1. In a high jump competition, jumpers are allowed three attempts to succeed at each height. For one particular height Imran estimates his chances of succeeding as follows.

- The probability that he will succeed on his first attempt is $\frac{4}{5}$.
- If he fails on his first attempt, the probability that he will succeed on his second attempt is $\frac{3}{4}$.
- If he fails on his first two attempts, the probability that he will succeed on his third attempt is $p$.

Use Imran's estimates to answer the following.
(i) Complete the below probability tree diagram for this situation.

## First attempt


(ii)

Find the probability that Imran succeeds on either his first or his second attempt.
(iii) Given that the probability that Imran succeeds at this particular height is $\frac{197}{200}$, , find $p$.
2. In a class of 30 students, each student studies exactly one modern language. 14 students study French, 9 students study Spanish and 7 students study German. A committee of 6 students is to be chosen from these 30 students. Find the number of ways of choosing the committee if it contains
(i) any 6 students from the class,
(ii) 2 students studying each language,
(iii) exactly 1 student studying French.
3. Sandra makes repeated, independent attempts to hit a target. On each attempt, the probability that she succeeds is 0.1 .
i. Find the probability that
a. the first time she succeeds is on her 5th attempt,
b. the first time she succeeds is after her 5th attempt,
c. the second time she succeeds is before her 4th attempt.

Jill also makes repeated attempts to hit the target. Each attempt of either Jill or Sandra is independent. Each time that Jill attempts to hit the target, the probability that she succeeds is 0.2. Sandra and Jill take turns attempting to hit the target, with Sandra going first.
ii. Find the probability that the first person to hit the target is Sandra, on her
a. 2nd attempt,
b. 10th attempt.
4. i. A bag contains 12 black discs, 10 white discs and 5 green discs. Three discs are drawn at random from the bag, without replacement. Find the probability that all three discs are of different colours.
ii. A bag contains 30 red discs and 20 blue discs. A second bag contains 50 discs, each of which is either red or blue. A disc is drawn at random from each bag. The probability that these two discs are of different colours is 0.54 . Find the number of red discs that were in the second bag at the start.
5. The probability distribution of a random variable $X$ is given in the table.

| $x$ | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: |
| $\mathrm{P}(X=x)$ | 0.6 | 0.3 | 0.1 |

Two values of $X$ are chosen at random. Find the probability that the second value is greater than the first.
6. A random variable $X$ has probability distribution given by

$$
\mathrm{P}(X=x)=\frac{1}{860}(1+x) \text { for } x=1,2,3, \ldots, 40 .
$$

(a) Find $\mathrm{P}(X>39)$.
(b) Given that $x$ is even, determine $\mathrm{P}(x<10)$.
7. The probability distribution of a random variable $X$ is given in the table.

| $x$ | 0 | 2 | 4 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(X=x)$ | $\frac{3}{8}$ | $\frac{5}{16}$ | $4 p$ | $p$ |

(a) Find the value of $p$.
(b) Two values of $X$ are chosen at random. Find the probability that the product of these values is 0 .
8. The discrete random variable $X$ takes values 1, 2, 3, 4 and 5 , and its probability distribution is defined as follows.

$$
\mathrm{P}(X=x)= \begin{cases}a & x=1, \\ \frac{1}{2} \mathrm{P}(X=x-1) & x=2,3,4,5, \\ 0 & \text { otherwise },\end{cases}
$$

where $a$ is a constant.

$$
\text { (a) Show that } a=\frac{16}{31} \text {. }
$$

The discrete probability distribution for $X$ is given in the table.

| $x$ | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(X=x)$ | $\frac{16}{31}$ | $\frac{8}{31}$ | $\frac{4}{31}$ | $\frac{2}{31}$ | $\frac{1}{31}$ |

(b) Find the probability that $X$ is odd.

Two independent values of $X$ are chosen, and their sum $S$ is found.
(c) Find the probability that $S$ is odd.
(d) Find the probability that $S$ is greater than 8 , given that $S$ is odd.

Sheila sometimes needs several attempts to start her car in the morning. She models the number of attempts she needs by the discrete random variable $Y$ defined as follows.

$$
\mathrm{P}(Y=y+1)=\frac{1}{2} \mathrm{P}(Y=y) \quad \text { for all positive integers } y .
$$

(e) Find $\mathrm{P}(Y=1)$.

Give a reason why one of the variables, $X$ or $Y$, might be more appropriate as a model for the number of attempts that Sheila needs to start her car.
9. Bag A contains 3 black discs and 2 white discs only. Initially Bag $B$ is empty. Discs are removed at random from bag $A$, and are placed in bag B, one at a time, until all 5 discs are in bag $B$.
(a) Write down the probability that the last disc that is placed in bag $B$ is black.
(b) Find the probability that the first disc and the last disc that are placed in bag B are both black.

Find the probability that, starting from when the first disc is placed in bag B, the
(c) number of black discs in bag $B$ is always greater than the number of white discs in bag $B$.
10. Each of the 30 students in a class plays at least one of squash, hockey and tennis.

- 18 students play squash
- 19 students play hockey
- 17 students play tennis
- 8 students play squash and hockey
- 9 students play hockey and tennis
- 11 students play squash and tennis
(a) Find the number of students who play all three sports.

A student is picked at random from the class.
(b) Given that this student plays squash, find the probability that this student does not play hockey.
Two different students are picked at random from the class, one after the other, without replacement.
(c) Given that the first student plays squash, find the probability that the second student plays hockey.
11. Joanne has five cards, numbered 1, 1, 1, 2, 2. She picks two cards at random, without replacement. The variable $X$ denotes the sum of the numbers on the two cards.

$$
\text { (a) Show that } \mathrm{P}(X=3)=\frac{3}{5} \text {. }
$$

The table shows the probability distribution of $X$.

| $x$ | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: |
| $\mathrm{P}(X=x)$ | $\frac{3}{10}$ | $\frac{3}{5}$ | $\frac{1}{10}$ |

Joanne replaces the two cards. Now Liam picks two cards at random from the five cards, without replacement. The variable $Y$ denotes the sum of the numbers on the two cards that Liam picks.
(b) Find $\mathrm{P}(X=Y)$.


| Question | Answer/Indicative content | Marks | Part marks and guidance |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\frac{4}{5}+\frac{1}{5} \times \frac{3}{4} \quad$ or $1-\frac{1}{5} \times \frac{1}{4}$ $=\frac{19}{20} \text { or } 0.95$ | M2 <br> A1 <br> [3] | $\frac{4}{5}+$ prod of <br> 2 P's or eg $\frac{4}{5}+\frac{1}{5} \times \frac{4}{5}$ <br> $1-$ prod of <br> 2 P's M1 or $1-\frac{1}{5} \times \frac{1}{5}$ <br> or $\frac{4}{5}+\frac{1}{5} \times \frac{3}{5}$  <br> No ft from or $1-\frac{1}{5} \times \frac{2}{5}$ <br> tree diag. M1M0A0 <br> Examiner's Comments <br> Many candidates answered this q uestion correctly. A few omitted the probability of succeeding on the first attempt and just <br> found $\square \square \frac{1}{5} \times \frac{4}{5}$. Others considered both first and second attempts, but incorrectly just <br> added $\square \square \frac{4}{5}+\frac{3}{4}$. |  |
|  | iii$1-\frac{1}{5} \times \frac{1}{4} \times(1-p)=\frac{197}{200}$ or $\frac{3}{200}$ seen <br> $\frac{1-p}{20}=\frac{3}{200}$ any correct <br> step, one <br> fract each side | M1 <br> M1d | or $0.95+\frac{1}{5} \times \frac{1}{4} \times p=\frac{197}{200}$ or $\frac{4}{5}+\frac{1}{5} \times \frac{3}{4}$ <br> or $\frac{7}{200}$ seen <br> $+\frac{1}{5} \times \frac{1}{4} \times p=\frac{197}{200}$ <br> eg $\frac{19+p}{20}=\frac{197}{200}$ eg $\frac{1}{20} p=\frac{7}{200}$ <br> or $\frac{1}{20} p=\frac{7}{200}$ oe in <br> decimals <br> Dep 1st M1  |  |



| Question |  | Answer/Indicative content | Marks | Part marks and guidance |
| :--- | :--- | :--- | :---: | :---: |
|  |  | Total | 8 |  |



| Question | Answer/Indicative content | Marks <br> M2 | Part marks and guidance |  |
| :---: | :---: | :---: | :---: | :---: |
| iii | 14 (or ${ }^{14} \mathrm{C}_{1}$ ) or $14 \times$ alone $\times{ }^{16} \mathrm{C}_{5} \quad 4368$ $=61152$ | M2 <br> A1 <br> [3] |  <br> Examiner's Comments Arithmetical errors were common in the otherwise correct, but very long, method of adding six products of combinations. Candidates who used the direct method ( ${ }^{14} \mathrm{C}_{1} \times{ }^{16} \mathrm{C}_{5}$ ) were more likely to obtain the correct answer. Some candidates, incorrectly, found ${ }^{14} \mathrm{C}_{1} \times{ }^{30} \mathrm{C}_{5}$ or ${ }^{14} \mathrm{C}_{1} \times$ ${ }^{29} \mathrm{C}_{5}$. Others added ${ }^{14} \mathrm{C}_{1}+$ ${ }^{16} \mathrm{C}_{5}$. |  |
|  | Total | 6 |  |  |



| Question | Answer/Indicative content | Marks | Part marks | guidance |
| :---: | :---: | :---: | :---: | :---: |
| i | $=0.028$ | A1 | Ans 0.027 probably M0M1M1A0 but check working <br> SC if no M-mks scored: SSF, SSS, FSS, SFS or SS, FSS, SFS seen or implied: B1 <br> Examiner's Comments <br> Only a few candidates used the simplest method which involves SS, FSS, SFS. <br> Few candidates answered this question totally correctly although many gave partially correct answers. Some gave only $0.1^{2} \times 0.9$. Many gave $3 \times$ $0.1^{2} \times 0.9$ but omitted + $0.1^{3}$. Many included terms such as $0.1 \times 0.9^{2}$. Some used the complement method, but most of these only gave $1-0.9^{3}$, omitting to subtract $3 \times 0.9^{2} \times 0.1$ also. | This method only scores using " 1 - ": $0.9^{3} ; 3 \times 0.9^{2} \times$ 0.1 no incorrect multiples MI; MI 1 - one or both terms with no further wking: M1 (dep M1) eg $1-0.9^{3}$ alone M1M0M1 |
| ii <br> ii | (a) $0.9 \times 0.8 \times 0.1$ $=\frac{9}{125} \text { or } 0.072$ | M1 A1 | alone or allow $\times 0.8$ (ie girls in wrong order) $(=0.0576)$ <br> Examiner's Comments <br> This question was well answered by most candidates. A few misread and thought Jill went first. Others included success for the wrong girl or for both girls. | NOT $0.9 \times 0.8 \times 0.1 \times 0.2=$ 0.0144: MOAO <br> NOT $0.9 \times 0.8 \times 0.2=$ <br> 0.144: MOAO |


| Question |  | Answer/Indicative content | Marks | Part marks and guidance |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | ii | (b) $0.9^{9 \text { or } 10} \times 0.8^{\text {yorlu }} \times 0.1$ (or $\times 0.2$, not $\times 0.1 \times 0.2$ ) $\begin{aligned} & (0.9 \times 0.8)^{9} \times 0.1 \text { oe } \\ & =5.2 \times 10^{-3} \text { or } 0.0052(2 \mathrm{sf}) \end{aligned}$ | M1 <br> M1 <br> A1 | allow $0.9^{9 \text { or } 10} \times 0.8^{9 \text { or }^{10} \times}$ $0.1 \times{ }^{18,19,20} C_{1}$ If ans $=$ 0.00360 or 0.0150 see SC below <br> fully correct <br> SC Consistent use of 0.8 for both girls: (ii)(a) 0.128 (ii)(b) 0.00360 or 0.9 for both girls: (ii)(a) 0.081 (ii)(b) 0.0150 If both these ans seen, allow (a) 0 (b) B1 <br> Examiner's Comments <br> Many candidates were confused as to how many failures were necessary for each girl. Others included success for the wrong girl or for both girls. |  |
|  |  | Total | 13 |  |  |






| Question |  | Answer/Indicative content | Marks | Part marks and guidance |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 7 | a | $\begin{aligned} & \frac{3}{8}+\frac{5}{16}+4 p+p=1 \\ & p=\frac{1}{16} \text { or } 0.0625 \end{aligned}$ | $\begin{gathered} \text { M1 } \\ (A O 1.1 a) \\ \text { A1 } \\ \text { (AO1.1) } \\ {[2]} \end{gathered}$ | $\begin{aligned} & \text { oe eg } \\ & 5 p=1-\left(\frac{3}{8}+\frac{5}{16}\right) \end{aligned}$ <br> Examiner's Comments <br> Most candidates answered this question correctly. A few tried to use $\Sigma x p$ instead of $\Sigma \mathrm{p}$. |  |
|  | b | $\frac{3}{8} \times \frac{5}{8}$ or $\frac{3}{8} \times \frac{3}{8}$ seen oe $\begin{aligned} & \frac{3}{8} \times \frac{5}{8}+\frac{5}{8} \times \frac{3}{8}+\frac{3}{8} \times \frac{3}{8} \text { oe } \\ & \left.=\frac{39}{64} \text { or } 0.609(3 \mathrm{sf})\right) \end{aligned}$ | M1 (AO1.1a) <br> M1 <br> (AO2.1) <br> A1 <br> (AO1.1) <br> [3] | or eg <br> $\frac{3}{8} \times \frac{5}{16}+\frac{3}{8} \times \frac{4}{16}+\frac{3}{8} \times \frac{1}{16}$ <br> ft their p or <br> $1-\left(\frac{5}{16}+\frac{1}{4}+\frac{1}{16}\right)^{2}$ <br> M2 <br> ft their p or <br> Allow0.61 $1-\left(\frac{5}{8}\right)^{2}$ <br> M 2 <br> Examiner's Comments <br> Most candidates scored only one mark because they omitted one or two of the three possible routes to obtaining a product of 0 . |  |
|  |  | Total | 5 |  |  |


| Question |  | Answer/Indicative content | Marks | Part marks and guidance |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | a | $a\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}\right)=1 \text { soi }$ $a=\frac{16}{31}$ | M1 (AO <br> 3.1a) <br> A1 (AO <br> 1.1) <br> [2] |  <br> This question was well answered on the whole, although a few candidates used the probabilities in the table, just finding $1-\left(\frac{8}{31}+\frac{4}{31}+\frac{2}{31}+\frac{1}{31}\right)$ |  |
|  | b | $\begin{array}{r} \mathrm{P}(X=1,3 \text { or } 5)=\frac{21}{31} \text { or } 0.677 \text { or } \\ 0.68(2 \mathrm{sf}) \end{array}$ | B1 (AO <br> 1.1a) <br> [1] | Examiner's Comments This question was well answered. |  |



| Question | Answer/Indicative content | Marks | Part marks and guidance |  |
| :---: | :---: | :---: | :---: | :---: |
| d | $\begin{aligned} & P(\text { Sum }>8 \text { \& odd })=P(\text { Sum } \\ & =9) \\ & =P(4,5)+P(5,4) \\ & =\frac{2}{31} \times \frac{1}{31}+\frac{1}{31} \times \frac{2}{31}\left(=\frac{4}{961}\right) \\ & \frac{P(\text { Sum }>8 \& \text { odd })}{P(\text { Sum odd })} \\ & ='^{\frac{4}{961}} '^{\prime} \frac{420}{961} \\ & =\frac{1}{0.0695}(2 \mathrm{sf}) \end{aligned}$ | M1 (AO <br> 1.1a) <br> M1 (AO <br> 2.4) <br> A1 (AO <br> 1.1) <br> [3] |  <br> Examiner's Comments Most candidates recognised the need to find $\mathrm{P}(S=9)$, but some omitted to include both 4, 5 and 5, 4 . Many then correctly divided by their answer to part (d). |  |
|  | $\begin{aligned} & S_{\infty}=\frac{p}{1-0.5}=1 \\ & \mathrm{P}(X=1)=0.5 \end{aligned}$ | M1 (AO 3.4) <br> A1 (AO 3.4) | Correct ans, no working M1A1 <br> Examiner's Comments <br> Some candidates recognised the need for an infinite series, but most could not cope with the fact that the first term is unknown. Many candidates thought that $Y$ cannot be 0 , hence $P(Y=0)=0$ and hence $P(Y=1)=0.5-0=0$ |  |


| Question |  | Answer/Indicative content | Marks | Part marks and guidance |
| :--- | :--- | :--- | :--- | :--- | :--- |




| Question |  | Answer/Indicative content | Marks | Part marks and guidance |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 11 | a | $\begin{aligned} & \frac{3}{5} \times \frac{1}{2} \text { or } \frac{2}{5} \times \frac{3}{4} \\ & \frac{3}{5} \times \frac{1}{2}+\frac{2}{5} \times \frac{3}{4} \\ & \left(=\frac{3}{5} \quad \text { AG }\right) \end{aligned}$ | M1 (AO1. <br> 1) <br> A1(AO1. <br> 1) <br> [2] | $\left.{ }_{\text {or } \frac{3}{3} \times \frac{1}{2} \times 2 \operatorname{cor} \frac{2}{3} \times \frac{3}{4} \times 2} \right\rvert\, \begin{aligned} & \text { Must see } \\ & \text { this step } \end{aligned}$ |  |
|  | b | $\begin{aligned} & \left(\frac{3}{5}\right)^{2}+\left(\frac{3}{10}\right)^{2}+\left(\frac{1}{10}\right)^{2} \\ & =\frac{23}{50} \text { or } 0.46 \end{aligned}$ | M1 (AO1. <br> 1a) <br> A1(AO1. 1) <br> [2] |  |  |
|  |  | Total | 4 |  |  |

