1.

(a) Find and simplify the first three terms in the expansion of $\left(3-\frac{x}{2}\right)^6$ in ascending powers of x. [3]

(b) Explain how the result in part (a) can be used to give an approximation to the value of $(2.95)^6$. [1]

2. i. Find the binomial expansion of $(2 + x)^5$, simplifying the terms.

[4]

ii. Hence find the coefficient of y^3 in the expansion of $(2 + 3y + y^2)^5$.

[3]

3. i. Find and simplify the first three terms in the expansion of $(2 + 5x)^6$ in ascending powers of x.

[4]

ii. In the expansion of $(3 + cx)^2(2 + 5x)^6$, the coefficient of x is 4416. Find the value of c.

[3]

4. i. Find and simplify the first three terms in the binomial expansion of $(2 + ax)^6$ in ascending powers of x.

[4]

ii. In the expansion of $(3 - 5x)(2 + ax)^6$, the coefficient of x is 64. Find the value of a.

[3]

5. i. Find the binomial expansion of $(3 + kx)^3$, simplifying the terms.

[4]

ii. It is given that, in the expansion of $(3 + kx)^3$, the coefficient of x^2 is equal to the constant term. Find the possible values of k, giving your answers in an exact form.

[2]

6. Given that the binomial expansion of $(1 + kx)^n$ is $1 - 6x + 30x^2 - \dots$, find the values of n and k. State the set of values of x for which this expansion is valid.

[6]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		a	$\left(3-\frac{x}{2}\right)^6 = 3^6 + {}^6C_1 3^5\left(-\frac{x}{2}\right) + {}^6C_2 3^4\left(-\frac{x}{2}\right)^2 + \ldots$ $= 729 - 729x + \frac{1215}{4}x^2 + \ldots$	M1 (AO1.1a) A1 (AO1.1) A1 (AO1.1) [3]	For x and x^2 terms as ${}^6C_1 k_1 x$ and ${}^6C_2 k_2 x^2$ For at least two terms correct (may be unsimplified) For all three terms correct and simplified	
		b	$3-\frac{x}{2} = 2.95 \Rightarrow x = 0.1,$ so substitute $x = 0.1$ into the expansion from part **(a)** to obtain an approximation for 2.95^6	E1 (AO2.4) [1]	Solve $3-\frac{x}{2} = 2.95$ and state that this value must be substituted into the	

					previous expansion	
			Total	4		
2		i	$(2 + x)^5 = 32 + 80x + 80x^2 + 40x^3 + 10x^4 + x^5$	M1*	Attempt expansion resulting in at least 5 terms – products of powers of 2 and x	Each term must be an attempt at a product, including binomial coeffs if used Allow M1 for no, or incorrect, binomial coeffs Powers of 2 and x must be intended to sum to 5 within each term (allow slips if intention correct) Allow M1 for powers of $\frac{1}{2}x$ from expanding $k(1 + \frac{1}{2}x)^5$, any k (allow if powers only applied to x and not $\frac{1}{2}$
		i		M1d*	Attempt to use correct binomial coefficients	At least 5 correct from 1, 5, 10, 10, 5, 1 – allow missing or incorrect (but not if raised to a power) May be implied rather than explicit Must be numerical eg ${}^5C_1$ is not enough They must be part of a product within each term The coefficient must be used in an attempt at the relevant term ie $5 \times 2^3 \times x^3$ is M0 Allow M1 for correct coeffs from $k(1 + \frac{1}{2}x)^5$, any k
		i		A1	Obtain at least 4 correct simplified terms	Either linked by ‘ + ’ or as part of a list
		i		A1	Obtain a fully correct expansion	Terms must be linked by ‘ + ’ and not just commas A0 if a correct expansion is subsequently spoiled by attempt to simplify, including division **SR for expanding brackets:** M2 – for attempt using all 5 brackets giving a quintic A1 – obtain at least 4 correct simplified terms A1 – obtain a fully correct expansion

						Examiner's Comments Virtually all candidates were able to carry out an efficient and accurate attempt at the binomial expansion with only the occasional slip resulting in the loss of a mark. A very small number of candidates omitted the binomial coefficients, or failed to write the terms as products. Some candidates attempted the C4 technique of first taking out a factor of 2^5, but this was often poorly executed and candidates would be well advised to select the method most appropriate to the question posed.
		ii	$80(3y + y^2)^2 + 40(3y + y^2)^3$ coeff of $y^3 = (80 \times 6) + (40 \times 27) = 1560$	M1	Attempt to use $x = 3y + y^2$	Replace x with $3y + y^2$ in at least one relevant term and attempt expansion, including relevant numerical coeff from (i) or from restart
		ii		A1	Obtain $480(y^3)$ or $1080(y^3)$	Could be with other terms, inc y^3
		ii		A1	Obtain 1560 (or $1560y^3$)	Ignore terms involving powers other than y^3
		ii	OR $(1 + y)^5(2 + y)^5$ $= (1 + 5y + 10y^2 + 10y^3 \ldots) \times$ $(32 + 80y + 80y^2 + 40y^3 \ldots)$ coeff of $y^3 = 320 + 800 + 400 + 40$			OR M1– attempt expansion of both $(1 + y)^5$ and $(2 + y)^5$ (allow powers higher than 3 to be discarded) and make some attempt at the product A1 – obtain at least 2 correct coeffs of y^3 A1 – obtain 1560 (or $1560y^3$)
		ii	OR $((2 + 3y) + y^2)^5$ $= (2 + 3y)^5 + 5(2 + 3y)^4y^2$ $= \ldots 10 \times 4 \times 27y^3 \ldots$ $+ 5 \times 4 \times 8 \times 3y \times y^2$ coeff of $y^3 = 1080 + 480 = 1560$			OR M1 – attempt expansion of at least one relevant term A1 – obtain $480(y^3)$ or $1080(y^3)$ A1 – obtain 1560 (or $1560y^3$) OR

						M1 – attempt expansion of all 5 brackets (allow powers higher than 3 to be discarded throughout method) A2 – obtain 1560 (or $1560y^3$) <u>Examiner's Comments</u> This proved to be one of the most challenging questions on the paper, and many candidates had little idea of how to formulate an appropriate strategy. A number of candidates felt unable to even make an attempt at the question, possibly because they had not seen anything similar on previous papers. The most common method attempted was to replace x with $3y + y^2$ in the expansion found in part (i). This often resulted in partial success, but common errors were to use $3y^3$ rather than $(3y^3)$ and to not appreciate that $(3y + y^2)^2$ would also provide a relevant term. A surprisingly popular method involved attempting the expansion of $\{(1 + y)(2 + y)\}^5$ but a common error was to omit the power of 5 from one of the brackets. The third main method was to attempt an alternative expansion such as $\{(2 + 3y + y^2)\}^5$, and there were also some candidates who attempted to multiply out all five brackets but this was very rarely successful. A number of solutions simply consisted of a jumble of numbers with no attempt to actually explain the working. If examiners cannot discern whether a valid method has been used then it is difficult to award credit.
			Total	7		

3		i	$(2 + 5x)^6 = 64 + 960x + 6000x^2$	M1	Attempt at least first 2 terms – products of binomial coeff and correct powers of 2 and $5x$	Must be clear intention to use correct powers of 2 and $5x$ Binomial coeff must be 6 soi; ${}^6C_1$ is not yet enough Allow BOD if 6 results from $^6/_1$ Allow M1 if expanding $k(1 + {}^5/_2x)^6$, any k
		i		A1	Obtain $64 + 960x$	Allow 2^6 for 64 Allow if terms given as list rather than linked by '+'
		i		M1	Attempt 3rd term – product of binomial coeff and correct powers of 2 and $5x$	Allow M1 for $5x^2$ rather than $(5x)^2$ Binomial coeff must be 15 soi; ${}^6C_2$ is not yet enough Allow M1 if expanding $k(1 + {}^5/_2x)^6$, any k $1200x^2$ implies M1, as long as no errors seen (including no working shown)
		i		A1	Obtain $6000x^2$ **Examiner's Comments** Candidates are becoming ever more adept at the binomial expansion, and the majority scored full marks on this question. As always, the most successful candidates made effective use of brackets. The most common error was to omit to square $5x$ in its entirety. Candidates usually used the correct binomial coefficients, but this was not always shown explicitly which made it difficult to award credit. A few candidates spoiled an otherwise correct answer by dividing through by a common factor.	A0 if an otherwise correct expansion is subsequently spoiled by attempt to simplify eg $4 + 60x + 375x^2$ **If expanding brackets:** Mark as above, but must consider all 6 brackets for the M marks (allow irrelevant terms to be discarded)
		ii	$(9 + 6cx \ldots)(64 + 960x + \ldots)$	M1*	Expand first bracket and attempt at least one relevant product	Expansion of first bracket does not have to be correct, but must be attempted so M0 if using $(3 + cx)(64 + 960x\ldots)$ No need to see third term in expansion of first bracket

						Must then consider a product and not just use 6c + 960 Expansion could include irrelevant / incorrect terms Using an incorrect expansion associated with part **(i)** can get M1 M1
		ii	(9 × 960) + (6c × 64) = 4416 8640 + 384c = 4416 384c = −4224	M1d*	Equate sum of the two relevant terms to 4416 and attempt to solve for c	Must now consider just the two relevant terms M0 if additional terms, even if error has resulted in kx BOD if presence of x is inconsistent within equation
		ii	c = −11	a1	Obtain c = −11 **Examiner's Comments** This part of the question proved to be more challenging. The higher-scoring candidates were able to identify the required products and find the value of c with ease. Other candidates could identify the two required products but then spoilt their answer with 8640 + 384c becoming 9024c. Some gained one mark for identifying one of the two required products. However, a number of candidates struggled to get started, with the most common error being to find the sum or the product of 6x and 960x.	A0 for c = −11x
			Total	**7**		
4		i	$(2 + ax)^6 = 64 + 192ax + 240a^2x^2$	B1	Obtain 64	Allow 2^6 but not $64x^0$
		i		B1	Obtain 192ax	Must be 192ax, not unsimplified equiv
		i		M1	Attempt 3rd term – product of 15, 2^4 and $(ax)^2$	Allow M1 for ax^2 rather than $(ax)^2$ Binomial coeff must be 15 soi; ${}^6C_2$ is not yet enough $240ax^2$ implies M1, even if no other

		i		A1	Obtain $240a^2x^2$ Examiner's Comments Most candidates were able to attempt the correct binomial expansion, and there were very few attempts involving a full expansion. The most common error was for the final term to appear as $240ax^2$ rather than the required $240a^2x^2$. In some solutions this was a result of failing to use brackets in the expansion, and in other solutions the brackets were initially stated but subsequently ignored. Some candidates attempted to take out a common factor and then use the expansion for $(1 + x)^n$. This was not always successful, and candidates should appreciate the need to use the most appropriate method for a given problem. Some, otherwise correct, solutions were subsequently spoiled by an attempt to simplify their final answer by dividing through by a common factor.	method shown Allow M1 if expanding $k(1 + {}^a/_2x)^6$, any k Or $240(ax)^2$ but A0 if this then becomes $240ax^2$ (ie no isw) Full marks can be awarded if terms are just listed rather than linked by '+' A0 if an otherwise correct expansion is subsequently spoiled by attempt to simplify eg $4 + 12ax + 15a^2x^2$ **If expanding brackets:** Mark as above, but must consider all 6 brackets for the M mark (allow irrelevant terms to be discarded)

		ii	(3 × 192a) + (−5 × 64)	M1	Attempt both relevant terms	M0 if additional terms used If a fuller expansion is attempted then it must be made clear which terms are being used Could be coefficients or terms still involving x, but must be consistent for both terms For M1 ignore what, if anything, the terms are equated to
		ii	576a − 320 = 64	A1FT	Equate to 64, to obtain any correct equation, possibly still unsimplified	Following their expansion in (i) (which must contain the two relevant terms), ie 3(their 192a) − 5(their 64) = 64 Presence / absence of 'x' must be consistent throughout eqn
		ii	576a = 384 a = ⅔	A1	Obtain a = ⅔ CWO **Examiner's Comments** Nearly all candidates appreciated the need to use their expansion from the previous part of the question and were then able to attempt the terms required for this part. Some candidates simply picked out the two relevant terms whereas others started with a fuller expansion. No credit was gained in this question until the required two terms, and no others, were identified. A surprisingly common error was to erroneously combine their two terms, with 576ax − 320x becoming 256ax. Some candidates were unsure as to whether to equate the entire terms or just the coefficients; both approaches were condoned as long as it was consistent throughout the entire equation, which was not always the case.	Fraction must be simplified so A0 for $^{384}/_{576}$ Allow exact decimal equiv only, so A0 for 0.666… etc

			Total	7		
5		i	$3^3 + (3 \times 3^2 \times kx) + (3 \times 3 \times (kx)^2) + (kx)^3$ $= 27 + 27kx + 9k^2x^2 + k^3x^3$	M1	Attempt expansion	Must attempt at least 3 of the 4 terms Each term must be an attempt at the product of the relevant binomial coeff soi, the correct power of 3 and the correct power of *kx* Allow M1 if powers used incorrectly with *kx* ie only applied to the *x* and not to *k* as well Binomial coeff must be numerical, so ${}^3C_2$ is M0 until evaluated Allow M1 for expanding $c(1 + \frac{kx}{3})^3$, any *c* Allow M1 for reasonable attempt to expand brackets
		i		A1	Obtain at least two correct terms	Allow 3^3 for 27 and 3^2 for 9 Allow $(kx)^2$ and/or $(kx)^3$ unless later incorrect Terms could just be listed
		i		A1	Obtain at least one further correct term	Allow 3^3 for 27 and 3^2 for 9 Allow $(kx)^2$ and/or $(kx)^3$ unless later incorrect Terms could just be listed
		i		A1	Obtain fully correct simplified expansion	Must now be 27 and 9, not still index notation Allow $(kx)^2$ and/or $(kx)^3$ unless later incorrect Must be a correct expansion, with terms linked by '+' rather than just a list of 4 terms No ISW if correct final answer is subsequently spoiled by attempt to 'simplify' eg dividing by 27 **Examiner's Comments** This part of the question was very well answered, and the majority of candidates gained all of the marks available. Most candidates used the binomial expansion and made efficient use of brackets in obtaining a fully correct solution. The

						most common error was to either not use brackets at all, or to ignore the brackets that had been used earlier, resulting in an expansion where the powers of k were incorrect. A few candidates expanded the three brackets, and this was usually also done correctly.
		ii	$9k^2 = 27$ $k^2 = 3$ $k = \pm\sqrt{3}$	M1	Equate their coeff of x^2 to their constant term and attempt to solve for k	Must be equating coefficients not terms - allow recovery if next line is $k^2 = 3$, but M0 if x^2 still present at this stage Must attempt k, but allow if only positive square root is considered If a division attempt was made in part (i) then allow M1 for using either their original terms or their 'simplified' terms
		ii		A1	Obtain k = $\pm\sqrt{3}$	Must have ±, or two roots listed separately Final answer must be given in exact form A0 for $\pm\sqrt{(^{27}/_9)}$ Must come from correct coefficients only, not from terms that were a result of a division attempt **SR** allow **B1** if $k = \pm\sqrt{3}$ is given as final answer, but inconsistent use of terms / coefficients within solution
		ii				**Examiner's Comments** By contrast, this part of the question proved to be challenging for all but the most able candidates. Most were able to correctly identify the coefficient of x^2 but were unsure as to the 'constant term', with 3 and k being the most common errors. The other common error was to equate entire terms, rather than just coefficients, resulting in x^2 only being present on one side of the equation. Both of these misconceptions demonstrate the importance of candidates being aware of the

						correct terminology as well as the correct processes.
			Total	6		
6			$nk = -6$ soi $\frac{n(n-1)k^2}{2!} = 30$ soi substitution of $n = \pm\frac{6}{k}$ or $k = \pm\frac{6}{n}$ or $k = \pm\sqrt{\frac{60}{n(n-1)}}$ oe eliminate one variable from their equations $n = -1.5$ oe $k = 4$ expansion is valid for $\lvert x\rvert < \frac{1}{4}$ or $-\frac{1}{4} < x < \frac{1}{4}$ isw	B1 B1 M1 A1 A1 B1FT	allow $nkx = -6x$ and /or $\frac{n(n-1)k^2}{2!}x^2 = 30x^2$ for first two marks allow omission of brackets eg allow $-\frac{6}{4}$ FT their k	NB $\frac{n(n-1)\times 36}{2\times n^2} = 30$ oe $(-\frac{6}{k})(\frac{-6}{k} - 1)k^2 = 60$ oe Examiner's Comments Many candidates knew what to do here. Most wrote down the correct equations and successfully substituted to eliminate either k or n. Many candidates made basic errors in the manipulation of the equations such as the substitution of $n = -k - 6$. Even when substitutions were correct, the algebra often went wrong. Many candidates either neglected the

						request to state the set of values for which the expansion is valid, or demonstrated a variety of misconceptions. Answers ranging from "x is any real number" to mod $x < -\frac{1}{4}$ were seen.
			Total	6		