1. i. Sketch the curve $y=(1+x)(2-x)(3+x)$, giving the coordinates of all points of intersection with the axes.
ii. Describe the transformation that transforms the curve $y=(1+x)(2-x)(3+x)$ to the curve $y=(1-x)(2+x)(3-x)$.
2. i. Sketch the curve $y=2 x^{2}-x-6$, giving the coordinates of all points of intersection with the axes.
ii. Find the set of values of $x$ for which $2 x^{2}-x-6$ is a decreasing function.
iii. The line $y=4$ meets the curve $y=2 x^{2}-x-6$ at the points $P$ and $Q$. Calculate the distance $P Q$.
3. 

i. Sketch the curve $y=\frac{2}{x^{2}}$.
[2]
ii. The curve $y=\frac{2}{x^{2}}$ is translated by 5 units in the negative $x$-direction. Find the equation of the curve after
it has been translated.
iii. Describe a transformation that transforms the curve $y=\frac{2}{x^{2}}$ to the curve $y=\frac{1}{x^{2}}$.
4.
i. Sketch the curve $y=2 x^{2}-x-3$, giving the coordinates of all points of intersection with the axes.
ii. Hence, or otherwise, solve the inequality $2 x^{2}-x-3>0$.
iii. Given that the equation $2 x^{2}-x-3=k$ has no real roots, find the set of possible values of the constant $k$.
5.
i. Sketch the curve $y=-\frac{1}{x}$.
ii. The curve $y=-\frac{1}{x}$ is translated by 2 units parallel to the $x$-axis in the positive direction. State the equation of the transformed curve.
[2]
iii. Describe a transformation that transforms the curve $y=-\frac{1}{x}$ to the curve $y=-\frac{1}{3 x}$.
6. The curve $y=f(x)$ passes through the point $P$ with coordinates $(2,5)$.
(i) State the coordinates of the point corresponding to $P$ on the curve $y=\mathrm{f}(x)+2$.
(ii) State the coordinates of the point corresponding to $P$ on the curve $y=\mathrm{f}(2 x)$.
(iii) Describe the transformation that transforms the curve $y=\mathrm{f}(x)$ to the curve $y=\mathrm{f}(x+4)$.
7.
i. Sketch the curve $y=x^{2}(3-x)$ stating the coordinates of points of intersection with the axes.
ii. The curve $y=x^{2}(3-x)$ is translated by 2 units in the positive direction parallel to the $x$ axis. State the equation of the curve after it has been translated.
[2]
iii. Describe fully a transformation that transforms the curve $y=x^{2}(3-x)$ to $y=\frac{1}{2} x^{2}(3-x)$.
8. The diagram below shows the graph of $y=f(x)$.

(a) On the diagram above, draw the graph of $y=\mathrm{f}\left(\frac{1}{2} x\right)$.
(b) On the diagram above, in a different colour, draw the graph of $y=\mathrm{f}(x-2)+1$.
9.
(a) Sketch the curves $y=\frac{3}{x^{2}}$ and $y=x^{2}-2$ on the axes provided below.

(b) In this question you must show detailed reasoning.

Find the exact coordinates of the points of intersection of the curves $y=\frac{3}{x^{2}}$ and

$$
y=x^{2}-2
$$

10. Part of the graph of $y=\mathrm{f}(x)$ is shown below, where $\mathrm{f}(x)$ is a cubic polynomial.

(a) Find $f(-1)$.
(b) Write down three linear factors of $f(x)$.

It is given that $f(x) \equiv a x^{3}+b x^{2}+c x+d$.
(c) Show that $a=-2$.
(d) Find $b, c$ and $d$.
11. Show in a sketch the region of the $x-y$ plane within which all three of the following inequalities are satisfied.

$$
3 y \geq 4 x \quad y-x \leq 1 \quad y \geq(x-1)^{2}
$$

You should indicate the region for which the inequalities hold by labelling the region $R$.
12. The cubic polynomial $f(x)$ is defined by $f(x)=2 x^{3}-7 x^{2}+2 x+3$.
(a) Given that $(x-3)$ is a factor of $f(x)$, express $f(x)$ in a fully factorised form.
(b) Sketch the graph of $y=\mathrm{f}(x)$, indicating the coordinates of any points of intersection with the axes.
(c) Solve the inequality $\mathrm{f}(x)<0$, giving your answer in set notation.
(d) The graph of $y=\mathrm{f}(x)$ is transformed by a stretch parallel to the $x$-axis, scale factor $\frac{1}{2}$. Find the equation of the transformed graph.
13.


The diagram shows the graph of $y=g(x)$.
Using the same scale as in this diagram, sketch, on the copy of the diagram below, the curves:
(a)

$$
y=\frac{3}{2} g(x)
$$



$$
y=g\left(\frac{1}{2} x\right)
$$


[2]
14. $f(x)$ is a cubic polynomial in which the coefficient of $x^{3}$ is 1 . The equation $f(x)=0$ has exactly two roots.
(a) Sketch a possible graph of $y=\mathrm{f}(x)$.


It is now given that the two roots are $x=2$ and $x=3$.
(b) Find, in expanded form, the two possible polynomials $f(x)$.

## Mark scheme

| Question |  | Answer/Indicative content | Marks | Part marks and guidance |
| :---: | :---: | :---: | :---: | :---: |
| 1 | i |  | B1 | -ve cubic with 3 distinct roots <br> $(0,6)$ labelled or indicated on $y$-axis - seen elsewhere not enough <br> $(-3,0),(-1,0)$ and $(2,0)$ labelled or indicated on $x$-axis and no other $x$-intercepts. <br> Examiner's Comments <br> Most candidates recognised that the cubic was negative and sketched an appropriate curve with correct $y$ - and $x$ - intercepts clearly labelled. Some candidates who multiplied out the full expression made an error in finding the $y$-intercept; others with an otherwise correct solution omitted this. Those who drew a positive cubic did not normally see that this contradicted the required value on one of their axes. |
|  | ii | Reflection <br> in the $y$ axis | B1 | Not mirrored / flipped etc. <br> or $x=0$. No / through / along etc. Must be "in". Cannot get $2^{\text {nd }} \mathrm{B} 1$ without some indication of a reflection e.g. flip etc. <br> Do not ISW if contradictory statement seen <br> Examiner's Comments <br> Although the majority of candidates recognised the transformation as being a reflection, their skills in describing this using correct |




\begin{tabular}{|c|c|c|c|c|}
\hline 3 \& $i$

i

i

i \&  \& B1 \& | Excellent curve for $y=\frac{2}{x^{2}}{ }_{\text {n either quadrant }}$ |
| :--- |
| Excellent curve for $y=\frac{2}{x^{2}}$ n other quadrant and no more. |
| SC B1 Reasonably correct curves in 1st and 2nd quadrants and no more |
| Examiner's Comments |
| Graph sketching continues to prove challenging for many candidates. In this case, both the shape and the choice of quadrants proved demanding with fewer than $60 \%$ of candidates securing both marks. It would help if candidates were to equip themselves with a ruler to draw axes as perhaps their intention with asymptotic graphs would then be clearer. With regard to the shape, too many candidates drew an " $L$ " shaped diagram with large sections parallel to, rather than approaching, the axes; this lost marks. | <br>

\hline \& ii \& $$
y=\frac{2}{(x+5)^{2}}
$$ \& M1

A1 \& | $\frac{2}{(x+5)^{2}} \text { or } \frac{2}{(x-5)^{2}} \text { seen }$ |
| :--- |
| Fully correct, must include " $y=$ " or " $\mathrm{f}(x)=$ " |
| Examiner's Comments | <br>

\hline
\end{tabular}




\begin{tabular}{|c|c|c|c|c|}
\hline \& \& \& \& Most candidates used their answer to part (i) and chose the correct outside region, although choosing the inside region was a frequently seen error. The notation used to describe the region was usually correct; incorrect language such as joining the two sections with the word 'and' lost the accuracy mark. <br>
\hline \& iii

iii

iii \& $$
b^{2}-4 a c=1^{2}-4 \times 2 \times-(3+k)
$$

$$
25+8 k<0
$$

\[
k<-\frac{25}{8}

\] \& | M1 |
| :--- |
| A1 |
| A1 | \& | Rearrangement and use of $b^{2}-4 a c<0$, must involve 3 and $k$ in constant term (not $3 k$ ) |
| :--- |
| $p+8 k<0$ oe found, any constant $p$. $p$ need not be simplified |
| Correct final answer |
| Examiner's Comments |
| This proved demanding for many candidates. Although some secured all three marks, many earned no credit as they either put the discriminant equal to zero or, as was frequently seen, to $k$, making no attempt to rearrange the given equation. Accuracy marks were often lost as candidates failed to deal with the minus signs both in the discriminant and in the expression for $c$. A few candidates found the turning point of their graph either by differentiation or by completing the square but these approaches were far less common. | <br>

\hline \& \& Total \& 9 \& <br>
\hline
\end{tabular}

\begin{tabular}{|c|c|c|c|c|}
\hline 5 \& i \&  \& B2 \& \begin{tabular}{l}
Excellent curve in both quadrants: \\
- correct shape, symmetrical, not touching axes \\
- asymptotes clearly the axes \\
- not finite \\
- allow slight movement away from asymptote at one end but not more. \\
Examiner's Comments \\
The vast majority of candidates chose the correct quadrants and basic shape for their sketch to earn one mark and many earned the second mark with a good sketch. Often the second mark was withheld due to inaccuracies such as not clearly indicating the graph tended towards the axes as asymptotes; many graphs ran parallel to the axes for a considerable portion of their length. Others touched or even crossed the axes.
\end{tabular} \\
\hline \& ii \& \[
y=-\frac{1}{x-2} \mathrm{oe}
\] \& M1

A1 \& | $(y=)-\frac{1}{x-2} \text { or }(y=)-\frac{1}{x+2}$ |
| :--- |
| Fully correct, must include " $y=$ " |
| Examiner's Comments |
| Although there was some confusion with signs amongst the lowest attaining candidates, the majority earned both marks for correctly stating the new equation of the curve. It was relatively rare to see the translation mistakenly performed vertically, which represents a considerable improvement on previous sessions. | <br>

\hline \& iii \& | Stretch |
| :--- |
| Scale factor $\frac{1}{3}{ }_{\text {Jarallel to the }}$ |
| $x$-axis (or $y$-axis) | \& B1

B1 \& | Stretch or "stretched" etc.; do not accept squashed, compressed, enlarged etc. |
| :--- |
| Correct description Condone just factor $\frac{1}{3}$ " ${ }^{\text {out }}$ |
| no reference to units. Must not follow e.g. "reflection" | <br>

\hline
\end{tabular}

|  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |








|  |  |  | Whilst this question asked for the coordinates, it is good practice to always provide them on a sketch graph even if not explicitly requested. <br> Key <br> Guidance to offer for future teaching and learning practice |  |
| :---: | :---: | :---: | :---: | :---: |
|  | c $\{x: x<-0.5\} \cup\{x: 1<x<3\}$ | M1 (AO 2.2a) <br> A1ft (AO 2.5) | Identify one set of values <br> Fully correct solution in set notation | ft their cubic roots in (b), even if not 3 real, distinct, roots <br> Allow notation using just inequalities Allow interval notation eg ( $-\infty,-0.5$ ) and/or ( 1,3 ) If both sets of values given then ignore linking sign for this mark <br> ft their cubic roots in (b) , as long as 3 real, distinct, roots <br> Each set should have the correct structure ie $\{x:\}$ with the sets linked by $u$ <br> Allow equivs $\begin{aligned} & \operatorname{eg}\{x: x<-0.5\} \cup\{x: x>1\} \cap\{x: x \\ & <3\} \\ & \text { eg }(-\infty,-0.5) \cup(1,3) \end{aligned}$ |



|  |  |  | $\begin{aligned} & \frac{(2 x+1)(x-1)(x-3)<0}{f(x) \in(-\infty,-0.5) \cup(1,3)} \end{aligned}$ <br> This response demonstrates an alternative, but equally, valid use of set notation and also gains full credit. $\qquad$ <br> This response gains one mark for identifying at least one correct set of values using inequalities. The use of $U$ shows some familiarity with set notation, but the structure is not fully correct so they do not get the second mark. |
| :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & y=2(2 x)^{3}-7(2 x)^{2}+2(2 x)+3 \\ & =16 x^{3}-28 x^{2}+4 x+3 \end{aligned}$ <br> OR | $\begin{gathered} \text { M1 } \\ \text { (AO 1.2) } \\ \\ \\ \text { A1 } \end{gathered}$ | Attempt $\mathrm{f}(2 x)$ or $\mathrm{f}(0.5 x)$ <br> Obtain correct equation <br> Condone lack of brackets as long as implied by later work MO if each term just multiplied or divided by 2 <br> Must have $y=\ldots$ <br> Condone $\mathrm{f}(2 x)=\ldots$, or $\mathrm{f}(x)=\ldots$ |


|  | $y=(2 x-3)(4 x+1)(2 x-1)$ |  |  | (AO 1.1) <br> [2] | Accept unsimplified equiv ISW an incorrect attempt to expand <br> Examiner's Comments <br> Most candidates were able to replace $x$ with $2 x$ to obtain a correct equation although a few gave it as an expression, omitting the ' $y=$ ' at the start. The best solutions made effective use of brackets to show $x$ being replaced with $2 x$, and then expanded them. Other candidates simply wrote down an equation; if this was wrong then there was no evidence to justify that they had been attempting the correct method, but with an error on substitution, as opposed to attempting to double each term. |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Total |  | 9 |  |
| 13 | a | Tails up parabola <br> vertex (2, -3) <br> cutting $x$-axis at $(0,0)$ and $(4,0)$ | $\frac{( \pm 2 \mathrm{~mm})}{( \pm 2 \mathrm{~mm})}$ | B1 (AO 1.1) <br> B1 (AO 1.1) <br> [2] |  <br> (scale: $1 \mathrm{~cm}=1$ unit) |
|  | b | Tails up parabola <br> vertex $(4,-2)$ <br> cutting $x$-axis at $(0,0)$ and $(8,0)$ | $\frac{( \pm 2 \mathrm{~mm})}{( \pm 2 \mathrm{~mm})}$ | B1 (AO 1.1) <br> B1 (AO 1.1) <br> [2] |  <br> (scale: $1 \mathrm{~cm}=1$ unit) |
|  |  | Total |  | 4 |  |



