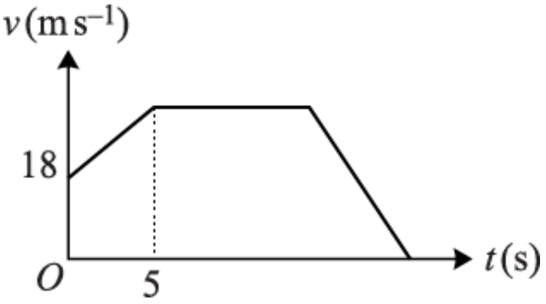
1.




The diagram shows the ($t$, $v$) graph of a car moving along a straight road, where $v$ m s$^{-1}$ is the velocity of the car at time $t$ s after it passes through the point $A$. The car passes through $A$ with velocity 18 m s$^{-1}$, and moves with constant acceleration 2.4 m s$^{-2}$ until $t = 5$. The car subsequently moves with constant velocity until it is 300 m from $A$. When the car is more than 300 m from $A$, it has constant deceleration 6 m s$^{-2}$, until it comes to rest.

i. Find the greatest speed of the car.

[2]

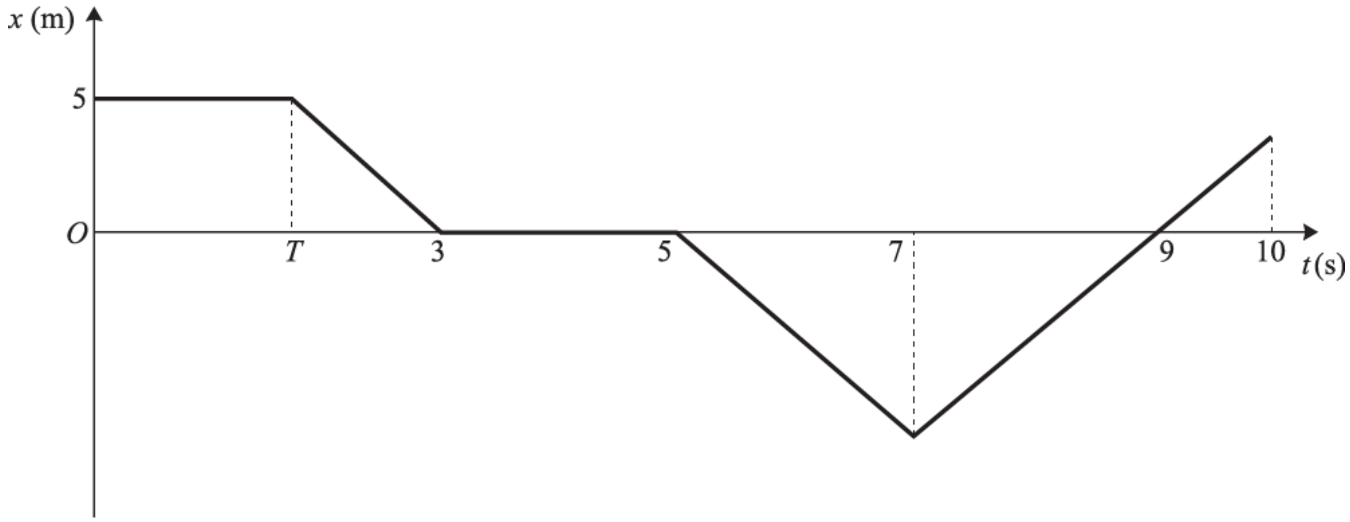
ii. Calculate the value of $t$ for the instant when the car begins to decelerate.

[5]

iii. Calculate the distance from $A$ of the car when it is at rest.

[3]

2.




A particle $P$ can move in a straight line on a horizontal surface. At time $t$ seconds the displacement of $P$ from a fixed point $A$ on the line is $x$ m. The diagram shows the $(t, x)$ graph for $P$. In the interval $0 \leqslant t \leqslant 10$, either the speed of $P$ is 4 $ms^{-1}$, or $P$ is at rest.

i. Show by calculation that $T = 1.75$.

[2]

ii. State the velocity of $P$ when

a. $t = 2$,

[1]

b. $t = 8$,

[1]

c. $t = 9$.

[1]

iii. Calculate the distance travelled by $P$ in the interval $0 \leqslant t \leqslant 10$.

[3]

For $t > 10$, the displacement of $P$ from $A$ is given by $x = 20t - t^2 - 96$.

iv. Calculate the value of $t$, where $t > 10$, for which the speed of $P$ is 4 $ms^{-1}$.

[4]

3. A particle $P$ is projected vertically downwards with initial speed 3.5 $ms^{-1}$ from a point $A$ which is 5 m above horizontal ground.

i. Find the speed of $P$ immediately before it strikes the ground.

[2]

After striking the ground, $P$ rebounds and moves vertically upwards and 0.87 s after leaving the ground $P$ passes through $A$.

ii. Calculate the speed of $P$ immediately after it leaves the ground.

[3]

It is given that the mass of $P$ is 0.2 kg.

iii. Calculate the change in the momentum of $P$ as a result of its collision with the ground.

[2]

4. A small ball is projected vertically upwards with speed 18 m $s^{-1}$ from a point $O$ on the ground. At the same instant a small object is released from rest at a point 27 m vertically above $O$.

(i) Verify that the ball and the object collide 1.5 s after they are set in motion. [4]

(ii) Find the velocities of the ball and the object immediately before they collide. [3]

The ball and the object have equal mass. When the ball and the object collide, they coalesce.

(iii) Find the time after their collision when they strike the ground at $O$. [5]

END OF QUESTION paper

# Mark scheme

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | |
|---|---|---|---|---|---|---|
| 1 | | i | $v = 18 + 2.4 \times 5$ | M1 | $v = u + at$ | |
| | | i | $v = 30$ | A1 | | |
| | | ii | Distance while accelerating = (18 + 30) × 5/2 | B1 | Or 30 × 5 − (30 − 18) × 5/2 etc = 120, or 45 + 75. Numerical. | |
| | | ii | Distance at constant speed = 30($t$ – 5) | B1 | Tolerate 30$t$. Algebraic. | |
| | | ii | | M1 | Adds their areas to get 300 | |
| | | ii | 30($t$ – 5) + (18 + 30) × 5/2 = 300 | A1 | 30$T$ = 300 − 120, 30$t$ + 45 + 75 = 300, etc | |
| | | ii | $t = 11$ | A1 | | |
| | | ii | *OR* | | | |
| | | ii | Distance while accelerating = (18 + 30) × 5/2 (= 120) | B1 | Or 30 × 5 − (30 − 18) × 5/2 etc = 120, or 45 + 75. Numerical. | |
| | | ii | Distance at constant speed = 300 − cv(120) | M1 | Subtracts their area from 300 | |
| | | ii | $\text{Time at constant speed} = \dfrac{(300 - \text{cv}(120))}{30}$ | B1 | Equivalent to "distance at constant speed algebraic" | |
| | | ii | Time at constant speed = 6 | A1 | | |
| | | ii | $t = 11$ | A1 | | |
| | | ii | *OR* | | | |
| | | ii | Distance = 30$t$ | B1 | Rectangle, comprising 300 + area of "missing triangle" | |
| | | ii | Distance = (30 − 18) × 5/2 | B1 | "Missing triangle", to be removed | |
| | | ii | 30$t$ – (30 − 18) × 5/2 = 300 | M1A1 | Subtracts their areas to get 300 | |
| | | ii | $t = 11$ | A1 | | |
| | | ii | *OR* | | | |
| | | ii | Distance while accelerating = (18 + 30) × 5/2 | B1 | 120 | |
| | | ii | Distance at constant speed = 30($t$ – 5) | B1 | May be implied. Tolerate 30$t$. Algebraic. | |
| | | ii | Distance at constant speed = 300 − 120 = 30($t$ – 5) | M1A1 | *OR* 180 = 30$t$ M1, $t$ = 6 A1 | |
| | | ii | $t = 11$ | A1 | | |
| | | ii | Splitting area horizointally | | | |

| | | | | | | |
|---|---|---|---|---|---|---|
| | | ii | Distance = 18$t$ | B1 | Lower portion of area | |
| | | ii | Distance = ($t$ + [$t$ – 5]) × (30 – 18)/2 | B1 | Upper portion of area | |
| | | ii | 18$t$ + ($t$ + [$t$ – 5]) × (30 – 18)/2 = 300 | M1A1 | 30$t$ – 30 = 300 | |
| | | ii | $t$ = 11 | A1 | | |
| | | iii | $S = 30^2/(2 \times (\pm 6))$ | M1 | $0^2 = 30^2 \pm 2 \times 6S$, with candidate's $v$(i) | |
| | | iii | $S$ = 75 | A1 | | |
| | | iii | Distance = 375 m | A1ft | 300 + cv(75) | |
| | | iii | *OR* | M1 | Accept $T$ = 5 if no working or from 30/–6, with candidate's $v$(i) | |
| | | iii | $T$ = 30/6 and $S$ = 30$T$/2 | | | |
| | | iii | $S$ = 75 | A1 | | |
| | | iii | Distance = 375 m | A1ft | 300 + cv(75)<br><br>**Examiner's Comments**<br><br>Many fully correct solutions were seen. The most frequent error was not realising in part (ii) that 300 m was the entire journey distance while both accelerating and moving with constant speed. Thus "300/30 = 10, 10 + 5 = 15 seconds" was the most common mistake seen. Very many candidates expressed their work in a very informal way, their solutions consisting predominantly of numbers (120, 180, 6, 11) without much explanation of what they meant.<br><br>Candidates could directly find in part (iii) the distance while decelerating (from $0^2 = 30^2 - 2 \times 6s$ ), and so those who first calculated a time while decelerating needed to use that time to find a distance before becoming eligible for any mark. | |
| | | | Total | 10 | | |
| 2 | | i | 5/($T$ – 3) = –4 *OR* 5/(3 – $T$) = 4 | M1 | Accept verification, 4 × (3 – 1.75) M1 | |
| | | i | $T$ = 1.75 | A1 | = 5 A1 *OR* 5/(3 – 1.75) M1 = 4 A1 | |
| | | ii | **(a)** –4 ms$^{-1}$ | B1 | | |
| | | ii | **(b)** 4 ms$^{-1}$ | B1 | | |
| | | ii | **(c)** 4 ms$^{-1}$ | B1 | | |
| | | iii | 2 × (–)4, 2 × 4, (1 ×)4 | M1* | Calculates any one unknown distance | Allow if only one |

| | | | | | | |
|---|---|---|---|---|---|---|
| | | iii | $d = (–)5 + (–)8 + 8 + 4$ | D*M1 | Adds 5 and "3 other" distances or –5 and "3 other" displacements | calc. correct<br><br>Note t = 5 to t = 9, t = 5 to t = 10 etc, may be one term |
| | | iii | $d = 25$ m | A1 | Correctly comes from 4 × (1.25 + 4 + 1) 3/3 | |
| | | iv | $v = \mathrm{d}(20t - t^2 - 96)/\mathrm{d}t$ | M1* | Differentiates $x$, accept $20 - t$ as "differentiation" | |
| | | iv | $v = 20 - 2t$ | A1 | | |
| | | iv | $20 - 2t = -4$ | D*M1 | $20 - 2t + c = -4$ is DM0 | |
| | | iv | $t = 12$ (ignore any solutions less than 10) | A1 | Only from $20 - 2t = -4$. This answer can arise fortuitously from solving $20t - t^2 - 96 = 0$.<br><br>**Examiner's Comments**<br><br>In recent examinations, a ($t$, $v$) graph has been presented to candidates. It was clear that a minority of candidates used methods inappropriate to a ($t$, $x$) diagram. Others wrongly used constant acceleration formulae, in a problem where changes of velocity are instantaneous. Only the best candidates were able to solve fully, as only they realised that a speed of 4 m s$^{-1}$ was consistent with $v = -4$. | SC Verifying that t = 12 gives v = –4 can gain final M1A1 (A special case of trial and refinement) |
| | | | **Total** | **12** | | |
| 3 | | i | $v^2 = 3.5^2 + 2g \times 5$ | M1 | Uses $v^2 = 3.5^2 +/- 2g5$ | Accept $-3.5^2$ for $(-3.5)^2$ etc |
| | | i | $v = 10.5$ ms$^{-1}$ | A1 | **Examiner's Comments**<br><br>Was almost always answered correctly. | |
| | | ii | | M1 | $+/-5 = 0.87u +/- g\,0.87^2/2$ | May come from $s = vt - gt^2/2$ |
| | | ii | $5 = 0.87u - g \times 0.87^2/2$ | A1 | | |
| | | ii | $u = 10.0$ ms$^{-1}$ | A1 | **Examiner's Comments**<br><br>This part was almost always answered correctly, save for a significant minority of candidates who | |

| Question | | | Answer | Marks | Guidance | |
|---|---|---|---|---|---|---|
| | | | | | had the wrong sign before the term involving *g*. One unusual feature was the high proportion of candidates who rearranged the standard *suvat* equation into a form which had *u* as its subject. | |
| | | iii | Change = 0.2 × 10.5 + 0.2 × 10 | M1 | Or +/– 0.2(Ans(i) +/– Ans(ii))<br><br>It is OK get –4.1 from correct work<br><br>**Examiner's Comments** | |
| | | iii | Change = 4.1(0) kg $ms^{-1}$ | A1 | Was nearly always answered by subtracting the magnitudes of the momentum on landing and on lift-off. A minority of candidates used the initial speed of 3.5 m $s^{-1}$ in their calculations. | |
| | | | Total | 7 | | |
| 4 | | i | Object fall = $9.8 \times 1.5^2/2$ | B1 | 11.025 m | **SC** 18x1.5 = 27 B1 only but $9.8 \times 1.5^2/2 + 18 \times 1.5 - 9.8 \times 1.5^2/2 = 27$ B1B1M1A1 |
| | | | Ball rise = $18 \times 1.5 - 9.8 \times 1.5^2/2$ | B1 | 15.975 m | |
| | | | Distance = 11.025 + 15.975 | M1 | Appropriate signs and full accuracy | |
| | | | Distance = 27 m AG | A1<br>[4] | | |
| | | | *OR* | | | |
| | | | Distance fallen = $9.8t^2/2$ | B1 | | $9.8t^2/2$ without the context of "distance fallen" is B0. Similarly for $18t - 9.8t^2/2$ |
| | | | Distance risen = $18t - 9.8t^2/2$ | B1 | | |
| | | | $9.8t^2/2 + 18t - 9.8t^2/2 = 27$ | M1 | Solves total of distances equation | |
| | | | $t = 1.5$ AG | A1 | **Examiner's Comments**<br><br>In part (i), candidates were asked to "verify" t =1.5 which indicated that finding the distances moved by the object and ball should first be calculated. The next stage was to demonstrate that these distances added together equalled the 27 m initial gap. This stage had to be done using exact arithmetic; the use of rounded figures could lead only to the conclusion that the ball and object were very close. Some candidates successfully set | |

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | | | up and solved and equation in t containing two squared terms (which subsequently cancelled) and so showed that t =1.5. | |
| | | ii | Object vel (=9.8 x 1.5) = 14.7 m $s^{-1}$ (down)<br>Ball vel =+/−(18 − 9.8 x 1.5)<br>Ball vel = 3.3 (upwards) | B1<br>M1<br>A1<br>[3] | Accept −14.7 m $s^{-1}$<br>Must be a difference expression<br>3.3 if $v$=18 or −3.3 if $v$ = −18<br>Candidates may find object velocity and ball velocity in (i). **These answers must be quoted here for 3 marks to be given.**<br>Examiner's Comments<br>Part (ii) was done well. | |
| | | iii | $14.7m - 3.3m = 2mu$<br>$u = 5.7$<br>$15.975 = 5.7t + 9.8t^2/2$<br>Solve $4.9t^2 + 5.7t - 15.975 = 0$<br>$t = 1.32$ s | M1<br>A1<br>M1*<br>D*M1<br>A1<br>[5] | Momentum conservation; after mass = 2xbefore mass<br>Must use coalesced velocity and s<27<br>3 term QE and evidence of method of solution if answer incorrect.<br>Disregard signs<br>$v^2 = 5.7^2 + 2 \times 9.8 \times 15.975$ and $v = 5.7 + 9.8t$ Create both M1*<br>Find $v$ (=18.537..), solve for $t$ D*M1 Answer t = 1.32 s<br>Examiner's Comments<br>Part (iii) was often accompanied by diagrams which showed a horizontal collision. The momentum equation proved awkward, with uncertainty about the direction of motion signs, both before and after coalescence, or not having the "after" mass doubled. | |

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|---|---|---|---|---|---|
| | | | Total | 12 | |