1. A curve has equation $y = 2x^2$. The points A and B lie on the curve and have x-coordinates 5 and $5 + h$ respectively, where $h > 0$.

   i. Show that the gradient of the line AB is $20 + 2h$.

   [3]

   ii. Explain how the answer to part (i) relates to the gradient of the curve at A.

   [1]

   iii. The normal to the curve at A meets the y-axis at the point C. Find the y-coordinate of C.

   [3]

2. Differentiate $f(x) = x^4$ from first principles. [5]

3.

   (a) Given that $f(x) = x^2 - 4x$, use differentiation from first principles to show that $f'(x) = 2x - 4$. [5]

   [3]

   (b) Find the equation of the curve through (2, 7) for which $\frac{dy}{dx} = 2x - 4$.

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	$y_1 = 50$, $y_2 = 2(5 + h)^2$	B1	Finds y coordinates at 5 and 5 + h	Need not be simplified
		i	$$\frac{(50+20h+2h^2)-50}{(5+h)-5}$$	M1	Correct method to find gradient of a line segment; at least 3/4 values correct	
		i	$20 + 2h$	A1	Fully correct working to give answer **AG**	**Examiner's Comments** Less than two-thirds of candidates secured all three marks for this part, and although some used the fact the answer was given to go back and correct slips in algebraic working, others obtained answers that were clearly "fiddled". Many attempted to use differentiation rather than the correct method to find the gradient of a line segment.
		ii	e.g. "As h tends to zero, the gradient will be 20" **Example responses to (ii)** h is zero so the gradient is 20 **B1** At A $x = 5$, $h = 0$ so gradient equals 20 **B1** As h approaches 0, the gradient of AB approaches 20 which is the gradient of A **B1** As h were infinitely small, $20 + 2h$ is the same as the gradient at A, otherwise it's greater than the gradient at A **B1** The smaller h is the closer the gradient of AB is to the gradient of the curve at A **B1** As h tends to zero the gradient gets closer and closer to the actual value **B1** The gradient of AB tends to the gradient of the tangent of the curve as h tends to zero **B1**	B1	Indicates understanding of limit	e.g. refer to h tending to zero or substitute $h = 0$ into $20 + 2h$ to obtain gradient at A **Examiner's Comments** Correct answers to this were rarely seen. An appreciation of the understanding of a limit was expected, but many just used differentiation to compare the values or, even more commonly, discussed the "negative reciprocal".

			The answer of (i) is converging towards the gradient at A **B1** The gradient at A is 20 **B0** The gradient at A is 20 so $h = 0$ **B0** At A, gradient is 20 so it's $2h$ more **B0** $\frac{dy}{dx} = 20$, so it is the gradient of A plus a bit more **B0** $2h + 20 = 20$ so $h = 0$ **B0** They're getting closer to each other **B0**			
		iii iii iii	Gradient of normal $= -\frac{1}{20}$ $y - 50 = -\frac{1}{20}(x-5), x = 0$ 50¼	B1 M1 A1	Gradient of line must be numerical negative reciprocal of their gradient at A through their A Correct coordinate in any form e.g. $\frac{201}{4}, \frac{1005}{20}$	Any correct method e.g. labelled diagram. **Examiner's Comments** Most candidates were able to access this question, although some still worked with the line segment rather than the point A and so were unable to earn credit. The arithmetical demand led to the loss of accuracy marks in many cases.
			Total	7		
2			$f(x + h) = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$	M1(AO1.1) M1(AO1.1)	Attempt at expansion with product of powers of x and h summing to 4 and some attempt at	

			$\dfrac{f(x+h)-f(x)}{h}=\dfrac{4x^3h+6x^2h^2+4xh^3+h^4}{h}$ $=4x^3+6x^2h+4xh^2+h^3$ As $h \longrightarrow 0$ all the terms in h tend to zero. Therefore $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=4x^3$	A1(AO1.1) A1(AO2.4) E1(AO2.1) [5]	coefficients, not necessarily correct Attempt $\dfrac{f(x+h)-f(x)}{h}$ Allow at most two errors All terms correct Accept some indication that as h tends to 0, the terms involving h vanish and leave $4x^3$ Award for good use of language, and of limit and function notation	Only requires the two M1 marks to be awarded.
			Total	5		
3		a	$f(x+h)-f(x)=\{(x+h)^2-4(x+h)\}-\{x^2-4x\}$ $=x^2+2xh+h^2-4x-4h-x^2+4x$ $=2xh+h^2-4h$ $\dfrac{f(x+h)-f(x)}{h}=\dfrac{2xh+h^2-4h}{h}$	M1(AO 2.1) A1(AO 2.5) M1(AO 1.1) A1(AO 2.1)	Attempt to simplify $f(x+h)-f(x)$ Correct expression for $f(x+h)-f(x)$	

			$= 2x + h - 4$ $$f'(x) = \lim_{h \to 0}(2x + h - 4) = 2x - 4$$	A1(AO 2.4) [5]	Attempt $\dfrac{f(x+h)-f(x)}{h}$ Obtain correct expression Complete proof by considering limit as $h \to 0$	
		b	$y = x^2 - 4x + c$ $7 = 4 - 8 + c$ $c = 11$ $y = x^2 - 4x + 11$	B1(AO 3.1a) M1(AO 1.1) A1(AO 1.1) [3]	Correct equation, including c Attempt to find c Obtain correct equation	
			Total	8		