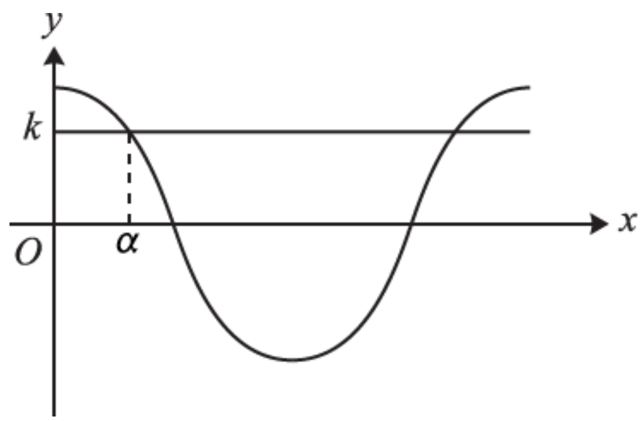
1.



The diagram shows part of the curve $y = 2\cos\frac{1}{3}x$, where x is in radians, and the line $y = k$.

i. The smallest positive solution of the equation $2\cos\frac{1}{3}x = k$ is denoted by α State, in terms of α,

a. the next smallest positive solution of the equation $2\cos\frac{1}{3}x = k$,

[1]

b. the smallest positive solution of the equation $2\cos\frac{1}{3}x = -k$.

[2]

ii. The curve $y = 2\cos\frac{1}{3}x$ is shown in the Printed Answer Book. On the diagram, and for the same values of x, sketch the curve of $y = \sin\frac{1}{3}x$.

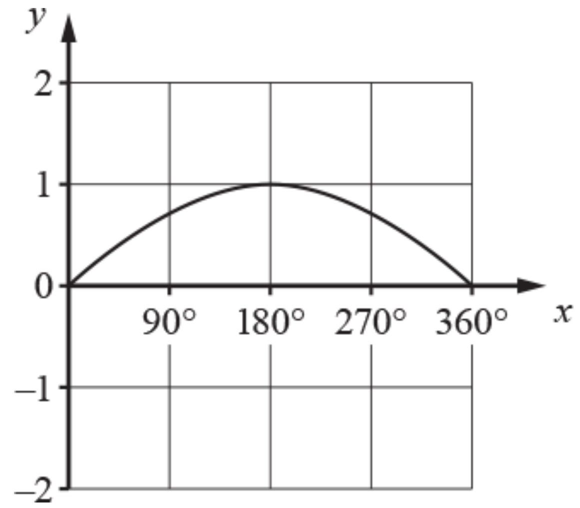
[2]

iii. Calculate the x-coordinates of the points of intersection of the curves in part **(ii)**. Give your answers in radians correct to 3 significant figures.

[4]

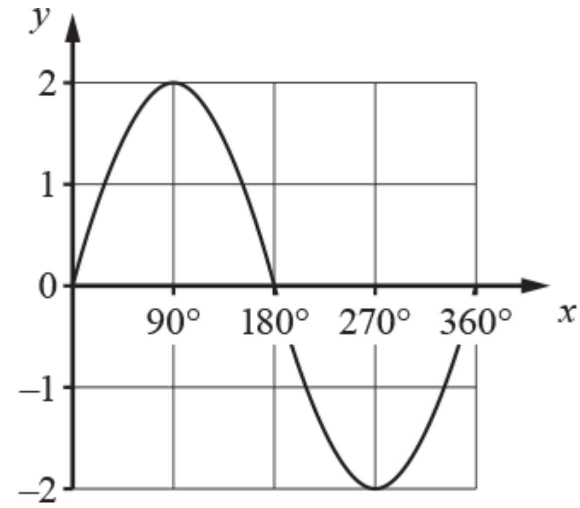
2. Each of the curves shown below is a transformation of part of the curve $y = \sin x$.

(a)



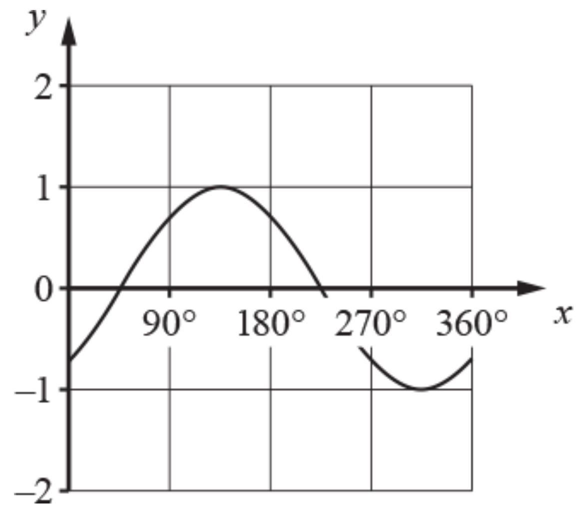
y = ...

(b)



y = ...

(c)

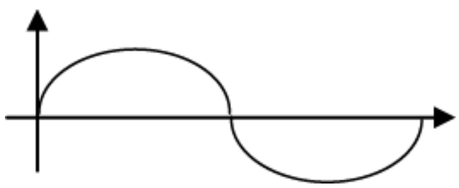


y = ...

Write the equation of each curve. [3]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	**(a)** 6π − α	B1	State 6π − α	Allow unsimplified equiv Allow in degrees ie 1080 − α, or equiv
		i	**(b)** 3π − α	M1	Use period of 6π to make valid attempt at solution	Allow any unsimplified equiv Allow in degrees ie 540 − α, or equiv
		i		A1	Obtain 3π − α	Must be simplified, and in radians Allow *a* or alpha for α
		i			<u>Examiner's Comments</u> Candidates found this question very challenging, and only a small proportion gained any credit. Solutions indicated that many candidates appreciated that the period was no longer 2π, but were still unable to identify what it now was. It was surprising that a number of candidates correctly marked the period as 6π on the graph in part (ii), but were unable to gain any marks in part (i). Partial credit was allowed for unsimplified answers and/or those in degrees, which helped a few candidates.	
		ii		M1	Correct graph shape for $y = k\sin\frac{1}{3}x$	Must be one complete (positive) sin cycle, starting at (0, 0) and clearly intended to have a final root at the same *x*-value as the end point of the given curve – use published overlay for guidance Allow the curve to extend beyond this final root Allow any amplitude Condone a slightly inaccurate *x*-intercept for the middle root Condone poor curvature, including overly straight sections and stationary values that are pointed rather than curved
		ii		A1	Fully correct graph	Curve should clearly be intended to have an amplitude that is half of the given curve, but explicit labels of 1 and −1 are not required A0 if an incorrect scale is given –

		ii			**Examiner's Comments** As with the other graph-sketching question on this paper, too many attempts lacked care or clarity which meant that examiners were unable to discern the true intent of the candidate. The first mark was for appreciating that the period of the curve that they were sketching would be the same as that of the given curve. Some leeway was given when making this judgment, but a sizeable minority offered curves that seemed to have no intention of finishing at the correct point. The second mark was for their curve to have an amplitude that was half of the given curve. Whilst some excellent solutions were seen, a number of curves did not make this intention clear and some had an amplitude that was not consistent throughout the curve.	such as drawing at correct height but then labelling with values other than 1 and −1 A smooth, symmetrical curve is now required, with correct *x*-intercepts clearly intended Ignore any scale, correct or incorrect, on the *x*-axis
		iii	$\tan\frac{1}{3}x = 2$	B1	Obtain $\tan\frac{1}{3}x = 2$ soi	Allow B1 for correct equation even if no, or an incorrect, attempt to solve Give BOD on notation eg $\frac{\sin}{\cos}\left(\frac{1}{3}x\right) = 2$, as long as correct equation is seen or implied at some stage If $\tan\frac{1}{3}x = 2$ s obtained fortuitously from incorrect algebra then mark as B0M1A0A0, even if required roots are seen
		iii	$\frac{1}{3}x = 1.107, 4.249$	M1	Attempt to solve $\tan\frac{1}{3}x = k$	Attempt $3\tan^{-1}(k)$, any (non-zero) numerical k M0 for $\tan^{-1}(3k)$ Allow if attempted in degrees not radians M1 could be implied rather than explicit

		iii		A1	Obtain 3.32	Must be radians and not degrees Allow answers in range [3.32, 3.33] A0 for answer given as a multiple of π Must be radians and not degrees Allow answers in range [12.7, 12.8] A0 for answer given as a multiple of π Max of 3/4 if additional solutions given in range [0, 6π] but ignore any solutions outside of this range Answer only, with no method shown, is 0/4
		iii	$x = 3.32, 12.7$	A1	Obtain 12.7	**Alt method:** **B1** Obtain $5\sin^2\frac{1}{3}x = 4$ or $5\cos^2\frac{1}{3}x = 1$ **M1** Attempt to solve $\sin^2\frac{1}{3}x = k$ or $\cos^2\frac{1}{3}x = k$ (allow M1 if just the positive square root used) **A1** Obtain 3.32 **A1** Obtain 12.7 (max 3/4 if additional solutions in range)
		iii			<u>Examiner's Comments</u> Most candidates could equate the two curves, but many then struggled to make any further progress. Whilst correct generic identities were quoted, they were not always applied accurately. The most common error was for the coefficients of x to be cancelled, resulting in $\tan x = 2$. The attempted use of $\sin^2 x + \cos^2 x = 1$ was rarely successful; in some attempts it was used as $\sin x + \cos x = 1$, whereas attempts at squaring often resulted in the coefficient of x also being squared. Nevertheless, just under half of the candidates did manage to obtain a correct equation, usually $\tan(\frac{1}{3}x) = 2$ but sometimes another acceptable alternative. Most of these candidates could then find at least one correct root, though only the best candidates were able to also find the second root. Some candidates spoiled an otherwise	

					correct solution by giving their answers in degrees not radians.	
			Total	9		
2		a	$y = \sin(\frac{1}{2}x)$	B1(AO1.1) [1]		
		b	$y = 2 \sin x$	B1(AO1.1) [1]		
		c	$y = \sin(x - 45°)$	B1(AO1.1) [1]	or $y = \sin(x + 315°)$ or $y = \cos(x - 135°)$ — or other correct	
			Total	3		