little more thought to the method required, and a number of correct solutions were seen. The most common method was to find the area of a segment and then double it, and a rather more long-winded method was to first find the unshaded area in triangle *ABC*. An efficient and elegant solution was to find the difference between the total area of two sectors and the area of the rhombus, though some candidates mistakenly believed it to be a square. Some,

						otherwise correct, solutions were spoiled by a loss of accuracy in the working and hence in the final answer.
			Total	9		
2		i	sector area = ½ × 16^2 × 0.8 = 102.4	M1*	Attempt area of sector using (½) $r^2\theta$, or equiv	Condone omission of ½, but no other errors Must have r = 16, not 7 M0 if 0.8π used not 0.8 M0 if (½) $r^2\theta$ used with θ in degrees Allow equiv method using fractions of a circle
		i	triangle area = ½ × 16 × 7 × sin 0.8 = 40.2	M1*	Attempt area of triangle using (½) $ab\sin C$ or equiv	Condone omission of ½, but no other errors Angle could be in radians (0.8 not 0.8π) or degrees (45.8°) Must have sides of 16 and 7 Allow even if evaluated in incorrect mode (gives 0.78) If using ½ × b × h, then must be valid use of trig to find b and h
		i		M1d*	Attempt area of sector – area of triangle	Using ½ × 16^2 × (0.8 – sin 0.8) will get M1 M0 M0
		i	area BDC = 62.2 cm^2	A1	Obtain 62.2, or better **Examiner's Comments** This question was very well answered, with the majority scoring full marks. Candidates quoted the relevant formulae accurately, and then work to an acceptable degree of accuracy. As always, a few candidates felt the need to work in degrees rather than radians; not only is this more long-winded but using rounded values can affect the accuracy of the final answer. A few candidates decided that the region specified was itself a sector of a different circle and pursued this method, seeming not fazed when their two radii then appeared to be of different lengths.	Allow answers in range [62.20, 62.25] if > 3sf

		ii	$BD^2 = (16^2 + 7^2 - 2 \times 16 \times 7 \times \cos 0.8)$ $BD = 12.2$	M1	Attempt length of BD using correct cosine rule	Must be correct cosine rule Allow M1 if not square rooted, as long as BD^2 seen M0 if 0.8π used not 0.8 Allow if evaluated in degree mode (gives 9.00) Allow if incorrectly evaluated – using $(16^2 + 7^2 - 2 \times 16 \times 7) \times \cos 0.8$ gives 7.51 Allow any equiv method, as long as valid use of trig Attempting the cosine rule in part **(i)** will only get credit if result appears in part **(ii)**
		ii		A1	Obtain 12.2, or better	Allow any answer rounding to 12.2, with no errors seen Could be implied in method rather than explicit
		ii	arc $BC = 16 \times 0.8 = 12.8$	B1	State or imply that arc BC is 12.8	Allow if 16 × 0.8 seen, even if incorrectly evaluated
		ii	per = 12.2 + 12.8 + 9 = 34.0 cm	A1	Obtain 34, or better **Examiner's Comments** This was equally well done, with many concise and elegant solutions seen. The arc length was invariably correct, and candidates recognised the need to use the cosine rule though some struggled to evaluate this correctly, either by inserting imaginary brackets or by using the incorrect calculator mode. A surprising minority thought that 16 – 7 = 11.	Accept 34 or 34.0, or any answer rounding to 34.0 if > 3sf
			Total	8		
3		i	$\text{arc} = 12 \times \frac{2\pi}{3}$	M1	Attempt use of $r\theta$	Allow M1 if using θ as ⅔ M1 implied by sight of 25.1, or better M0 if $r\theta$ used with θ in degrees M1 for equiv method using fractions of a circle, with θ as 120°

		i	$= 8\pi$	A1	Obtain 8π only **Examiner's Comments** Most candidates could quote the relevant formula and then obtain the correct length of the arc, though a number spoiled their answer by then giving the decimal approximation. Some candidates used more cumbersome methods involving fractions of a circle, which were usually correct though rarely resulted in an exact answer. A significant minority found the length of the chord *AB* instead, either through not reading the question carefully or through a lack of understanding of the terminology.	Given as final answer – A0 if followed by 25.1
		ii	$\text{sector} = \frac{1}{2} \times 12^2 \times \frac{2\pi}{3} = 48\pi$	M1*	Obtain area of sector using $\frac{1}{2}r^2\theta$	Must be correct formula, including $\frac{1}{2}$ Must have $r = 12$ Allow M1 if using θ as ⅔ M0 if $\frac{1}{2}r^2\theta$ used with θ in degrees M1 for equiv method using fractions of a circle, with θ as 120° M1 implied by sight of 151 or better
		ii	$\text{triangle} = \frac{1}{2} \times 12^2 \times \sin\frac{2\pi}{3} = 36\sqrt{3}$	M1*	Attempt area of triangle using $\frac{1}{2}r^2\sin\theta$	Must be correct formula, including $\frac{1}{2}$ Must have $r = 12$ Allow M1 if using θ as ⅔ Allow even if evaluated in incorrect mode (2.63 or 41.8) If using ½ × b × h, then must be valid use of trig to find b and h M1 implied by sight of 62.4, or better
		ii	segment = $48\pi - 36\sqrt{3}$	M1d*	Correct method to find segment area	Area of sector - area of triangle M0 if using θ as ⅔ Could be exact or decimal values
		ii		A1	Obtain either $48\pi - 36\sqrt{3}$ or 88.4	Allow decimal answer in range [88.44, 88.45] if > 3sf

		ii		A1	Obtain $48\pi - 36\sqrt{3}$ only Examiner's Comments Once again candidates could quote the relevant formulae and accurately substitute into them. It was common to see two separate exact values which then became a decimal once combined, possibly because of using a calculator. Candidates must appreciate that if an exact answer is requested then this should be the only final answer provided, and expect to get penalised if the decimal approximation is also given. The other common error was not ensuring that the calculator was in the correct mode when evaluating the area of the triangle.	Given as final answer - A0 if followed by 88.4
			Total	7		
4		i	sector = $\frac{1}{2} \times 8^2 \times 1.2$ (= 38.4)	M1*	Attempt area of sector using $\frac{1}{2} r^2\theta$, or equiv	Must be correct formula, including ½ M0 if 1.2π used not 1.2 M0 if $\frac{1}{2} r^2\theta$ used with θ in degrees Allow equiv method using fractions of a circle
		i	$\frac{1}{2} \times 2.6 \times 5.2 \times \sin 1.2$ (= 6.3)	M1*	Attempt area of triangle using $\frac{1}{2} ab\sin C$ or equiv	Must be correct formula, including ½ Angle could be in radians (1.2 not 12π) or degrees (68.8°) Must have sides of 2.6 and 5.2 Allow even if evaluated in incorrect mode If using $\frac{1}{2} \times b \times h$, then must be valid use of trig to find b and h
		i	38.4 – 6.3 = 32.1	M1d*	Attempt area of sector – area of triangle	Using $\frac{1}{2} \times 8^2 \times (1.2 - \sin 1.2)$ will get M1 M0 M0 Need area of sector > area of triangle
		i		A1	Obtain 32.1, or better Examiner's Comments This question was very well-answered, and the vast	Allow final answers rounding to 32.10 if > 3sf

					majority of candidates gained all of the marks available. The most efficient method was to work in radians throughout, though there were inevitably candidates who decided to use an equivalent method involving degrees instead. As long as there was no subsequent loss of accuracy, then this method was accepted. A few candidates simply assumed that triangle *OCD* was isosceles, and the other common error was to use the formula for the area of a segment rather than considering carefully the question posed.	
		ii	$8 \times 1.2 = 9.6$	M1*	Attempt use of $r\theta$, or equiv	Allow if 8×1.2 seen, even if incorrectly evaluated
		ii	$CD^2 =$ $2.6^2 + 5.2^2 - 2 \times 2.6 \times 5.2 \times \cos 1.2$	M1*	Attempt use of cosine rule, or equiv, to find *CD*	Must be correct cosine rule Allow M1 if not square rooted, as long as CD^2 seen M0 if 1.2π used not 1.2 Allow if incorrectly evaluated, inc mode Allow any equiv method, as long as valid use of trig
		ii	$CD = 4.90$ or $\sqrt{24}$	A1	Obtain $CD = 4.90$ or $\sqrt{24}$	Allow any answer in range [4.89, 4.90], with no errors seen Could be implied in method rather than explicit
		ii	perimeter = 2.8 + 4.9 + 5.4 + 9.6	M1d*	Attempt perimeter of region	(8 – 5.2) + (8 – 2.6) + their *AB* + their *CD* (not their CD^2)
		ii	= 22.7	A1	Obtain 22.7, or better __Examiner's Comments__ Candidates struggled a little more with this part of the question, but it was still very well-answered. Once again, many efficient and effective solutions were seen. Most candidates gained the first mark for finding correctly the required arc length, and could then attempt the cosine rule. Given that this rule is given in the formula book, it	Accept any answer in range [22.69, 22.70] if > 3sf

					was disappointing to see errors when it was initially quoted. Some candidates made errors when evaluating the expression, and these included treating ($b^2 + c^2 - 2bc$) as the coefficient of cosA and omitting to square root the evaluated expression. There was also a loss of accuracy in some solutions from premature approximation, and candidates would be well-advised to make efficient use of their calculator and only round when giving their final answer.	
			Total	9		
5		a	$r\theta = 15$ $\frac{1}{2}r^2\theta = 45$ $\frac{1}{2}r(15) = 45$ $r = 6$ and $\theta = 2.5$	B1(AO1.1) B1(AO1.1) M1(AO3.1a) A1(AO1.1) [4]	Accept any method for solving the equations simultaneously	
		b	$\frac{1}{2}(6)^2\sin\left(\frac{5}{2}\right)$ $45 - \text{their } \frac{1}{2}(6)^2\sin\left(\frac{5}{2}\right)$ 34.2(cm^2)	B1FT(AO1.1) M1(AO1.1) A1FT(AO1.1) [3]	FT their r and θ FT their r and θ	
			Total	7		

6			$r\theta = 6$ $\frac{1}{2}r^2\theta = 24$ $\frac{1}{2}r \times 6 = 24$ $r = 8$, $\theta = 0.75$ segment area $= 24 - \frac{1}{2} \times 8^2 \times \sin 0.75$ $= 2.19$	B1* B1* M1d* A1 M1	State $r\theta = 6$ State $\frac{1}{2}r^2\theta = 24$ Attempt to solve simultaneously to find r or θ	Or exact equiv from using a fraction of the circle Or exact equiv from using a fraction of the circle Allow B1 for $\frac{1}{2}r \times$ arc = 24 Stating both $\frac{1}{2}r^2\theta = 24$ and $\frac{1}{2}r^2\sin\theta = 24$ is B0 unless only the correct equation is subsequently used B1 B1 can be implied by a correct equation in a single variable As far as attempting r or θ, using a valid method (but allow slips) Must be using the two correct equations in r

				A1 [6]	Obtain $r = 8$, $\theta = 0.75$ (aef) Attempt area of segment Obtain 2.19, or better	and θ Both values required 24 – area of triangle, using $½r^2\sin\theta$ or equiv Allow if evaluated in degree mode (gives 23.58) Allow M1 for attempting $½r^2(\theta - \sin\theta)$ with their r and θ, even if this does not give area of sector as 24 Allow final answer in range [2.187, 2.188] if > 3sf Could use variables other than r and θ **Alt method for working in degrees** B1 - state	

					<table><tr><td></td><td>$^{\theta}/_{360} \times 2\pi r = 6$ B1 - state $^{\theta}/_{360} \times \pi r^2 = 24$ M1 - attempt to solve simultaneously A1 - obtain $r = 8$, $\theta = 43.0°$ or better (42.97…) M1 - attempt area of segment NB using $½r^2(\theta – \sin\theta)$ with θ in degrees is M0 as incorrect attempt at area of sector A1 - obtain 2.19 or better</td></tr></table> Examiner's Comments This question was also very well answered, with many fully correct solutions being seen. Candidates coped well with the lack of structure in this question and were able to formulate an effective method to find the area of the segment. Most candidates could state two relevant equations, either by quoting the relevant formulae or by working in fractions of a circle. An uncertainty over terminology resulted in some candidates using 24 as the area of the triangle rather than the sector. Having obtained two correct equations, most candidates were able to solve them simultaneously although a significant minority were	

					unable to simplify the indices involved. Candidates were familiar with the process for finding the area of a segment and many gained credit for doing so, even if there had been errors earlier on. A minority of candidates elected to work in degrees rather than radians throughout. A number manged to do so successfully, but the awkwardness of the numbers involved proved too much for others. Whilst there is nothing wrong with this approach, candidates should always attempt to use the most efficient method.	
			Total	6		
7		a	$0.5 \times 10 \times 7 \times \sin 0.8 = 25.107$ $0.5 \times r^2 \times 0.8 = 12.55$ $r = 5.6$ cm	M1 (AO 3.1a) M1 (AO 1.1) A1 (AO 1.1) [3]	Attempt area of triangle Equate area of sector to half of area of triangle and attempt to find r Obtain correct value for r — 5.6 or better (5.60217…)	
		b	$CD = 5.6 \times 0.8 = 4.48$ $AB = \sqrt{7^2 + 10^2 - 2 \times 7 \times 10 \times \cos 0.8} = 7.17$ $ABCD = 7.17 + (10 - 5.6) + 4.48 + (7 - 5.6)$ $= 17.5$ cm	M1 (AO 3.1a) M1 (AO 1.1) M1 (AO 1.1) A1 (AO 1.1) [4]	Attempt arc length using $r\theta$ Attempt AB using cosine rule Attempt	

					perimeter of *ABCD* Obtain 17.5 cm | Allow 17.4 www	
			Total	7		