1. In a factory, an inspector checks a random sample of 30 mugs from a large batch and notes the number, X, which are defective. He then deals with the batch as follows.

- If $X < 2$, the batch is accepted.
- If $X > 2$, the batch is rejected.
- If $X = 2$, the inspector selects another random sample of only 15 mugs from the batch. If this second sample contains 1 or more defective mugs, the batch is rejected. Otherwise the batch is accepted.

It is given that 5% of mugs are defective.

i.

a. Find the probability that the batch is rejected after just the first sample is checked.

[3]

b. Show that the probability that the batch is rejected is 0.327, correct to 3 significant figures.

[5]

ii. Batches are checked one after another. Find the probability that the first batch to be rejected is either the 4th or the 5th batch that is checked.

[3]

2. Froox sweets are packed into tubes of 10 sweets, chosen at random. 25% of Froox sweets are yellow.

i. Find the probability that in a randomly selected tube of Froox sweets there are

a. exactly 3 yellow sweets,

[3]

b. at least 3 yellow sweets.

[2]

ii. Find the probability that in a box containing 6 tubes of Froox sweets, there is at least 1 tube that contains at least 3 yellow sweets.

[3]

3. (a) Write down and simplify the first four terms in the expansion of $(x+y)^7$.
Give your answer in ascending powers of x. [2]

(b) Given that the terms in x^2y^5 and x^3y^4 in this expansion are equal, find the value of $\frac{x}{y}$. [2]

(c) A hospital consultant has seven appointments every day. The number of these appointments which start late on a randomly chosen day is denoted by L. The variable L is modelled by the distribution $\mathrm{B}\left(7, \frac{3}{8}\right)$. Show that, in this model, the hospital consultant is equally likely to have two appointments start late or three appointments start late. [3]

4. On average, 40% of candidates pass a certain test on the first attempt.

Three candidates take the test. The number who pass on the first attempt is denoted by X.

(a) State an appropriate model for X, including the values of any parameters. [1]

(b) State an appropriate model for X, including the values of any parameters. [2]

(c) Suggest a reason why one of these assumptions might not be true in practice. [1]

You should now assume that both these assumptions are true.

(d) Find the probability that exactly 2 of the 3 candidates pass the test. [1]

All candidates who fail the test take a re-test and, on average, 60% of these candidates pass. Assume that the same two assumptions are satisfied as for the original test.

(e) Find the probability that all three candidates pass, either on the test or on the re-test. [3]

5. A bag contains 100 black discs and 200 white discs. Paula takes five discs at random, without replacement. She notes the number X of these discs that are black.

(a) Find P(X = 3). [2]

Paula decides to use the binomial distribution as a model for the distribution of X.

(b) Explain why this model will give probabilities that are approximately, but not exactly, correct. [3]

(c) Paula uses the binomial model to find an approximate value for P(X = 3). Calculate the percentage by which her answer will differ from the answer in part **(b)**. [2]

Paula now assumes that the binomial distribution is a good model for X. She uses a computer simulation to generate 1000 values of X. The number of times that X = 3 occurs is denoted by Y.

(d) Calculate estimates of the limits between which two thirds of the values of Y will lie. [3]

6. The probability that Janice sees a kingfisher on any particular day is 0.3. She notes the number, X, of days in a week on which she sees a kingfisher.

(a) State one necessary condition for X to have a binomial distribution. [1]

Assume now that X has a binomial distribution.

(b) Find the probability that, in a week, Janice sees a kingfisher on exactly 2 days. [1]

Each week Janice notes the number of days on which she sees a kingfisher.

(c) Find the probability that Janice sees a kingfisher on exactly 2 days in a week during at least 4 of 6 randomly chosen weeks.

[3]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	(a) X~B(30, 0.05) seen or implied	B1	eg by 0.8122 or 1 − 0.5535 or $0.95^r \times 0.05^s$ (r, s > 1) Allow B(30, 0.95) or B(30, 0.5) for B1 30 × 0.05 alone insufficient for B1	If n = 15; B(15, 0.05) B1
		i	P(X > 2) = 1 − 0.8122 alone or 1 − (0.95^{30} + 30 × 0.95^{29} × 0.05 + ${}^{30}C_2$ × 0.95^{28} × 0.05^2)	M1	${}^nC_r$ insufficient for B1	1 − (0.95^{15} + 15 × 0.95^{14} × 0.05 + ${}^{15}C_2$ × 0.95^{13} × 0.05^2) M1
		i	= 0. 1878 or 0.188 (3 sfs)	A1	<u>Examiner's Comments</u> A few misread the question, using n = 15 throughout, instead of both n = 30 and n = 15. The problem here was not generally the use of the binomial distribution but the need to see through the context and recognise that a binomial calculation was appropriate. In this part there was a distinct advantage to using tables rather than the formula (which is a much longer method in this case). Some candidates chose to use the formula and made arithmetical errors. Using either method, some candidates used B(30, 0.95) instead of B(30, 0.05). Others used the correct distribution but, in order to find P(X < 2), they found 1 − P(X = 1) instead of 1 − P($X \leq$ 1). Some omitted the binomial coefficients.	= 0.0362 A0
		i	(b) Addition method: X~B(30, 0.05) & Y~B(15, 0.05) stated or implied	B1	NB eg 0.0362 implies B(15, 0.05) see below	Subtraction methods: X~B(30,0.05)&Y~B(15,0.05) stated or impl B1
		i	P(X = 2) = (0.8122 − 0.5535) or ${}^{30}C_2$ × 0.95^{28} × 0.05^2 or 0.2587/6 OR P(Y > 1) = (1 − 0.95^{15}) or 0.5367	M1		P(X = 2) = (0.8122 − 0.5535) or ${}^{30}C_2$ × 0.95^{28} × 0.05^2 or 0.2587/6

						OR $P(Y = 0) = 0.95^{15}$ or 0.4633 M1
		i	"0.2587/6" **x** "0.5367" or 0.1388	M1	fully correct method for $P(X = 2) \times P(Y \geq 1)$	fully correct method for $P(X = 2) \times P(Y = 0)$ "0.2587" × "0.4633" or 0.1199/8 M1
		i	$P(X > 2) + P(X = 2) \times P(Y > 1)$ = "0.1878" + "0.1388" alone	M1	[their (a) + any p] alone, but dep 1st M1	1 − (P(X = 0,1) + P(X = 2) × P(Y = 0)) = 1 − ("0.5535" + "0.1199") OR P(X > 2) − P(X = 2) × P(Y = 0)) = (1 − "0.5535") − "0.1199" dep 1st M1 M1
		i	= 0. 327 (3 sf) **AG**	A1		= 0. 327 (3 sf) **AG** A1
		i	For A1 must see correct wking or 0.3265/6…		If ans 0.327, check whether it comes from a correct method (possibly not in MS) or clearly comes from an incorrect method eg (0.4465 + 0.2587) × 0.4633 = 0.327 (ie (P(X > 2) + P(X = 2)) **x** P(Y = 0) B1M1M0M0A0	Do not use marks from a mixture of 3rd column and other columns. Decide which column would give most marks and mark according to that method.
		i				If n = 15 for both distr's, see next page NB If 0.1392 seen, it comes from given answer – (i)(a) (ie 0.3270 – 0.1878).
		i	Alternative scheme for the case where n = 15 is used for both distr's			If n = 15 for both distr's B(15, 0.05) B0 $P(X = 2) = {}^{15}C_2 \times 0.05^2 \times 0.95^{13}$ or 0.1348 OR $P(Y > 1) = 1 - 0.95^{15}$ or 0.5367 M1 "0.1348"x"0.5367" or 0.0723 correct method M1 their **(i)(a)** + "0.0732" Dep 1st M1 M1 = 0.1085 A0 NB Also mark subtraction methods if seen.
		i			**Examiner's Comments**	

					A few misread the question, using $n = 15$ throughout, instead of both $n = 30$ and $n = 15$. Few candidates were able to work their way through this part correctly. Some candidates found $P(Y \geq 1) = 1 - P(Y = 0 \text{ or } 1)$ or $P(Y \geq 1) = 1 - P(Y = 1)$. Some candidates were close to being correct, but found $P(X > 2) + P(X = 2) \times P(Y = 0)$ instead of $P(X > 2) + P(X = 2) \times (1 - P(Y = 0))$. Many complicated matters by using the same letter (X) for the number of defective mugs in the second sample as well as in the first.	
		ii	Any use of 0.327 or their **(i)(b)** for 1st M1	M1		
		ii	$(1 - 0.327)^3 \times 0.327 + (1 - 0.327)^4 \times 0.327$ Allow "correct" use of their **(i)(a)** or **(i)(b)** for 2nd M1	M1	$(0.5535 + 0.2586 \times 0.4633)^3 \times 0.327 +$ $(0.5535 + 0.2586 \times 0.4633)^4 \times 0.327$	$1 - 0.673^5 - (1 - 0.673^3)$ oe Allow any use of their **(i)(b)** for 1st M1 then if "correct" use, also 2nd M1
		ii	= 0.167 (3 sf)	A1		Allow use of their **(i)(a)** in "correct" method for M0M1A0 No marks for use of 0.95 & 0.05
		ii			<u>Examiner's Comments</u> A few misread the question, using $n = 15$ throughout, instead of both $n = 30$ and $n = 15$. This is an example of a common, "two-layered" type of question, requiring the use of a previously obtained figure as the value of p in a geometric (or binomial) calculation. Candidates would benefit from being taught to look out for such questions. Many candidates used the correct geometric structure but with $p = 0.05$ or with a probability equal to their answer to part (i)(a) instead of (i)(b). Others attempted to use a binomial distribution instead of a geometric.	
			Total	11		

2		i	**(a)** Binomial seen or implied	B1	by tables or $^{10}C_3$ or $^{10}C_7$	or by $0.25^a \times 0.75^b$ ($a + b = 10$)
		i	$0.7759 - 0.5256$ or $^{10}C_3 \times (1 - 0.25)^7 \times 0.25^3$	M1		
					Allow 0.25	
					Examiner's Comments	
		i	$= 0.250$ (3 sf)	A1	This question was usually answered correctly, usually by the formula and sometimes using tables. A few candidates omitted the binomial coefficient while others only read one value (0.7759) from the table, i.e P($X \le 3$)). Others found 1 - 0.7759.	
		i	**(b)** $1 - 0.5256$ or	M1	or P(X = 3,4,5,6,7,8,9,10) all correct terms	NOT $1 - 0.7759$ (P($X > 3$) from table)
					Allow $^{10}C_8$ instead of $^{10}C_2$	
					Examiner's Comments	
		i	$1 - ((1 - 0.25)^{10} + 10(1 - 0.25)^9 \times 0.25$ $+ {}^{10}C_2(1 - 0.25)^8 \times 0.25^2)$ $= 0.4744$ or 0.474 (3 sf)	A1	A few candidates found 1 - 0.7759 (i.e. P($X > 3$), or just gave 0.7759 as the answer. Others found 1 - P($X = 2$). Some (correctly) used the formula to find 1 - P(X = 0, 1 or 2). This is not a particularly long method, but it is significantly longer than finding 1 - P($X \le 2$) using the tables. Others attempted various methods not involving the binomial distribution at all.	
		ii	0.4744 or 0.474) or 0.5256 or 0.526 seen	M1	Their (i)(b) seen, or result of 1−(i)(b) seen	eg B(6, 0.474) or P($X \ge 3$) = 0.474
		ii	$1 - (1 - \text{“}0.4744\text{”})^6$ oe	M1	or P(X = 1,2,3,4,5,6) all correct terms seen	
					ft from **(i)(b)**	
		ii	$= 0.979$ (3 sf)	A1f	**Examiner's Comments**	
					A good number of candidates answered this question	

					correctly. Many, however, found this very typical question difficult, not appreciating its two-layered structure. These candidates gave answers such as $\frac{1}{6}\times{}^{10}C_3\times0.25^3\times0.75^7$. Other candidates used their answer to part (i)(b) (thus gaining one mark) but used it in incorrect ways, such as dividing it by 6 or by raising it to the power of 6. Some subtracted it from 1, which is correct, but failed to take the next two steps correctly. A few candidates used their answer to part (i)(a) instead of (i)(b).	
			Total	8		
3		a	$y^7 + 7xy^6 + 21x^2y^5 + 35x^3y^4$	B2 (AO1.1 1.1) [2]	B1 for three terms correct	
		b	$21x^2y^5 = 35x^3y^4$ $\dfrac{x}{y} = \dfrac{3}{5}$ or 0.6	M1 (AO3.1a) A1 (AO1.1) [2]	Equate their terms in x^2y^5 and x^3y^4	
		c	$\mathrm{P}(L=2) = {}_7C_2\left(\frac{3}{8}\right)^2\left(\frac{5}{8}\right)^5$ and $\mathrm{P}(L=3) = {}_7C_3\left(\frac{3}{8}\right)^3\left(\frac{5}{8}\right)^4$ Let $p=\frac{3}{8}$ and $1-p=\frac{5}{8}$, then $\dfrac{p}{1-p}=\dfrac{\frac{3}{8}}{\frac{5}{8}}=\dfrac{3}{5}$	B1 (AO3.3) M1 (AO3.4) E1 (AO2.1)	Connect to part (b)	

			so from part (b) $21p^2(1-p)^5 = 35p^3(1-p)^4$ and P(L = 2) =P(L = 3)	[3]	AG	
			Total	7		
4		a	Bin(3, 0.4)	B1(AO 3.3) [1]		
		b	P(Pass) same for all candidates Each candidate's result is indep of the other results	B1(AO 2.4) B1(AO 2.4) [2]	oe oe Not just "Independent"	
		c	eg Some candidates may be more able than others	B1(AO 3.5b) [1]	oe; or eg 'work harder'	

		d	0.288	B1(AO 3.4) [1]	BC	
		e	0 & 3, 1 & 2, 2& 1, 3 seen or implied 0.216×0.216 + 0.432×0.36 + 0.288×0.6 + 0.064 = 0.439 (3 sf)	M1(AO 3.1a) A1(AO 3.4) A1(AO 1.1) [3]	all correct	
			Total	8		
5		a	$\mathrm{P}(X=3) = {}^5\mathrm{C}_3 \times \frac{100}{300} \times \frac{99}{299} \times \frac{98}{298} \times \frac{200}{297} \times \frac{199}{296}$ = 0.164318883 = 0.164 (3 sf)	M1(AO 1.1a) A1(AO 1.1) [2]	or equiv methods	
		b	P(disc is black) changes each trial (because no replacement) oe But change in prob is small oe Hence bin gives approx, but not exact, probs oe	E1(AO 2.4) E1(AO 2.4) E1(AO 3.5b) [3]		
		c	$\mathrm{P}(X=3 \text{ using bin}) = {}^5\mathrm{C}_3 \times \left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^3$ (= 0.164609053)			

			$\frac{'0.164609053'-'0.164318883'}{'0.164318883'} \times 100$ = 0.177%	M1(AO 3.4) A1(AO 1.1) [2]	ft their values for M1 cao	
		d	μ = 1000 × 0.164609053 (= 164.609053) σ^2 = 1000 × 0.164609053 × (1 – 0.164609053) (= 137.5129127) X ~ Normal $164.609053 \pm \sqrt{137.5129127}$ = 152.88 to 176.34 Estimated limits are 153 to 176	M1(AO 3.3) M1(AO 1.1a) A1(AO 1.1) [3]	both np and npq correct method ft their μ and $\sqrt{\sigma^2}$. Allow rounding to 3 sf Allow (150 – 155) to (174 – 180) ft their 0.164.6….. Allow use of 0.1643…. $165 \pm \sqrt{138}$ or better	
			Total	10		

6		a	Prob of seeing a kingfisher is the same each day OR Seeing a kingfisher on one day is independent of other days	B1 (AO3.5b) [1]	<table><tr><td>oe In context</td><td>Not: Prob of seeing kingfisher is indep</td></tr></table> **<u>Examiner's Comments</u>** Despite almost identical questions having been asked in many examples of paper 4732 over many years, most candidates did not score the mark. Many quoted from their textbooks, to no avail, for example "Repeated trials" or "Only two possible outcomes, seeing or not seeing a kingfisher" or "There must be a fixed number of days". Many gave a correct answer, but not in context, such as "Independent trials" or "Constant probability of success." None of the above scored the mark. Some came a little nearer, but confused the two correct conditions, e.g. "The probability of seeing a kingfisher on a certain day is independent of other days."	
		b	0.318 (3 sf)	B1 (AO3.4) [1]	<table><tr><td>BC Allow 0.32 or 0.317</td><td></td></tr></table> **<u>Examiner's Comments</u>** Most candidates understood how to calculate a binomial probability. Some used the formula, while others used the calculator function, giving only the answer. This is acceptable because "detailed reasoning" is not asked for.	

		c			Many candidates truncated, rather than rounded, their three significant figure answer. A few candidates just found 0.3^2.	
		c	0.318 or their (b) used in a calculation 1 – P(X ≤ 3) using p = 0.318 or their (b) = 0.0854	M1 (AO3.1b) M1 (AO1.1) A1 (AO1.1) [3]	or B(6, their (b)) stated or P(X = 4, 5, 6) attempted using p = 0.318 or their (b) Allow 0.0845 to 0.0875 Allow 2 sf **BC** <u>Examiner's Comments</u> Many candidates understood that they needed to use their answer from part (b), but many did not do so correctly. Some used the formula without the relevant coefficients. Candidates should be aware that they can use the binomial distribution function on their calculator, thus avoiding the possibility arithmetic errors in binomial problems. Some candidates found P(exactly 4 days) instead of P(at least 4 days). Others just found 0.318^4. A few candidates misunderstood the question and found P(at least 8 days out of 42).	

			Total	5	