1. (a) A student suggests that, for any prime number between 20 and 40, when its digits are squared and then added, the sum is an odd number.

For example, 23 has digits 2 and 3 which gives $2^2 + 3^2 = 13$, which is odd. [2]

Show by counter example that this suggestion is false.

(b) Prove that the sum of the squares of any three consecutive positive integers cannot be divided by 3. [3]

2. (a) Jack makes the following claim.
"If n is any positive integer, then $3^n + 2$ is a prime number."
Prove that Jack's claim is incorrect. [3]

(b) Jill writes the following statement.

$$x = 3 \Leftrightarrow x^2 = 9$$

(i) Explain why Jill's statement is incorrect. [1]

(ii) Write a corrected version of Jill's statement. [1]

3. Prove by exhaustion that if the sum of the digits of a 2-digit number is 5, then this 2-digit number is not a perfect square. [3]

4. In each of the following cases choose one of the statements

$$P \Rightarrow Q \qquad P \Leftarrow Q \qquad P \Leftrightarrow Q$$

to describe the relationship between P and Q.

(a) P: $y = 3x^5 - 4x^2 + 12x$

Q: $\dfrac{dy}{dx} = 15x^4 - 8x + 12$ [1]

[1]

(b) P: $x^5 - 32 = 0$ where x is real
Q: $x = 2$

(c) P: $\ln y < 0$
Q: $y < 1$

[1]

5. [5]

N is an integer that is not divisible by 3. Prove that N^2 is of the form $3p + 1$, where p is an integer.

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Guidance	
1		a	31 gives $3^2 + 1^2 = 10$ 10 is even and hence the suggestion is false	M1(AO2.1) E1(AO2.1) [2]		OR **M1** 37 gives $3^2 + 7^2 = 58$ **E1** 58 is even and hence the suggestion is false
		b	$n^2 + (n+1)^2 + (n+2)^2$ $3n^2 + 6n + 5$ $3(n^2 + 2n + 1) + 2$ which always leaves a remainder of 2 and so cannot be divided by 3	M1(AO2.1) A1FT(AO1.1) E1(AO2.1) [3]	Any valid expressions for three consecutive integers FT *their* expressions Correct conclusion .	
			Total	5		
2		a	At least one correct calc'n of $3^n + 2$ with $n \geq 1$ $3^5 + 2 = 245$ 245 is div by 5, so statement incorrect	M1(AO1.1a) A1(AO2.1) E1(AO2.1) [3]	or eg $3^6 + 2 = 731$ 731 is div by 17, so statement incorrect	One contradiction seen Must see this line oe
		b	i) $(-3)^2 = 9$ or $x = -3$ gives $x^2 = 9$	B1(AO2.3) [1]	oe	
		b	ii) $x = 3 \Rightarrow x^2 = 9$ or $x = \pm 3 \Leftrightarrow x = 9$	B1(AO2.1) [1]	Enter text here.	
			Total	5		

3			14, 41, 23, 32, 50 or 16, 25, 36, 49 None of these 2-digit numbers is a perfect square or None of these squares has digit sum = 5	M1(AO2.1) A1(AO2.1) E1dep(AO2.4) [3]	$\geq$ 4 of these or $\geq$ 3 of these One set all correct oe dep M1A1 — Ignore sq nos above 49
			Total	3	
4		a	$P \Rightarrow Q$	B1 (AO 1.1) [1]	Examiner's Comments The most common error was to give $P \Leftrightarrow Q$. $P \rightarrow Q$ was condoned.
		b	$P \Leftrightarrow Q$	B1 (AO 1.1) [1]	Examiner's Comments The most common error was $P \Rightarrow Q$. $P \leftrightarrow Q$ was condoned.
		c	$P \Rightarrow Q$	B1 (AO 1.1) [1]	Examiner's Comments The most common error was $P \Leftrightarrow Q$. $P \Leftarrow Q$ was also seen, perhaps because candidates thought each statement would be used once in this question. Note this was not the case and is not inevitable. $P \rightarrow Q$ was condoned. This part was found to be the most difficult.
			Total	3	
5			$N = 3k + 1$ or $N = 3k + 2$ (where k is an integer) $(3k + 1)^2$ | $(3k + 2)^2$ $= 9k^2 + 6k + 1$ | $= 9k^2 + 12k + 4$	M1 (AO3.1a) M1 (AO1.1) A1 (AO2.1) A1 (AO2.4)	**One** of these. Allow without "N = " — Any letter other than p Attempt **one** of these — Allow p **Both** correct — Allow p Or $9k^2$+$6k$ div — or similar in

			$= 3(3k^2 + 2k) + 1$ or $= 3(3k^2 + 4k + 1) + 1$ **Both** these are of form $3p + 1$, p an integer	E1 (AO2.2a) [5]	<table><tr><td>by 3 or $9k^2+12k+3$ div by3 **One** of these Must say p is integer or $3k^2 + 2k$ and $3k^2 + 4k + 1$ are integers Similar marks for method using $N = 3k + 1$& $N = 3k - 1$</td><td>words. Allow p Dep on M1M1A1A1 $N = 3p + 1$: max M0M1A1A1E0</td></tr></table> Examiner's Comments This question tests "proof by exhaustion" as included in paragraph 1.01a of the specification. This method of proof involves either considering all possible values or all possible categories of values. This question tests the latter. It was not well answered on the whole. Many candidates started with, for example, N = 3k + 1 and gave a partly correct argument based on this (although most omitted to say "where k is an integer"). Then many of these omitted to consider either N = 3k + 2 or N = 3k – 1 as well. Some candidates started with N = 3p + 1 and gave an otherwise correct argument, ignoring the use of "p" in the question. Some candidates tried to work from $N^2 = 3p + 1$. These all failed. Some verified the result in a few numerical cases. These scored no marks.
			Total	5	