1. Find

i. $\int \left(2-\frac{1}{x}\right)^2 \mathrm{d}x$,

ii. $\int (4x+1)^{\frac{1}{3}} \mathrm{d}x$

[5]

2. Use the substitution $u = 2x + 1$ to evaluate $\int_0^{\frac{1}{2}} \frac{4x-1}{(2x+1)^5} \mathrm{d}x$.

[7]

3. Use the substitution $u = 1 + \ln x$ to find $\int \frac{\ln x}{x(1+\ln x)^2} \mathrm{d}x$.

[6]

4.
i. Show that $\frac{1}{1-\tan x} - \frac{1}{1+\tan x} \equiv \tan 2x$.

[2]

ii. Hence evaluate $\int_{\frac{1}{12}\pi}^{\frac{1}{6}\pi} \left(\frac{1}{1-\tan x} - \frac{1}{1+\tan x}\right) \mathrm{d}x$, giving your answer in the form $a \ln b$.

[5]

5. By first using the substitution $t = \sqrt{x+1}$, find $\int \mathrm{e}^{2\sqrt{x+1}} \mathrm{d}x$.

[6]

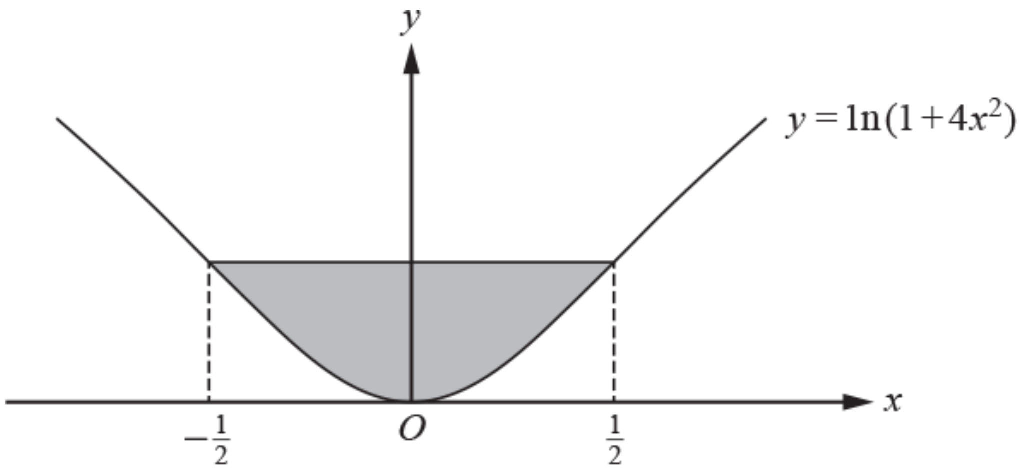
6. Use the substitution $u = x^2 - 2$ to find $\int \frac{6x^3+4x}{\sqrt{x^2-2}} \mathrm{d}x$.

[6]

7. (a) Find $\int 5x^3\sqrt{x^2+1}\,\mathrm{d}x$. [5]

(b) Find $\int \theta \tan^2\theta\,\mathrm{d}\theta$. You may use the result $\int \tan\theta\,\mathrm{d}\theta = \ln|\sec\theta| + c$. [5]

8. (a) Show that the two non-stationary points of inflection on the curve $y =$ 1n $(1 + 4x^2)$ are at $x = \pm\frac{1}{2}$. [6]



The diagram shows the curve $y = \ln(1 + 4x^2)$ The shaded region is bounded by the curve and a line parallel to the x-axis which meets the curve where $x = \frac{1}{2}$ and $x = -\frac{1}{2}$.

(b) Show that the area of the shaded region is given by

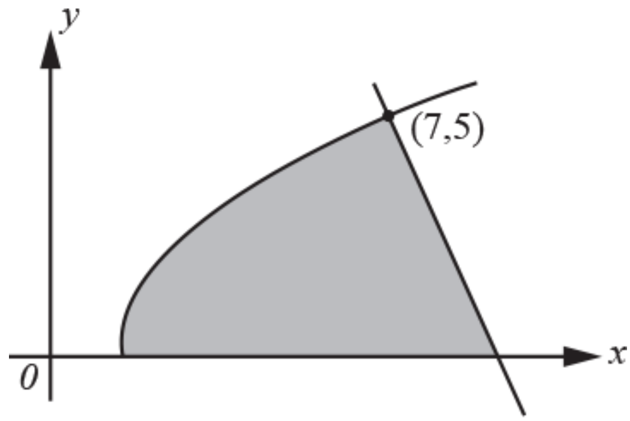
$$\int_0^{\ln 2} \sqrt{e^y - 1}\,\mathrm{d}y.$$ [3]

(c) Show that the substitution $e^y = \sec^2\theta$ transforms the integral in part **(b)** to $\int_0^{\frac{1}{4}\pi} 2\tan^2\theta\,\mathrm{d}\theta$. [2]

(d) Hence find the exact area of the shaded region. [3]

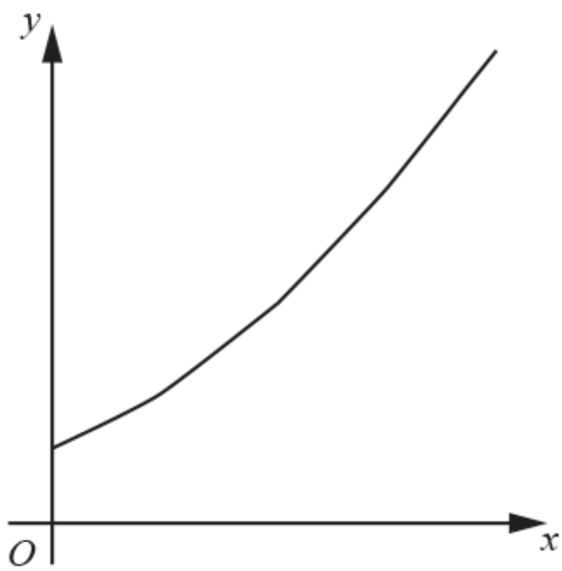
9. Use the substitution $u = 1 + \ln x + x$ to find y $\int \frac{3(x+1)(1-\ln x - x)}{x(1+\ln x + x)}\,\mathrm{d}x$. [6]

10.



The diagram shows the curve $y = \sqrt{4x-3}$ and the normal to the curve at the point (7, 5). The shaded region is bounded by the curve, the normal and the *x*-axis. Find the exact area of the shaded region. [8]

11.



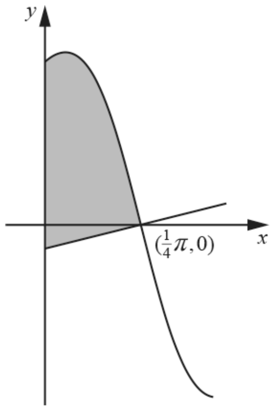
The diagram shows the curve $y = e^{\sqrt{x+1}}$ for $x \geq 0$.

(a) Use the substitution $u^2 = x + 1$ to find $\int e^{\sqrt{x+1}}\,dx$. [6]

(b) Make x the subject of the equation $y = e^{\sqrt{x+1}}$. [1]

(c) Hence show that $\int_{e}^{e^4} ((\ln y)^2 - 1)\,dy = 9e^4$. [4]

12. **In this question you must show detailed reasoning.**



The diagram shows the curve $y = \dfrac{4\cos 2x}{3 - \sin 2x}$, for $x \geq 0$, and the normal to the curve at the point $(\frac{1}{4}\pi, 0)$. Show that the exact area of the shaded region enclosed by the curve, the normal to the curve and the y-axis is $\ln\frac{9}{4} + \frac{1}{128}\pi^2$. [10]

13. Use a suitable trigonometric substitution to find $\int \dfrac{x^2}{\sqrt{1-x^2}}\,dx$. [7]

END OF QUESTION paper

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	Expand to produce form $k_1 + \frac{k_2}{x} + \frac{k_3}{x^2}$	M1	For non-zero constants k_1, k_2, k_3 ; allow if middle term appears as two, so far, unsimplified terms	
		i	Obtain $4x - 41\text{N } x - \frac{1}{x}$ or $4x - 41\text{n}x - x^{-1}$	A1	Condoning absence of modulus signs but A0 if expression involves |1n x| or |4 1n x|	
		ii	Integrate to obtain form $k(4x+1)^{\frac{4}{3}}$	M1	Any non-zero constant k	
		ii	Obtain $\frac{3}{16}(4x+1)^{\frac{4}{3}}$	A1	With coefficient simplified	
		ii	Include … + c or … + k at least once anywhere in answer to question 2	B1	Even if associated with incorrect integral **Examiner's Comments** It was disappointing that this question involving two routine integration requests did not result in greater success. Only 39% of the candidates recorded full marks. The main problem was with part (i) where many candidates did not appreciate that the first step had to be expansion of the integrand; there were many attempts featuring $(2 - \frac{1}{x})^3$. Amongst those who did realise that expansion was needed, there were errors with signs. Candidates fared far better with part (ii) and the only errors to occur with any frequency were an incorrect power of $\frac{2}{3}$ and an incorrect coefficient of $\frac{3}{4}$. One mark was available for inclusion of the constant of integration and most candidates did earn this mark.	

Question			Answer/Indicative content	Marks	Part marks and guidance
			Total	5	

Question			Answer/Indicative content	Marks	Part marks and guidance	
2			Attempt diff to connect d*u* & d*x*	M1	or find $\frac{du}{dx}$ or $\frac{dx}{du}$	
			Correct result e.g. $\frac{du}{dx} = 2$ or d*u* = 2 d*x*	A1		
			Indef integ in terms of $u = \frac{1}{2}\int \frac{2u-3}{u^5}(du)$	A1	Must be completely in terms of *u*.	
			Integrate to $\frac{u^{-3}}{-3} - \frac{3u^{-4}}{-8}$ oe	A1A1	or (using 'by parts') $\frac{(2u-3)u^{-4}}{-8} - \frac{u^{-3}}{12}$	Award B1,B1 for $\frac{4u^{-3}}{-3} - \frac{3u^{-4}}{-2}$ or for $\frac{2u^{-3}}{-3} - \frac{3u^{-4}}{-4}$ or for $\frac{(2u-3)u^{-4}}{-2} - \frac{u^{-3}}{3}$ or for $\frac{(2u-3)u^{-4}}{-4} - \frac{u^{-3}}{6}$
			Use correct variable & correct values for limits	M1	Provided minimal attempt at $\int$f(*u*)du made	

Question			Answer/Indicative content	Marks	Part marks and guidance	
			$= \frac{-23}{384}$ oe $(-0.059895 \ldots..)$ [ISW,e.g. changing to $\frac{23}{384}$]	M1	Accept decimal answer only if minimum of first 3 marks scored **Examiner's Comments** This question was relatively straightforward provided candidates were meticulous in the presentation of their work and in any algebraic manipulation that was necessary. Many did not achieve the transformation of the numerator $4x - 1$ into $2u - 3$. A not insignificant number cancelled the '2' in this numerator with the '2' produced by $dx = \frac{1}{2}du$. The denominator of the transformed integral, $2u^5$, frequently became $(2u)^{-5}$ in the numerator. The '$\frac{1}{2}$' was frequently overlooked. Even though it was easy to separate the transformed integral into two relatively simple parts, many used the idea of integration by parts again. It was interesting to note that some candidates managed to apply the correct limits to a correct function, only to get an incorrect answer. Were they using their calculators incorrectly, particularly with negative values?	
			Total	7		

Question			Answer/Indicative content	Marks	Part marks and guidance	
3					**Examiner's Comments** The process of integration by substitution was well known and most candidates managed to get to the first stage of needing to integrate $\frac{u-1}{u^2}$. Although most had an idea of what to do, the integration of $-\frac{1}{u^2}$ proved harder than expected. The place where the majority fell down was at the end, when they forgot to re-substitute; perhaps candidates were more used to substitution being used with a definite, rather than indefinite, integral.	
			Find d*u* in terms of d*x* (or vv) or $\frac{\mathrm{d}u}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}u}$	M1	An attempt – not necessarily accurate No evidence of *x* at this A1 stage	
			Substitute, changing given integral to $\int \frac{u-1}{u^2}(\mathrm{d}u)$	A1		
			Provided of form $\frac{au+b}{u^2}$, <u>either</u> split as $\frac{au}{u^2}+\frac{b}{u^2}\ldots$	M1	or use 'parts' with '*u*' = *au* + *b*, 'd*v*' = $\frac{1}{u^2}$	
			Integrate as ln $u+\frac{1}{u}$ or FT as $a\ln u-\frac{b}{u}$ [=F(*u*)]	√A1	or $-(au+b)\frac{1}{u}+a\ln u$ FT [= G(*u*)]	

Question			Answer/Indicative content	Marks	Part marks and guidance	
			Re-substitute u = 1 + ln x in G(u) $\ln(1+\ln x)+\dfrac{1}{1+\ln x}$ (+ c) ISW	M1 A1	Re-substitute u = 1 + ln x in F(u) or $\ln(1+\ln x)-\dfrac{\ln x}{1+\ln x}$ (+ c) ISW	
			Total	6		

Question			Answer/Indicative content	Marks	Part marks and guidance	
4		i	$\frac{(1+\tan x)-(1-\tan x)}{(1-\tan x)(1+\tan x)}$	M1	Combine (or write as 2 separate fractions) using common denominator	Accept with / without brackets in num Accept 1– tan *x*.1 + tan *x* in denom
		i	$= \frac{2\tan x}{1-\tan^2 x} = \tan 2x$ Answer Given	A1	$\frac{2\tan x}{1-\tan^2 x}$ essential stage N.B. If tan *x* changed into $\frac{\sin x}{\cos x}$ before manipulation, apply same principles **Examiner's Comments** This was shown successfully by most although, as the answer was given, special notice was taken of each stage and for some (1 – tan *x*)(l + tan *x*) = 1 + $\tan^2 x$ was sometimes in evidence. This earned the method mark (for knowing the approach to be taken) but not the accuracy mark.	A0 for omission of any necessary brackets
		ii	$\int \tan 2x \, dx = \lambda \ln(\sec 2x)$ or $\mu \ln(\cos 2x)$ [= F(x)]	M1		
		ii	$\lambda = \frac{1}{2}$ or $\mu = -\frac{1}{2}$	A1		
		ii	their $F[\frac{\pi}{6}]$ – their $F[\frac{\pi}{12}]$	M1	dependent on attempt at integration………	……i.e. not for $\tan\left(\frac{\pi}{3}\right) - \tan\left(\frac{\pi}{6}\right)$
		ii	$\frac{1}{2}\ln 2 - \frac{1}{2}\ln\frac{2}{\sqrt{3}}$ oe	A1	i.e. any correct but probably unsimplified numerical version	

Question			Answer/Indicative content	Marks	Part marks and guidance	
		ii	$\frac{1}{2}\ln\sqrt{3}$ or $\frac{1}{4}\ln 3$ or $\ln 3^{\frac{1}{4}}$ or $\frac{1}{2}\ln\frac{6}{2\sqrt{3}}$ oe ISW	+A1	i.e. any correct version in the form *a* ln *b* **Examiner's Comments** The obvious errors were made here and the correct multiples of ln(sec 2 *x*) or ln(cos 2*x*) were frequently missing. The logarithmic work was usually well done.	
			Total	7		

Question			Answer/Indicative content	Marks	Part marks and guidance	
5			$\frac{dt}{dx} = k(x+1)^{-\frac{1}{2}}$ or $\frac{dx}{dt} = 2t$ from $x = t^2 \pm 1$ oe	M1	or eg $k\,dt = \frac{dx}{\sqrt{x+1}}$ oe	
			$\int kte^{2t}dt$	M1*	k is any non-zero constant	
			$kt \times \frac{1}{2}e^{2t} \pm k\int \frac{1}{2}e^{2t}dt$	M1dep*		
			$te^{2t} - \int e^{2t}dt$	A1	may be implied by the next **A1**	
			$te^{2t} - \frac{1}{2}e^{2t}$	A1		
			$\sqrt{x+1}e^{2\sqrt{x+1}} - \frac{1}{2}e^{2\sqrt{x+1}} + c$ cao www	A1	+ c may be seen in previous line only for **A1** <u>**Examiner's Comments**</u> Most candidates earned the first two marks. Thereafter a surprising number either didn't recognise the need to use integration by parts, or attributed the variables the wrong way round and made no further progress. The many candidates who did use integration by parts usually went on to score five marks in total – most either missed off the constant of integration, or neglected to substitute back in for x.	if dt is not seen in the integral at some point impose a penalty of 1 mark from total mark of 2 or more
			Total	6		

Question			Answer/Indicative content	Marks	Part marks and guidance	
6			$\frac{du}{dx} = 2x$ oe or $\frac{dx}{du} = \frac{1}{2}(u \pm 2)^{-\frac{1}{2}}$ oe	M1		
			$\frac{Ax^2 + B}{2}$ or better from replacing dx NB $\frac{6x^3 + 4x}{2x} = \frac{6x^2 + 4}{2}$	M1		or substitution of $x = (u \pm 2)^{\frac{1}{2}}$ in denominator from $\frac{dx}{du}$
			substitution of $x^2 = u \pm 2$ or $x = (u \pm 2)^{\frac{1}{2}}$ in numerator	M1	**NB** $3(u+2) + 2$ or $3(u+2)^{3/2} + 2(u+2)^{\frac{1}{2}}$	
			$\int (\frac{3u+8}{\sqrt{u}})[du]$ oe	A1	$\frac{3(u+2)+2}{\sqrt{u}}$ or better	
			$\frac{3u^{\frac{3}{2}}}{\frac{3}{2}} + \frac{8u^{\frac{1}{2}}}{\frac{1}{2}}$ oe	A1	or $6u^{3/2} + 16u^{\frac{1}{2}} - 4u^{3/2}$ from integration by parts	

Question			Answer/Indicative content	Marks	Part marks and guidance	
			$2(x^2-2)^{\frac{3}{2}}+16(x^2-2)^{\frac{1}{2}}+c$ cao	A1	allow $2(x^2 - 2)^{½} (x^2 + 6) + c$ for final mark, **A0** if *du* not seen at some stage in the integral	must see constant of integration here or in previous line and coefficients must be simplified for final **A1** **Examiner's Comments** Most candidates understood the drill for integration by substitution, and most laboriously expressed d*x* in terms of *u* and d*u* rather than factorising the numerator and cancelling out 2*x*. The method marks were achieved by most, but poor algebra often led to the loss of the accuracy marks. Of those who successfully integrated the correct expression in *u*, a significant minority lost the final accuracy mark by omitting "+ *c*" or by failing to substitute back in terms of *x*.
			Total	6		

Question			Answer/Indicative content	Marks	Part marks and guidance		
7		a	$u = x^2 + 1$ $du = 2x dx$ $\frac{5}{2}\int(u-1)u^{\frac{1}{2}}du$ $\frac{5}{2}\int\left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)du$ $u^{\frac{5}{2}} - \frac{5}{3}u^{\frac{3}{2}} + c$ $\left(x^2+1\right)^{\frac{5}{2}} - \frac{5}{3}\left(x^2+1\right)^{\frac{3}{2}} + c$	M1(AO 1.1a) M1(AO1.1) A1(AO1.1) M1(AO1.1) A1(AO1.1) **[5]**	Attempt a substitution of x and dx Replace as far as $k\int(u-1)u^{\frac{1}{2}}du$ Integrate their integral if in u Do not condone missing $+c$ in both **(a)** and **(b)**	**M0** for du = dx	
		b	$\int\tan^2\theta\,d\theta = \int\left(\sec^2\theta - 1\right)d\theta$ $= \tan\theta - \theta$ $u = \theta$, $dv = \tan^2\theta$ So $\int\theta\tan^2\theta\,d\theta = \theta(\tan\theta-\theta) - \int(\tan\theta-\theta)\,d\theta$ $-\frac{1}{2}\theta^2 + \theta\tan\theta - \ln\lvert\sec\theta\rvert + c$	M1(AO 1.1) A1(AO1.1) M1(AO3.1a) A1(AO1.1) A1(AO1.1) **[5]**	Award for sight of the intermediate result Recognise integration by parts with appropriate choice of u and dv Obtain correct intermediate result	**OR** **M1** $\int\theta\tan^2\theta\,d\theta = \int\theta\left(\sec^2\theta-1\right)d\theta$ **A1** $= \int\theta\sec^2\theta\,d\theta - \int\theta\,d\theta$ **M1** $u = \theta$, $dv = \sec^2\theta$ $\int\theta\tan^2\theta\,d\theta$ $= \theta\tan\theta - \int\tan\theta\,d\theta - \frac{1}{2}\theta^2$ **A1** $= -\frac{1}{2}\theta^2 + \theta\tan\theta - \ln\lvert\sec\theta\rvert + c$	

Question			Answer/Indicative content	Marks	Part marks and guidance
			Total	10	

Question			Answer/Indicative content	Marks	Part marks and guidance	
8		a	$\frac{dy}{dx} = \frac{1}{1+4x^2} \times 8x$ Attempt use of quotient rule or equivalent $\frac{d^2y}{dx^2} = \frac{8-32x^2}{(1+4x^2)^2} = 0 \Rightarrow x = \ldots$ $x^2 = \frac{1}{4} \Rightarrow x = \pm\frac{1}{2}$ When $x = \pm\frac{1}{2},\ \frac{dy}{dx} = \pm 2 \neq 0$ and there is a sign change in the second derivative on either side of x so these points are therefore non-stationary points of inflection	B1(AO1.1) B1(AO1.1) M1*(AO3.1a) dep* M1(AO1.1) E1(AO2.1) E1(AO2.4) [6]	For $\frac{1}{1+4x^2}$ For $8x$ Condone only one slip in differentiating their 1st derivative, but if the quotient rule is used it must have subtraction in the numerator Equate 2nd derivative to 0 and attempt to solve for x AG	Condone absence of necessary brackets

Question			Answer/Indicative content	Marks	Part marks and guidance		
		b	Area $= 2\int_0^{\lambda} \mathrm{f}(y)\,\mathrm{d}y$ $y = \ln(1+4x^2) \Rightarrow 4x^2 = \mathrm{e}^y - 1 \Rightarrow \mathrm{f}(y) = \frac{1}{2}\sqrt{\mathrm{e}^y - 1}$ $\lambda = \ln\left(1+4\left(\frac{1}{2}\right)^2\right) = \ln 2$	E1(AO2.1) E1(AO2.1) E1(AO2.1) [3]	Correct integral stated for required area Sufficient working for Sufficient working for top limit of integral		
		c	$\mathrm{e}^y = \sec^2\theta \Rightarrow \mathrm{d}y = 2\tan\,\theta\,\mathrm{d}\theta$ $= 2\int_0^{\frac{1}{4}\pi} \sqrt{\sec^2\theta - 1}\,\tan\theta\,\mathrm{d}\theta = 2\int_0^{\frac{1}{4}\pi} \tan^2\theta\,\mathrm{d}\theta$	M1(AO3.1a) A1(AO2.2a) [2]	Allow for any genuine attempt to differentiate the given substitution and express integral entirely in terms of θ AG; must show evidence for change of limits	Ignore limits for this mark	
		d	Area $= 2\int_0^{\frac{1}{4}\pi} (\sec^2\theta - 1)\,\mathrm{d}\theta$ $= 2[\tan\theta - \theta]_0^{\frac{1}{4}\pi} = 2\left\{\left(\tan\frac{1}{4}\pi - \frac{1}{4}\pi\right) - (\tan 0 - 0)\right\}$ $= 2\left(1 - \frac{1}{4}\pi\right)$	M1(AO3.1a) A1ft(AO1.1) A1(AO1.1) [3]	Reducing to form $\int(a\sec^2\theta + b)\,\mathrm{d}\theta$ Correctly integrating their $a\sec^2\theta + b$ with correct use of limits		
			Total	14			

Question			Answer/Indicative content	Marks	Part marks and guidance	
9			$\frac{du}{dx} = 1 + \frac{1}{x}$	B1		
			x + ln x = ±u ±1 oe substituted into the numerator	M1	allow slip in substitution	
			dx replaced by *their* $\left(\frac{1}{\frac{1}{x}+1}\right)$[d$u$] in integrand oe	M1		
			$\int\left(\frac{3(1-(u-1))}{u}\right)$[d$u$] oe	A1	may be simplified	$\int\left(\frac{6}{u}-3\right)du$
			Aln$u$ + Bu (+ c)	M1dep	following $\int\left(\frac{A}{u}+B\right)du$	
			6ln(1 + ln$x$ + x) – 3(1 + ln$x$ + x) + c oe isw	A1		if du and/or ∫ and/or + c not seen at some stage, withhold the final **A1**
				[6]	**Examiner's Comments** It was pleasing to see so many near perfect solutions to this tricky substitution question. The majority of candidates were able to make a start, although the amount of progress, but many candidates were not able to make much progress. Most correctly differentiated u, but often failed to rearrange the expression correctly. Substitution in the numerator for 1 – ln x – x defeated many and bracket and sign errors were commonplace.	
			Total	6		

Question			Answer/Indicative content	Marks	Part marks and guidance	
10			Differentiate to obtain $k(4x-3)^{-\frac{1}{2}}$	M1	For any non-zero constant *k*	
			Obtain correct $2(4x-3)^{-\frac{1}{2}}$	A1	Or unsimplified equiv	
			Use negative reciprocal of gradient to find intersection of normal with *x*-axis	M1	Using their attempt at first derivative; either using equation of normal $(y=-\frac{5}{2}x+\frac{45}{2})$or relevant right-angled triangle	
			Obtain $-\frac{5}{2}$ for gradient of normal and hence *x* = 9 or equiv such as base of triangle is 2	A1		
			Integrate to obtain $p(4x-3)^{\frac{3}{2}}$	M1	For any non-zero constant *p*	
			Obtain correct $\frac{1}{6}(4x-3)^{\frac{3}{2}}$	A1	Or unsimplified equiv	
			Use limits $\frac{3}{4}$ and 7 to obtain $\frac{125}{6}$for area under curve	A1	Allow calculation apparently using only upper limit	
			Use triangle area to obtain $\frac{155}{6}$for shaded area	A1		
				[8]	**Examiner's Comments** Many solutions to this question were impressive with candidates negotiating their way through the various steps with assurance. Approximately half of the candidates recorded full marks on this question. Most candidates handled the differentiation and integration accurately and appreciated the need to find the point of intersection of the normal with the x-axis. There was some uncertainty over the limits to be used for the integration and a few candidates concluded by subtracting the area of the triangle from $\frac{125}{6}$ rather than adding it to	

Question			Answer/Indicative content	Marks	Part marks and guidance	
					$\frac{125}{6}$. It was surprising how many candidates calculated the area of the triangle by integration rather than carrying out a simple $\frac{1}{2}\times 2\times 5$calculation.	
			Total	8		

Question			Answer/Indicative content	Marks	Part marks and guidance		
11		a	$2u\mathrm{d}u = \mathrm{d}x$ $\int 2u\mathrm{e}^u \mathrm{d}u$ $2u\mathrm{e}^u - \int 2\mathrm{e}^u \mathrm{d}u$ $2u\mathrm{e}^u - 2\mathrm{e}^u$ $2\sqrt{x+1}\mathrm{e}^{\sqrt{x+1}} - 2\mathrm{e}^{\sqrt{x+1}} + c$	B1(AO1.1) M1(AO3.1a) A1(AO2.4) M1(AO1.1a) A1(AO1.1) A1(AO1.1) [6]	Correct relationship soi Convert to integrand in terms of u Fully correct, including du Attempt integration by parts Fully correct in terms of u Fully correct in terms of x	A0 if du never seen, but all remaining marks available Condone no + c + c now required	
		b	$x = (\ln y)^2 - 1$	B1(AO2.1) [1]	Correct equation in form x = f(y)		

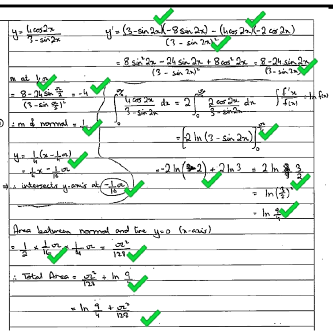
Question			Answer/Indicative content	Marks	Part marks and guidance		
		c	the equation in (b) gives the area between curve and y-axis y = $e^4 \Rightarrow x = 15$, $y = e \Rightarrow x = 0$ area between curve and x -axis is $(8e^4 – 2e^4) – (2e – 2e) = 6e^4$ area of rectangle is $15e^4$ hencereqd area is $15e^4 – 6e^4 = 9e^4$ A.G.	B1(AO2.4) B1(AO2.1) M1(AO2.2a) A1(AO2.4) [4]	Identify geometrical relationship Identify x limits Use x limits in integral from (a) Conclude convincingly		
			Total	11			

Question			Answer/Indicative content	Marks	Part marks and guidance		
12			DR $\frac{dy}{dx} = \frac{(-8\sin 2x)(3-\sin 2x)-(4\cos 2x)(-2\cos 2x)}{(3-\sin 2x)^2}$	M1 (AO 3.1a)	Attempt use of quotient rule	Correct structure, including subtraction in numerator Could be equivalent using the product rule	
				A1 (AO 1.1)	Obtain correct derivative	Award A1 once correct derivative seen even subsequently spoiled by simplification attempt	
			EITHER when $x = \frac{1}{4}\pi$, gradient $= \frac{-16-0}{4} = -4$	M1 (AO 2.4)	DR Attempt to find gradient at $\frac{1}{4}\pi$	EITHER State that $x = \frac{1}{4}\pi$ is being used, and show their fraction with each term (including 0) explicitly evaluated before being simplified ie $x = \frac{1}{4}\pi$, gradient = –4 is M0	
			OR $\frac{(-8\sin\frac{\pi}{2})(3-\sin\frac{\pi}{2})-(4\cos\frac{\pi}{2})(-2\cos\frac{\pi}{2})}{(3-\sin\frac{\pi}{2})^2} = -4$			OR Substitute	
			gradient of normal is $\frac{1}{4}$	B1ft (AO 2.1)	Correct gradient of normal		
			area of triangle is $\frac{1}{2} \times \frac{1}{16}\pi \times \frac{1}{4}\pi (= \frac{1}{128}\pi^2)$	M1 (AO 2.1)	Attempt area of triangle ie		

Question			Answer/Indicative content	Marks	Part marks and guidance		
				M1* (AO 3.1a)		$\frac{1}{4}\pi$ into their derivative and evaluate	
			$\int \frac{4\cos 2x}{3-\sin 2x}\,dx = -2\ln\lvert 3-\sin 2x\rvert$	A1 (AO 1.1)	Obtain integral of form *k*ln|3 – sin2*x*|	ft their gradient of tangent *y* coordinate could come from using equation of normal, $y=\frac{1}{4}(x-\frac{1}{4}\pi)$, or from using gradient of normal	
			$\int_0^{\frac{1}{4}\pi} \frac{4\cos 2x}{3-\sin 2x}\,dx = (-2\ln 2)-(-2\ln 3)$	M1d* (AO 2.1)	Obtain correct integral	Could integrate equation of normal	
				A1 (AO 1.1)	Attempt use of limits		
			$2\ln 3 - 2\ln 2 = \ln 9 - \ln 4 = \ln\frac{9}{4}$ OR $2\ln 3 - 2\ln 2 = 2\ln\frac{3}{2} = \ln\frac{9}{4}$	A1 (AO 2.1)		Condone brackets not modulus Allow any method, including substitution, as long as integral of correct form	
			hence total area is $\ln\frac{9}{4}+\frac{1}{128}\pi^2$ A.G.		Correct area under curve		
				[10]	Obtain correct total area	Possibly with unsimplified coefficient Using $\frac{1}{4}\pi$ and 0;	

Question			Answer/Indicative content	Marks	Part marks and guidance		
						correct order and subtraction (oe if substitution used) Must see a minimum of –2 ln2 + 2ln3 Must be exact At least one log law seen to be used before final answer Any equivalent exact form AG so method must be fully correct A0 if the gradient of –4 results from an incorrect derivative having been used A0 if negative area of triangle not dealt with convincingly	
					Examiner's Comments This was a question requiring detailed reasoning, so candidates were expected to show sufficient evidence of		

Question			Answer/Indicative content	Marks	Part marks and guidance	
					method throughout which not seen on all scripts. Candidates were generally successful when using the quotient rule to find the gradient of the curve. They were then expected to show evidence of finding the gradient at the given point; whilst some showed clear detail, others just stated the gradient with no justification. There were a variety of methods seen when finding the area of the triangle; the most common was to find the y intercept from the equation of the normal whereas others used integration. Some candidates identified that the $\frac{1}{128}\pi^2$ in the given answer must be the area of the triangle and attempted to fudge their answer to obtain this, in some cases deleting correct work and replacing it with a solution that was now worth less credit. Most candidates were able to correctly integrate the equation of the curve, some by inspection and others by using a substitution of their choosing. The limits were usually used correctly, but not all candidates provided sufficient evidence of use of logarithms before the appearance of $\ln\frac{9}{4}$. **Exemplar 8**	

Question			Answer/Indicative content	Marks	Part marks and guidance	
					 This response gained full credit for a correct solution with sufficient justification seen throughout. The derivative is correct, and there is clear evidence of substitution to find the gradient at $x = \frac{1}{4}\pi$. Had the gradient of –4 just been stated, but with no evidence, then this would have been penalised. The equation of the normal is used to find the intercept on the *y*-axis, and this is used to find the area of the triangle. The integration is also correct and limits are used correctly. There is then clear evidence of at least one log law being used to obtain the correct area under the curve. The two areas are then summed to justify the given answer.	
			Total	10		

Question			Answer/Indicative content	Marks	Part marks and guidance		
13			Let $x = \sin\theta$ $I = \int \frac{\sin^2\theta}{\cos\theta}\cos\theta\,d\theta$ $= \int \sin^2\theta\,d\theta$ $= \int \frac{1-\cos 2\theta}{2}\,d\theta$ $= \frac{1}{2}(\theta - \frac{\sin 2\theta}{2}) \quad (+c)$ $= \frac{1}{2}(\sin^{-1}x - x\sqrt{1-x^2}) + c$	M1 (AO 3.1a) A1 (AO 1.1) A1 (AO 1.1) M1 (AO 1.2) A1 (AO 1.1) A1 (AO 2.1) B1 (AO 2.5) [7]	Attempt use $\cos 2\theta = 1 - 2\sin^2\theta$ Correct integral in terms of θ Correct integral in terms of. $x + c$	Correct method with $x = \cos\theta$ scores similarly	
			Total	7			