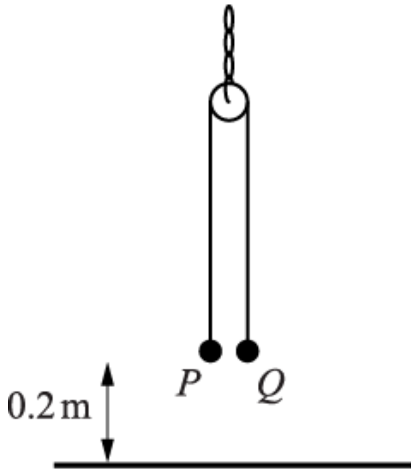
1.



A small smooth pulley is suspended from a fixed point by a light chain. A light inextensible string passes over the pulley. Particles *P* and *Q*, of masses 0.3 kg and *m* kg respectively, are attached to the opposite ends of the string. The particles are released from rest at a height of 0.2 m above horizontal ground with the string taut; the portions of the string not in contact with the pulley are vertical (see diagram). *P* strikes the ground with speed 1.4 m s^{-1}. Subsequently *P* remains on the ground, and *Q* does not reach the pulley.

i. Calculate the acceleration of *P* while it is in motion and the corresponding tension in the string.

[4]

ii. Find the value of *m*.

[3]

iii. Calculate the greatest height of *Q* above the ground.

[4]

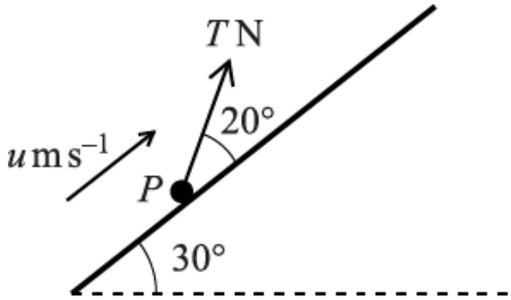
iv. It is given that the mass of the pulley is 0.5 kg. State the magnitude of the tension in the chain which supports the pulley

a. when *P* is in motion,

[2]

b. when *P* is at rest on the ground and *Q* is moving upwards.

[1]

2. 

A particle P of mass 0.25 kg moves upwards with constant speed u m s^{-1} along a line of greatest slope on a smooth plane inclined at 30° to the horizontal. The pulling force acting on P has magnitude T N and acts at an angle of 20° to the line of greatest slope (see diagram). Calculate

i. the value of T,

[3]

ii. the magnitude of the contact force exerted on P by the plane.

[3]

The pulling force T N acting on P is suddenly removed, and P comes to instantaneous rest 0.4 s later.

iii. Calculate u.

[4]

3. A particle P is projected with speed u m s^{-1} from the top of a smooth inclined plane of length $2d$ metres. After its projection P moves downwards along a line of greatest slope with acceleration 4 m s^{-2}. At the instant 3 s after projection P has moved half way down the plane. P reaches the foot of the plane 5 s after the instant of projection.

i. Form two simultaneous equations in u and d, and hence calculate the speed of projection of P and the length of the plane.

[6]

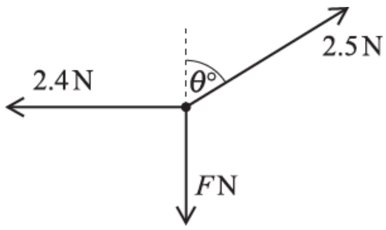
ii. Find the inclination of the plane to the horizontal.

[2]

iii. Given that the contact force exerted on P by the plane has magnitude 6 N, calculate the mass of P.

[2]

4.



A particle rests on a smooth horizontal surface. Three horizontal forces of magnitudes 2.5 N, F N and 2.4 N act on the particle on bearings $\theta°$, 180° and 270° respectively (see diagram). The particle is in equilibrium.

i. Find θ and F.

[4]

The 2.4 N force suddenly ceases to act on the particle, which has mass 0.2 kg.

ii. Find the magnitude and direction of the acceleration of the particle.

[3]

5. A particle P is projected down a line of greatest slope on a smooth inclined plane. P has velocity 5 m s^{-1} at the instant when it has been in motion for 1.6 s and travelled a distance of 6.4 m. Calculate

i. the initial speed and the acceleration of P,

[5]

ii. the inclination of the plane to the vertical.

[3]

6. A child is trying to throw a small stone to hit a target painted on a vertical wall. The child and the wall are on horizontal ground. The child is standing a horizontal distance of 8 m from the base of the wall. The child throws the stone from a height of 1 m with speed 12 m s^{-1} at an angle of 20° above the horizontal.

i. Find the direction of motion of the stone when it hits the wall.

[6]

The child now throws the stone with a speed of V m s^{-1} from the same initial position and still at an angle of 20° above the horizontal. This time the stone hits the target which is 2.5 m above the ground.

ii. Find V.

[6]

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7.

In this question $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ denote unit vectors which are horizontal and vertically upwards respectively.

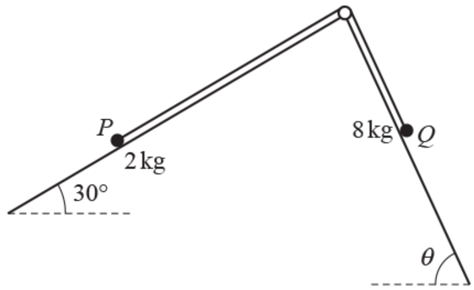
A particle of mass 5 kg, initially at rest at the point with position vector $\begin{pmatrix}2\\45\end{pmatrix}$ m, is acted on by gravity and also by

two forces $\begin{pmatrix}15\\-8\end{pmatrix}$ N and $\begin{pmatrix}-7\\-2\end{pmatrix}$ N.

(a) Find the acceleration vector of the particle. [3]

(b) Find the position vector of the particle after 10 seconds. [3]

8.



Two particles *P* and *Q*, with masses 2 kg and 8 kg respectively, are attached to the ends of a light inextensible string. The string passes over a small smooth pulley which is fixed at a point on the intersection of two fixed inclined planes. The string lies in a vertical plane that contains a line of greatest slope of each of the two inclined planes. Plane Π_1 is inclined at an angle of 30° to the horizontal and plane Π_2 is inclined at an angle of θ to the horizontal. Particle *P* is on Π_1 and *Q* is on Π_2 with the string taut (see diagram).

Π_1 is rough and the coefficient of friction between *P* and Π_1 is $\frac{\sqrt{3}}{3}$.

Π_2 is smooth.

The particles are released from rest and *P* begins to move towards the pulley with an acceleration of
$g \cos \theta$ m s^{-2}.

(a) Show that θ satisfies the equation

$$4 \sin \theta - 5 \cos \theta = 1.$$ [8]

(b) By expressing $4 \sin \theta - 5 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where $R > 0$ and $0 > \alpha > 90°$, find, correct to 3 significant figures, the tension in the string.

[7]

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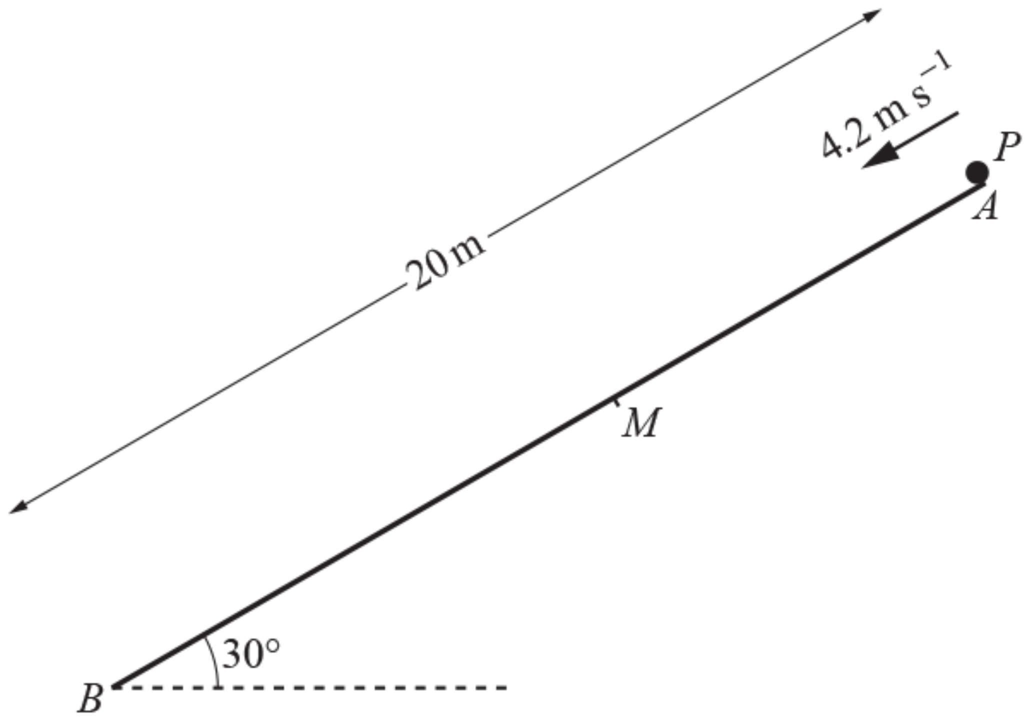
9.



A and *B* are points at the upper and lower ends, respectively, of a line of greatest slope on a plane inclined at 30° to the horizontal. The distance *AB* is 20 m. *M* is a point on the plane between *A* and *B*. The surface of the plane is smooth between *A* and *M*, and rough between *M* and *B*.

A particle *P* is projected with speed 4.2ms^{-1} from *A* down the line of greatest slope (see diagram). *P* moves down the plane and reaches *B* with speed 12.6ms^{-1}. The coefficient of friction between *P* and the rough part of the plane is $\frac{\sqrt{3}}{6}$.

(a) Find the distance *AM*. [8]

(b) Find the angle between the contact force and the downward direction of the line of greatest slope when *P* is in motion between *M* and *B*. [3]

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10. A ball B is projected with speed V at an angle α above the horizontal from a point O on horizontal ground. The greatest height of B above O is H and the horizontal range of B is R. The ball is modelled as a particle moving freely under gravity.

(a) Show that

(i) $H = \frac{V^2}{2g}\sin^2\alpha,$ [2]

(ii) $R = \frac{V^2}{g}\sin 2\alpha.$ [3]

(b) Hence show that $16H^2 - 8R_0H + R^2 = 0$, where R_0 is the maximum range for the given speed of projection. [5]

(c) Given that $R_0 = 200$m and $R = 192$m, find

(i) the two possible values of the greatest height of B, [2]

(ii) the corresponding values of the angle of projection. [3]

(d) State one limitation of the model that could affect your answers to part (c). [1]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	$1.4^2 = 2 \times a \times 0.2$	M1	Any use of $a = g$ is M0	
		i	*OR*			
		i	$0.2 = (0 + 1.4)t/2$ and $1.4 = 0 + at$		$t = 2/7$ hence $1.4 = a \times 2/7$	
		i	$a = 4.9$ m s^{-2}	A1		
		i	$0.3g - T = +/- 0.3 \times 4.9$	M1	N2L diff of weight and tension. Any use of $a = g$ is M0	
		i	$T = 1.47$ N	A1	**Examiner's Comments** A minority of scripts included the assertion that $u = 1.4$, but most contained a correct calculation of acceleration. Some candidates failed to calculate the tension in the string, while others erred in confusing the signs of their terms, and found $T = 4.41$ N.	
		ii	$+/-4.9m = 1.47 - mg$	M1	N2L for Q using values from (i), a not g; accept $a = g\Delta M/\Sigma M$	
		ii	$4.9m = 1.47 - mg$	A1ft	Diff cv(T) and mg correct way round; ft cv(T,a)	
		ii			$4.9 = g(0.3 - m)/(0.3 + m)$ M1A1; ftcv(a)	
		ii	$m = 0.1$	A1	**Examiner's Comments** Again this was answered well, though sign errors led to some wrong answers. Finding $m = 0.3$ seldom seemed to surprise a candidate.	
		iii	$1.4^2 = 2gs$	M1	Accn = g	
		iii	$s = 0.1$	A1	may be implied (eg $H = 0.3$) BoD sign uncertainty	
		iii	$H = 0.2 + 0.2 + 0.1$	M1	Needs 0.2 twice	

		iii	$H = 0.5$ m	A1	**Examiner's Comments** In only a few scripts did candidates demonstrate the upward velocity of Q at the instant when P struck the ground, or assume that the deceleration of P would continue to be 4.9. Fully correct work was often seen.	
		iv	*(a)* Tension = $0.5g + 2 \times 1.47$	M1		
		iv	Tension = 7.84 N	A1	**Examiner's Comments** This was the first major area of difficulty for many candidates, who used particle masses rather than the tension in the string.	
		iv	*(b)* Tension (= $0.5g$) = 4.9 N	B1	**Examiner's Comments** Though some candidates made this a complex situation, for many the answer was simple.	
			Total	**14**		
2		i	TCor S20 = $0.25g$Cor S30	M1	Equates cmpt T and cmpt wt / plane (doubt, see diagram and / or (ii))	
		i	$T\cos 20 = 0.25g\sin 30$	A1	1.225	
		i	$T = 1.3(0)$	A1	**Examiner's Comments** Though most scripts contained solutions based on resolving parallel to the plane, a significant minority resolved perpendicular to it, getting T = 6.20 as the answer. In this question, candidates having a clear diagram with forces were at a significant advantage. However, having 0.2 as the mass, or $T\cos 30$ as the component of T, were common misreads.	
		ii	R +/− TCorS20 = +/− $0.25g$CorS30	M1	Resolves perp plane, accept letter T	
		ii	$R + 1.3\sin 20 = 0.25g\cos 30$	A1 ft	ft(cv(T))	

		ii	$R = 1.68$ N	A1	**Examiner's Comments** Correct solutions were common. Some candidates who had used correct data in (i) used incorrect values in (ii). It was also quite usual for resolving perpendicular to the plane to be used again in (ii) after it had been (wrongly) used in (i).	
		iii	(m)accn = +/– (m)9.8sin30	M1*	N2L with single force a cmpt wt (accept cos)	
		iii	a = +/–4.9	A1		
		iii	u = +/–9.8sin30 × 0.4	D*M1		
		iii	$u = 1.96$	A1	Must be +ve (accept loss of – sign) **Examiner's Comments** Candidates who made errors in (i) or (ii) were able to gain full marks here, and most did. In this simple context candidates were able to avoid gross sign errors.	
			Total	10		
3		i	$d = 3u + 4 \times 3^2/2$ $(= 3u + 18)$	B1		
		i	$2d = 5u + 4 \times 5^2/2$ $(= 5u + 50)$	B1	*OR* $d = (5 - 3)(u + 3 \times 4) + 4 \times 2^2/2$ for lower half of slope ($d = 2u + 32$)	
		i	$6u + 36 = 5u + 50$	M1	Attempts to solve 2 SE in u and d, at least one with 3 terms. Tolerate u, d switch to x, y for solving reasons	
		i	$u = 14$ ms^{-1}	A1		
		i	$2d = 5 \times 14 + 4 \times 5^2/2$ *OR* $d = 3 \times 14 + 18$ *OR* $d = 2 \times 14 + 32$	M1	Substitutes in 3 term eqn, starts *suvat* again, or solves SEs again. If u is negative, allow substitution of +ve equivalent.	
		i	Length = 120 m	A1		
		i			Candidates who use s or x instead of d can be given marks BoD M1 for getting to a single equation with one unknown having used two	

					different equations. Candidates who find $2d = 120$ from their SE and then give a final answer $d = 60$ get A0. Candidates who find $2d = 120$ from their SE and then give no further work get A1.	
		ii	$4(m) = (m)g\sin\theta$	M1	Mass may be omitted on both sides. Allow $4(m) = (m)g\cos\theta$	
		ii	$\theta = 24.1°$	A1		
		ii			Value for mass assumed or wrong, allow M1A0 fortuitous.	
		iii	$6 = mg\cos 24.1$	M1	Or $6 = mg\sin 24.1$, uses numerical answer referring to (ii) www Examiner's Comments 5(i) was well attempted by nearly all, with many correct solutions seen. However, it was common for candidates to lose the last mark by giving a value for d but not giving the length of the plane. 5(ii) and 5(iii) were often omitted, and only the best candidates knew how to answer each part. In 5(iii) a version of Newton's Second Law ($4m = 6$) was sometimes offered as the solution. In nearly all cases when (iii) was correctly answered, candidates used their answer to part (ii). However, a few chose a different route, namely $(mg)^2 = (4m)^2 + 6^2$. The unfamiliar way in which this topic (component of force perpendicular to an inclined plane) was tested led to a rash of unexpected errors. The two most frequent were resolving 6, rather than the weight, and using sine function, rather than cosine.	
		iii	$m = 0.671$ kg	A1		
		iii			Candidates who use cos in (ii) and sin in (iii) will fortuitously get the correct mass. However (iii) is given M1A0	
			Total	10		

4		i	$2.5\sin\theta = 2.4$	M1	$2.5\text{CorS}\theta = 2.4$	$2.5\cos\theta = 2.4$ M1 hence
		i	$\theta = 73.7$	A1	Accept 74	$\theta = 16.3$ A0
		i	$2.5\cos\theta = F$	M1	$F = 2.5\,\text{SorC}\theta$, opposite to that above	$2.5\sin\theta = F$ M1 hence
		i	$F = 0.7$	A1	Exact, but allow 0.702 (3 sf) θ = 73.7	$F = 0.7(00)$ A1 SC
		i	*OR*			
		i	$2.4^2 + F^2 = 2.5^2$ or $F^2 = 2.5^2 - 2.4^2$	M1		
		i	$F = 0.7$	A1	**Examiner's Comments** The familiarity of the material gave rise to many correct answers. One error was using the value of 2.5cos73.7 as F. This gives 0.702 as the answer which is wrong correct to 3 significant figures; however there was no penalty for the mistake on this occasion.	F can then be used to find θ
		ii	$2.4 = 0.2a$	M1	N2L, Any horizontal force other than F, 0.7, 2.5 (Do not treat removing / using 2.5 as a MR)	Including g, automatically M0
		ii	$a = 12\text{ ms}^{-2}$	A1	12.0 from 2.5sin73.7 /0.2	
		ii	Bearing (0)90° *OR*			Horizontal is B0 (ambiguous)
		ii	"To right"," opposite old 2.4 N force" etc	B1	Angle value other than exactly 90° or 0° B0 Allow B1 for force dirn, if accn not found **Examiner's Comments** Was unfamiliar, and in consequence badly answered. There was little understanding that removal of the 2.4 N force from a system in equilibrium must create a resultant force of equal magnitude with the opposite sense. Indeed, calculations often ended when the resultant of the two remaining forces had been found	
			Total	7		

5		i		M1	Uses $s = (u + v)t/2$ or a combination of two other formulae	
		i	$6.4 = (u + 5)/2 \times 1.6$	A1	$5^2 = u^2 + 2 \times 6.4a$ M1	
		i	$u = 3$ m s^{-1}	A1	$5 = u + 1.6a$ M1 Accurate equation in one variable A1	
		i	$5 = 3 + 1.6a$	M1	$u = 3$ m s^{-1} A1	
		i	$a = 1.25$ m s^{-2} *OR*	A1	$a = 1.25$ m s^{-2} A1 Candidates may find a first (see below)	
		i	$6.4 = 5 \times 1.6 - a1.6^2/2$	M1	$s = vt +/- at^2/2$	
		i	$a = 1.25$ m s^{-2}	A1	Must be from $s = vt - at^2/2$	
		i		M1		
		i	$5 = u + 1.25 \times 1.6$	A1		
					SC Do not accept $a = 1.25$ from $6.4 = 5 \times 1.6 + a1.6^2/2$ but allow subsequent use of $a = 1.25$ in $5 = u + 1.25 \times 1.6$	
		i	$u = 3$ m s^{-1}	A1	**Examiner's Comments** Most candidates scored full marks. Usually appropriate *suvat* equations were selected, though a minority of candidates solved (correctly) two simultaneous equations.	
		ii	$1.25(m) = (m)g$CorSθ	M1	Resolves g or weight, $a \neq g$	
		ii	$1.25(m) = (m)g\cos\theta$ *OR* $1.25(m) = (m)g\sin\theta$	A1√	ft cv(1.25) from (i)	
					Must be angle with vertical	
		ii	Angle with vertical = 82.7°	A1	**Examiner's Comments**	
					By far the most common approach error was to give the angle with the	

					horizontal, so 2/3 was the most frequent score. A significant minority of candidates did not link the acceleration in (i) to the inclination in (ii).	
			Total	8		
6		i	$v_x = 12\cos20$	*B1	11.27631…..	
		i	$8 = 12t\cos20$	B1	Using suvat to find expression in t only. ($t = 0.70945…$)	
		i		*M1	Attempt at use of $v = u + at$	
		i	$v_y = 12\sin20 - g\text{cv}(t)$	A1	-2.84838……	
		i	$\tan\theta = v_y / v_x$	Dep**M1	Use trig to find a relevant angle	
		i	14.2° below horizontal	A1	14.1763… (75.8° downward vertical) **Examiner's Comments** Many good solutions were seen to this question. Although candidates are getting better at describing the direction relative to a fixed direction, there is still room for improvement. A simple 'below the horizontal' accompanying the angle would have been sufficient. A few candidates lost marks because they were unable to rearrange $8 = 12t\cos20$ correctly to obtain the value of t. A more common error was to use $v^2 = u^2 + 2as$ instead of $v = u + at$ to find the vertical component of velocity without justifying the sign taken when square rooting to find v.	
		ii	$8 = Vt\cos20$	B1		
		ii		*M1	Attempt at use of $s = ut + ½ at^2$	
		ii	$1.5 = Vt\sin20 - gt^2/2$	A1		
		ii	Eliminate t	dep*M1	OR Eliminate V and solve for t	
		ii	Attempt to solve a quadratic for V	dep*M1	AND Sub value for t and solve for V	
		ii	$V = 15.9$	A1	$V = 15.8606…$	

		ii	OR $y = x\tan\theta - gx^2\sec^2\theta/2u^2$	*B1	Use equation of trajectory	
		ii	Substitute values for y, x, θ	dep*M1		
		ii	$1.5 = 8\tan 20 - g8^2\sec^2 20/2V^2$	A1		
		ii	Attempt to solve a quadratic for V	dep*M2	SC M1 for solving for V^2 $V = 15.8606\ldots$	
		ii	$V = 15.9$	A1	**Examiner's Comments** This was well done in terms of the candidates knowing what was required, but in some cases the algebra wasn't always equal to the task. A small minority of candidates made the unfortunate assumption that the target was hit at the highest point of the trajectory.	
			Total	12		
7		a	$\mathbf{g} = \begin{pmatrix} 0 \\ -9.8 \end{pmatrix}$ $\begin{pmatrix} 15 \\ -8 \end{pmatrix} + \begin{pmatrix} -7 \\ -2 \end{pmatrix} + 5\begin{pmatrix} 0 \\ -9.8 \end{pmatrix} = 5\mathbf{a}$ $\mathbf{a} = \begin{pmatrix} 1.6 \\ -11.8 \end{pmatrix}$ or $\begin{pmatrix} 1.6 \\ -g-2 \end{pmatrix}$	B1(AO 1.2)E M1(AO 3.3)E A1(AO 3.4)C [3]	Use of **F** = *ma* with correct *m* and two terms of **F** correct __Examiner's Comments__ A significant number of candidates either forgot to include a gravity term or wrote	

					that $\begin{pmatrix}15\\-8\end{pmatrix}+\begin{pmatrix}-7\\-2\end{pmatrix}+5\begin{pmatrix}0\\-9.8\end{pmatrix}=5\mathbf{a}$ therefore forgetting to multiply the gravity term by the mass of the particle.	
		b	$\mathbf{s}=\frac{1}{2}\begin{pmatrix}1.6\\-11.8\end{pmatrix}(10)^2$ $\mathbf{s}=\begin{pmatrix}80\\-590\end{pmatrix}$ Position vector is $\begin{pmatrix}82\\-545\end{pmatrix}$	M1(AO 3.4)E A1ft(AO 1.1)E A1(AO 1.1)C [3]	Use of $\mathbf{s}=\mathbf{u}t+\frac{1}{2}\mathbf{a}t^2$ with t = 10 50a **Examiner's Comments** The majority of candidates correctly applied the vector form of $\mathbf{s}=\mathbf{u}t+\frac{1}{2}\mathbf{a}t^2$ n an attempt to find the position vector of the particle after 10 seconds, and it was generally only the errors mentioned in part (a) that stopped candidates from scoring full marks in this part.	
			Total	6		
8		a	$R_1 = 2g\cos 30$	B1 (AO1.1) M1	Resolve perpendicular to Π_1 where R_1 is the normal contact force on P — $R_1 = g\sqrt{3}$	

			$F=\frac{\sqrt{3}}{3}\times g\sqrt{3}$ $T-F-2g\sin 30=2(g\cos\theta)$ $8g\sin\theta-T=8(g\cos\theta)$ $8g\sin\theta-T=g-2g\sin 30=10g\cos\theta$ $8\sin\theta-1-1=10\cos\theta\Rightarrow 4\sin\theta=1$	(AO3.3) M1 (AO3.3) A1 (AO1.1) M1 (AO3.3) A1 (AO1.1) M1 (AO3.4) A1 (AO2.2a) [8]	Use of $F=\mu R$ Applying N II parallel to the plane for P Allow with a Applying N II parallel to the plane for Q Allow with a Combining simultaneous equations to eliminate T (dependent on all previous M marks) AG | $F=g$ The M marks for N II parallel to a plane require the correct number of terms and the weight resolved; allow sign errors and sin/cos confusion	
		b	$R=\sqrt{41}$ $R\cos\alpha=4$, $R\sin\alpha=5$ $\tan\alpha=\frac{5}{4}\Rightarrow\alpha=51.3$	B1 (AO1.1) M1 (AO1.1) A1 (AO1.1) M1 (AO1.1)	Forming two equations in R and α (allow sign errors or sin/cos confusion) | 6.403124… This mark is implied if correct α seen 51.340191… $\theta-51.34\ldots=$	

			$\theta - \alpha = \sin^{-1}\left(\frac{1}{R}\right)$ $\theta = 60.3$ $T = 8g\sin 60.3 - 8g\cos 60.3$ $T = 29.3\text{N}$	A1 (AO1.1) M1 (AO3.4) A1 (AO2.2a) [7]	Correct method for finding θ If θ is obtained by calculator with none of the above marks earned, allow SC **B2** for 60.3 or better Using their θ to evaluate T	8.9848… 60.325068… 29.303539…	
			Total	15			
9		a	Acceleration component = $g\sin 30°$ $v_M^2 = 4.2^2 + 2(g\sin 30°)x$ $R = mg\cos 30°$ $F = \frac{\sqrt{3}}{6}mg\cos 30°$ $mg\sin 30° - F = ma$	B1 (AO 1.2) M1 (AO 3.3) B1 (AO 3.3) M1 (AO 3.4) M1* (AO 3.3)	Correct acceleration component seen Use of $v^2 = u^2 + 2as$ for the motion from A to M Resolving perpendicular to the plane Use of $F = \mu R$ for the motion of P between M and B Use of Newton's	x is the distance AM and v_M is the speed of P at M R is the normal contact force between P and the plane, m is the mass of P	

			Answer	Marks	Guidance		
			$12.6^2 = v_M^2 + 2g\left(\sin 30° - \frac{\sqrt{3}}{6}\cos 30°\right)(20 - x)$	M1dep* (AO 3.4)	2nd Law for the motion of P between M and B		
			$12.6^2 = 4.2^2 + 2(g \sin 30°)x$ $+ 2g(20 - x)\left(\sin 30° - \frac{\sqrt{3}}{6}\cos 30°\right)$	M1 (AO 2.1)	Correct use of $v^2 = u^2 + 2as$ for the motion from M to B with their a and correct s Substitute their expression for v_M to obtain an equation in x only		
			$x = 8.8$ so the distance AM is 8.8m	A1 (AO 2.2a) [8]	BC		
		b	$\tan \alpha = \frac{R}{F} = \frac{mg \cos 30°}{\frac{\sqrt{3}}{6} mg \cos 30°}$	M1* (AO 3.1b) M1dep* (AO 1.1)	Equates ratio of contact forces to tan		
			angle = 180° − α = 106.1°	M1 (AO 1.1) [3]	Correct answer (to at least 3 sf)	106.102 113…	
			Total	11			

10	a	(i)	$0 = (V\sin\alpha)^2 + 2(-g)H$ $H = \frac{V^2}{2g}\sin^2\alpha$	M1 (AO 3.3) A1 (AO 1.1) [2]	Use of $v^2 = u^2 + 2as$ vertically **AG** – sufficient working must be shown	
		(ii)	$R = (V\cos\alpha)t$ and $0 = (V\sin\alpha)t + \frac{1}{2}(-g)t^2$ $(V\sin\alpha) - \frac{1}{2}g\left(\frac{R}{V\cos\alpha}\right) = 0$ $gR = 2V^2\sin\alpha\cos\alpha \Rightarrow R = \frac{V^2}{g}\sin 2\alpha$	M1* (AO 3.3) M1dep* (AO 1.1) A1 (AO 2.2a) [3]	Use of $s = ut + \frac{1}{2}at^2$ horizontally and vertically Re-arranging and eliminating t **AG** – sufficient working must be shown	**Alternatively**: Finding t from $0 = V\sin\alpha - gt$ and using double the value, oe
	b		$R_0 = \frac{V^2}{g}$ $\sin^2\alpha = \frac{2gH}{V^2},\ \cos^2\alpha = \frac{V^2 - 2gH}{V^2}$	B1 (AO 3.3) M1* (AO 3.1a) M1dep* (AO 2.1)	Correct expression for maximum range Obtain expressions for $\sin^2\alpha$ and $\cos 2\alpha$ using $\sin^2 I + \cos 2 1 \equiv I$ Eliminate α	

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			$R = \frac{2V^2}{g} \times \frac{\sqrt{2gH}}{V} \times \frac{\sqrt{V^2 - 2gH}}{V}$ $R = \frac{2}{g}\sqrt{2gH}\sqrt{R_0 g - 2gH}$ $\Rightarrow R^2 = \frac{4}{g^2}(2gH)(R_0 g - 2gH)$ $\Rightarrow 16H^2 - 8R_0 H + R^2 = 0$	M1 (AO 3.4) A1 (AO 2.2a) [5]	Eliminate V using maximum range and remove square roots – dependent on both previous M marks **AG** – sufficient working must be shown	
	c		(i) $16H^2 - 1600H + 36\ 864 = 0$ $H = 64$m or 36m	M1 (AO 1.1) A1 (AO 1.1) [2]	Substitute given values to obtain a quadratic in H **BC**	
			(ii) $\sin 2\alpha = \frac{192}{200}$ $\alpha = 36.9°$ $\alpha = 53.1°$	M1 (AO 1.1) A1 (AO 1.1) A1 (AO 1.1) [3]	Use given values to obtain a trigonometric equation in α 0.644 rad 0.927 rad	

		d	The model has not considered the possibility of: • Air resistance • The ball would have dimensions • Wind • The possible spin of the ball	B1 (AO 3.5b) [1]	Any one correct limitation identified	
			Total	16		

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