1. A uniform solid cylinder of height 12 cm and radius *r* cm is in equilibrium on a rough inclined plane with one of its circular faces in contact with the plane.

i. The cylinder is on the point of toppling when the angle of inclination of the plane to the horizontal is 21°. Find *r*.

[3]

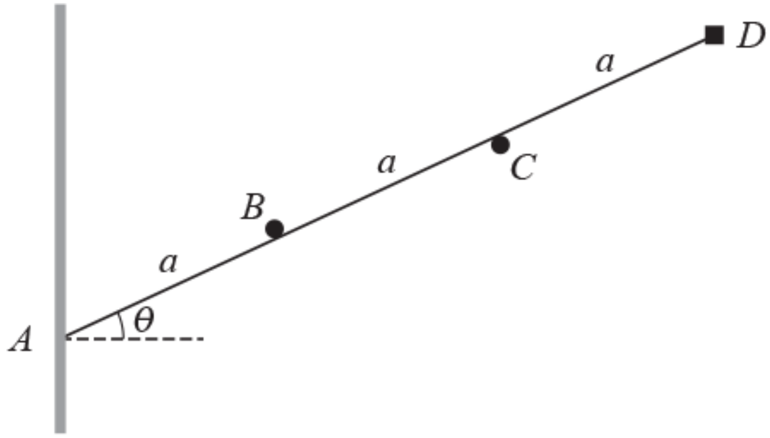
The cylinder is now placed on a different inclined plane with one of its circular faces in contact with the plane. This plane is also inclined at 21° to the horizontal. The coefficient of friction between this plane and the cylinder is μ.

ii. The cylinder slides down this plane but does not topple. Find an inequality for μ.

[2]

2. A uniform ladder *AB* of mass 35 kg and length 7 m rests with its end *A* on rough horizontal ground and its end *B* against a rough vertical wall. The ladder is inclined at an angle of 45° to the horizontal. A man of mass 70 kg is standing on the ladder at a point *C*, which is *x* metres from *A*. The coefficient of friction between the ladder and the wall is $\frac{1}{3}$ and the coefficient of friction between the ladder and the ground is $\frac{1}{2}$.
The system is in limiting equilibrium.
Find *x*. [8]

3.



A thin light rod AD has length $3a$. The end A is in contact with a smooth vertical wall which is perpendicular to the vertical plane containing the rod. The rod carries a load of weight W at the end D. The rod is held in equilibrium by two fixed smooth pegs B and C, where $AB = BC = CD = a$. The rod passes under peg B and over peg C, and makes an angle i with the horizontal (see diagram).

(a)

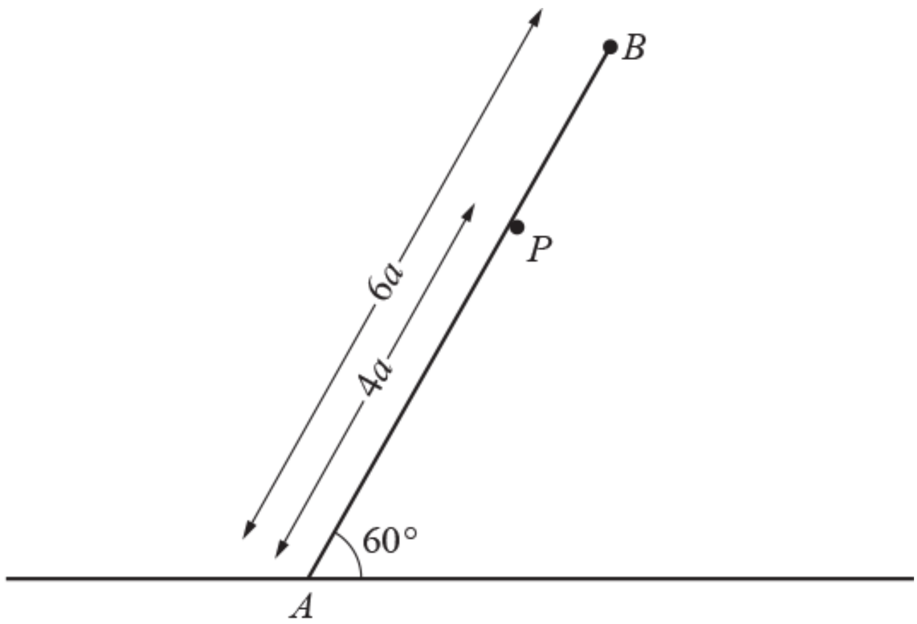
(i) Show that the normal contact force at C may be expressed as $W\left(\dfrac{3\cos^2\theta-1}{\cos\theta}\right)$. [5]

(ii) Find the normal contact force at B in terms of W and θ. [1]

(b) Hence show that the value of θ is at most 35.3°, correct to 3 significant figures. [2]

(c) Show that it is not possible for the magnitude of the reaction at A to equal the magnitude of the reaction at C. [6]

4.



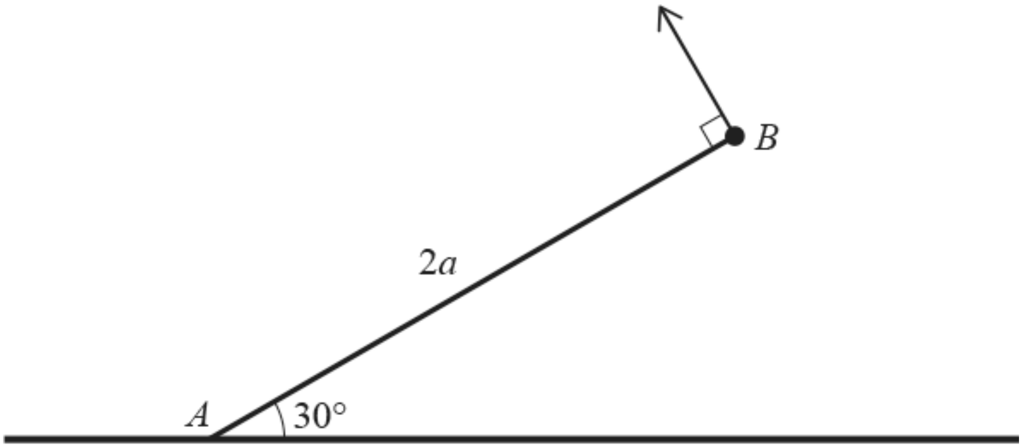
A uniform rod AB of mass m and length $6a$ rests in a vertical plane with A on rough horizontal ground. A particle of mass km, where k is a constant, is attached to the rod at B. The rod makes an angle of 60° with the horizontal and is supported by a small smooth peg P. The distance AP is $4a$ (see diagram).

(i) Calculate, in terms of m, g and k, the magnitude of the force exerted by the peg on the rod. [4]

The coefficient of friction between the rod and the ground is $\frac{1}{3}\sqrt{3}$.

(ii) Find the greatest value of k for which the rod remains in equilibrium. [5]

5.

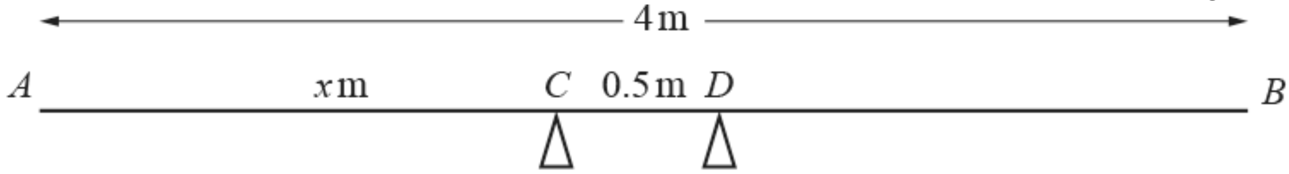


A uniform rod *AB*, of weight W N and length $2a$ m, rests with the end *A* on a rough horizontal table. A small object of weight $2W$ N is attached to the rod at *B*. The rod is maintained in equilibrium at an angle of 30° to the horizontal by a force acting at *B* in a direction perpendicular to the rod in the same vertical plane as the rod (see diagram).

(a) Find the least possible value of the coefficient of friction between the rod and the table. [7]

(b) Given that the magnitude of the contact force at *A* is $\sqrt{39}$ N, find the value of W. [2]

6. A uniform plank *AB* has weight 100 N and length 4 m. The plank rests horizontally in equilibrium on two smooth supports *C* and *D*, where $AC = x$ m and $CD = 0.5$ m (see diagram).



The magnitude of the reaction of the support on the plank at *C* is 75 N. Modelling the plank as a rigid rod, find

(a) the magnitude of the reaction of the support on the plank at *D*, [1]

(b) the value of x. [3]

A stone block, which is modelled as a particle, is now placed at the end of the plank at *B* and the plank is on the point of tilting about *D*.

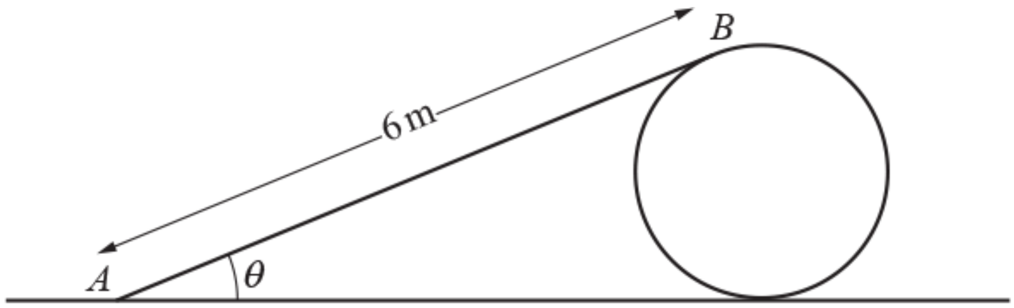
(c) Find the weight of the stone block. [3]

(d) Explain the limitation of modelling

(i) the stone block as a particle, [1]

(ii) the plank as a rigid rod. [1]

7.



The diagram shows a plank of wood *AB*, of mass 10 kg and length 6 m, resting with its end *A* on rough horizontal ground and its end *B* in contact with a fixed cylindrical oil drum. The plank is in a vertical plane perpendicular to the axis of the drum, and the line *AB* is a tangent to the circular cross-section of the drum, with the point of contact at *B*. The plank is inclined at an angle θ to the horizontal, where $\tan\theta = \frac{5}{12}$.

The plank is modelled as a uniform rod and the oil drum is modelled as being smooth.

(a) Find, in terms of *g*, the normal contact force between the drum and the plank. [3]

(b) Given that the plank is in limiting equilibrium, find the coefficient of friction between the plank and the ground.

[5]

8. A uniform ladder *AB*, of weight 150 N and length 4 m, rests in equilibrium with the end *A* in contact with rough horizontal ground and the end *B* resting against a smooth vertical wall. The ladder is inclined at an angle θ to the horizontal, where tan $\theta = 3$. A man of weight 750 N is standing on the ladder at a distance *x* m from *A*.

(a) Show that the magnitude of the frictional force exerted by the ground on the ladder is $\frac{25}{2}(2+5x)$ N. [4]

The coefficient of friction between the ladder and the ground is $\frac{1}{4}$.

(b) Find the greatest value of *x* for which equilibrium is possible. [3]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Guidance
1		i		M1	Attempt to use trigonometry to form equation for r
		i	$r / 6 = \tan 21$	A1	
		i	$r = 2.3(0)$	A1	$r = 2.30318…$ Examiner's Comments The majority of candidates appreciated the position of the centre of mass of the cylinder had to be vertically above the pivot point and proceeded accordingly to get a correct r value. The errors arose from the incorrect use of the height as 12 cm or 3 cm in a rightangled triangle, which included r.
		ii	$\mu <$ cv(r)/6 or $\mu mg\cos 21 < mg\sin 21$	M1	Attempt comparison between weight comp and max friction.
		ii	$\mu < 0.384$ or $\tan 21$	A1	$\mu < 0.38386…$ or $0.38333….$ (from 2.3); allow $\le$ Examiner's Comments The approach to solution of this was to compare the component of the weight down the slope with the maximum friction available. The majority used this approach. However a significant number of candidates used $F = \mu R$, and ended with an inequality for the cylinder NOT to slide down the plane.
			Total	5	
2			Let F_G be the frictional force at ground level and R_G the reaction Let F_W be the frictional force at the wall and R_W the reaction Let x be the distance the man can ascend before the ladder slips $F_G = \frac{1}{2} R_G$ and $F_W = \frac{1}{3} R_W$ Resolve horizontally and vertically: $F_G = R_W$ $R_G + F_W = 105g$	B1(AO2.1) B1(AO3.3) B1(AO3.1b) M1(AO1.1)	Either on a diagram or in words, B1 is awarded for a clear definition of the force variables used Both statements required Both resolutions required Or similarly about the top of the ladder

			$F_W = 15g$ $R_W = 45g = F_G$ $R_G = 90g$ Moments about the foot of the ladder: $35g(3.5\cos 45) + (70g\cos 45)x = 45g(7\cos 45) + 15g(7\sin 45)$ $x = 4.25$	B1(AO3.2a) M1(AO3.3) A1(AO3.4) A1(AO1.1) [8]	Accept numerical value of g used Attempt to solve the 4 equations simultaneously to obtain at least two numerical values for the variables. May be implied by later working **B1** for either F_W and R_W or F_G and R_G Allow sign errors and sin / cos confusion Correct statement cao
			Total	8	
3		a	**(a)** Moments @ A: $R_B a + 3aW\cos\theta = 2aR_C$ Resolve vertically: $W + R_B\cos\theta = R_C\cos\theta$ $W + (2R_C - 3W\cos\theta)\cos\theta = R_C\cos\theta$ $R_C = W\left(\frac{3\cos^2\theta - 1}{\cos\theta}\right)$	M1(AO3.3) M1(AO3.3) A1(AO1.1) M1(AO3.4) A1(AO1.1) [5]	Allow sign errors and sin / cos confusion Allow sign errors and sin / cos confusion — Or moments @ B For both equations correct — Or resolve ∥ rod Attempt solution of simultaneous equations to

					find RC in terms of W and θ AG; sufficient working must be shown	
		a	**(b)** $R_B = W\left(\dfrac{3\cos^2\theta - 2}{\cos\theta}\right)$	B1(AO1.1) [1]	oe, e.g. $R_B = W(3\cos\theta - 2\sec\theta)$	
		b	For equilibrium, $R_B \geq 0$ and $R_C \geq 0$ Critical case is $R_B = 0$, as this gives lower limit for θ so $\cos^2\theta = \frac{2}{3} \Rightarrow \theta_{\max} = 35.3°$ (correct to 3sf)	M1(AO2.1) E1(AO2.2a) [2]	For either considered; allow = for ≥ AG; sufficient reasoning required	
		c	Resolve ‖ rod: $R_A\cos\theta = W\sin\theta$ Obtain $R_A = W\tan\theta$ $W\tan\theta = W\left(\dfrac{3\cos^2\theta - 1}{\cos\theta}\right)$ $3\sin^2\theta + \sin\theta - 2 = 0$ $\sin\theta = \frac{2}{3}$ only, as $\sin\theta \neq -1$ $\theta = 41.8°$, but as this is greater than 35.3° it is not possible that R_A and R_C are equal	M1(AO3.3) A1(AO2.1) M1(AO2.1) M1(AO2.2a) A1(AO2.3) E1(AO2.4) [6]	Allow sin / cos confusion R_A in terms of W and θ correct in any form Equate expressions for R_A and R_C Use of trig identities to form 3-term quadratic equation in $\sin\theta$ BC; the negative value must be seen and not given as a final answer	Or moments @ C

					For correct argument justifying given result
			Total	14	
4		i	$3a(mg\cos 60) + 6a(kmg\cos 60) = 4aR$ $R = \frac{3}{8}mg(1+2k)$	M1 A1 A1 A1 [4]	Moments about *A* (oe) – 3 terms but 4 terms if components of *R* used. Moments about other points needs a complete method. Allow with omission of *g*. A1 for two terms correct AEEF **Examiner's Comments** Many good solutions were seen to this request, with a few cases where *g* was omitted completely or *km* used with *mg*. Pleasingly most had the force at the peg acting in the correct direction – just a few taking it as vertical. Most realised that the way forward with this question was taking moments about the point *A*, to eliminate the need to take into the forces at that point.
		ii	$X = R\sin 60$ $Y + R\cos 60 = kmg + mg$ $[\frac{3\sqrt{3}}{16}mg(1+2k) = \frac{1}{16}mg\mu(13+10k)\ ]$ $k_{max} = \frac{1}{2}$	B1 M1 A1 M1 A1 [5]	Resolving horizontally Resolving vertically, 4 terms, component of *R* Award if taking moments (all relevant forces included) about any point (not A). If Horizontal resolution is replaced this way, give M1A1 to either the vertical or a moments equation and B1 to the other. Similarly if 2 moments equations. Use of $X = \mu Y$ with cv(*R*) and μ substituted and $Y \neq R$ from (i) Allow $k \leq ½$ **Examiner's Comments** Candidates often do not use the most efficient method to solve moments questions. The majority of requests can be solved by taking moments once and resolving twice. As moments have already been used in part (i), the best attack for this question is to resolve twice, usually horizontally and vertically.

					Those who did this were usually the most successful. Candidates who attempted a second moments equation were usually less successful, either omitting the moment of a force or having incorrect distances.
			Total	9	
5		a	Moments about A: $WA\cos30° + 2W(2a\cos 30°) = 2R_B a$ $R_B = \frac{5}{4}\sqrt{3}\,W$ Resolve vertically: $R_A + R_B \cos 30° = W + 2W$ $R_A = \frac{9}{8}W$ Resolve horizontally: $R_A + R_B \sin 30°$ $F_A \le \mu R_A \Rightarrow \mu \ge \ldots$ $\mu \ge \frac{5}{9}\sqrt{3}$ so the least value of μ is $\frac{5}{9}\sqrt{3}$ Alternative solution Moments about B: $Wa\cos30° + F_A(2a\sin 30°) = R_A(2a\cos 30°)$ Resolve along AB: $R_A\cos 60° + F_A\cos 30° = W\cos 60° + 2W\cos 60°$ Both equations correct Solve simultaneous equations for R_A and F_A $R_A = \frac{9}{8}W$ and $F_A = \frac{5}{8}\sqrt{3}\,W$ $F_A \le \mu R_A \Rightarrow \mu \ge \ldots$	M1 (AO 3.3) A1 (AO 1.1) M1 (AO 3.3) A1ft (AO 1.1) B1 (AO 3.3) M1 (AO 3.3) A1 (AO 2.2a) M1 M1 A1 M1 A1 M1	Correct number of terms and attempt at component/ perp. dist. for W and $2W$ Four terms, with attempt at component of the force at B Consistent with their R_B Dependent on all previous M marks Correct number of terms and attempt at components/ perp. | $F_A = \frac{5}{8}\sqrt{3}\,W$ Allow equals here

			$\mu \geq \frac{5}{9}\sqrt{3}$ so the least value of μ is $\frac{5}{9}\sqrt{3}$	A1 [7]	distances Four terms, with components attempted Unsimplified Dependent on both previous M marks (Allow equals here) Both correct, from correct equations Dependent on all previous M marks
		b	$R^2_A + F^2_A = 39$ $W = 4$	M1 (AO 3.4) A1 (AO 1.1) [2]	Use of their R_A and F_A in terms of W in equation for magnitude of resultant cao
			Total	9	
6		a	25 N	B1(AO 3.4)E [1]	**Examiner's Comments** Nearly all candidates correctly stated that the magnitude of the reaction of the support on the plank at *D* was 25N
		b	$2(100) = 75x + (x + 0.5)(25)$	M1(AO 3.3)E A1ft(AO 1.1)C	eg moments about *A* – correct number of terms

			$x = 1.875$	A1(AO 1.1)C [3]	Follow through their 25 only **Examiner's Comments** The most common point on the plank to take moment about in this part was *A* and most who did so were successful in finding *x*. Of those that considered moments about a different point, while some were successful, too often examiners noted that candidates did not include all the required terms.
		c	$(x + 0.5 - 2)(100) = W(4 - 0.5 - x)$ $W = 23.1\text{N}$	M1(AO 3.3)E A1ft(AO 1.1)C A1(AO 1.1)A [3]	moments about *D* – correct number of terms – oe (leading to an equation in *W*) Follow through their *x* only Accept 23 or better 23.076923… **Examiner's Comments** Many candidates struggled with this part and did not realise that if the plank was about to tilt about D then this was the point to take moments about as the reaction at C is therefore zero in this limiting case. Candidates are strongly advised when taking moments to make it clear to the examiners which point (in this case on the plank) they are taking moments about. A number of candidates attempted to take moments about another point together with resolve forces vertically; these attempts were usually unsuccessful.

					AFL Guidance to offer for future teaching and learning practice.
	d	(i) Modelling the stone as a particle assumes that the weight of the stone block acts exactly at *B* therefore the block's dimensions (or the distribution of the mass of the block) have not been taken into consideration	B1(AO 3.5b)A [1]	Accept 'uniform' **Examiner's Comments** It was pleasing that candidates were well prepared for this type of question with many correctly stating that the modelling the stone as a particle assumes that the weight of the block acts exactly at *B* and therefore the block's dimensions (or the distribution of the mass of the block) has not been taken into account. A number of candidates incorrectly stated that modelling the block as a particle implied that the block had negligible mass.
	d	(ii) Modelling the plank as a rigid rod assumes that the plank remains in a straight line and does not bend	B1(AO 3.5b)A [1]	**Examiner's Comments** This part was also answered extremely well, with many correctly stating that modelling the block as a rigid rod assumes that the plank remains in a straight line and does not bend. A number of candidates, however, assumed that friction and/or air resistance had some relevance to this part.
		Total	9	
7	a	$\sin\theta = \frac{5}{13}$ and $\cos\theta = \frac{12}{13}$ or $\theta = 22.6\ldots$ Moments about *A*: $3 \times 10g \times \cos\theta = 6R_B$ $\Rightarrow R = \frac{60g}{13}$	B1 (AO1.1) M1 (AO3.3) A1 (AO1.1) [3]	May be seen anywhere in solution Allow sin/cos confusion R_B is the force at *B*

		b	Vertically : $R_A + R_B\cos\theta = 10g$ Horizontally: $F = R_B \sin\theta$ Using $\mu = \dfrac{F}{R_A}$ $\mu = \dfrac{30}{97}$ or 0.309	M1* (AO3.3) A1 (AO1.1) B1 (AO1.1) dep*M1 (AO3.3) A1 (AO2.2a) [5]	Resolving vertically – allow sign errors and sin/cos confusion Where R_A is the normal contact force acting on the plank at A Resolving horizontally – F is the frictional force at the ground With all angles replaced in their F and R_A	$R_A = \dfrac{970g}{169}$ $F = \dfrac{300g}{169}$ 0.309 278…
			Total	8		
8		a	$2(150\cos\theta) + x(750\cos\theta) = 4(R_B\sin\theta)$ $R_B = F_A \Rightarrow F_A = \frac{25}{2}(2+5x)$	M1 (AO 3.1b) A2 (AO 1.1,1.1) A1 (AO 1.1) [4]	Attempt moments e.g. about A **A1** for any two terms correct **AG** – sufficient working must be shown to justify the given answer	R_B is the normal contact force at the wall F_A is the frictional force at the ground
		b	$R_A = 150 + 750$ $\frac{25}{2}(2+5x) \le \frac{1}{4}(900)$ $x \le 3.2$ so the greatest value of x is 3.2	B1 (AO 3.3) M1 (AO 3.4) A1 (AO 2.2a)	Resolving vertically Correct use of $F_A \le \mu R_A$	R_A is the normal contact force at the ground

				[3]	Must have maximum value of x explicitly stated. / Allow equals throughout
			Total	7	