

Trigonometry Questions

- 3 It is given that $3 \cos \theta - 2 \sin \theta = R \cos(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$.
- (a) Find the value of R . *(1 mark)*
- (b) Show that $\alpha \approx 33.7^\circ$. *(2 marks)*
- (c) Hence write down the maximum value of $3 \cos \theta - 2 \sin \theta$ and find a **positive** value of θ at which this maximum value occurs. *(3 marks)*
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- 6 (a) Express $\cos 2x$ in the form $a \cos^2 x + b$, where a and b are constants. *(2 marks)*
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- 4 (a) (i) Express $\sin 2x$ in terms of $\sin x$ and $\cos x$. *(1 mark)*
- (ii) Express $\cos 2x$ in terms of $\cos x$. *(1 mark)*
- (b) Show that
- $$\sin 2x - \tan x = \tan x \cos 2x$$
- for all values of x . *(3 marks)*
- (c) Solve the equation $\sin 2x - \tan x = 0$, giving all solutions in degrees in the interval $0^\circ < x < 360^\circ$. *(4 marks)*
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- 3 (a) Express $\cos 2x$ in terms of $\sin x$. *(1 mark)*
- (b) (i) Hence show that $3 \sin x - \cos 2x = 2 \sin^2 x + 3 \sin x - 1$ for all values of x . *(2 marks)*
- (ii) Solve the equation $3 \sin x - \cos 2x = 1$ for $0^\circ < x < 360^\circ$. *(4 marks)*
- (c) Use your answer from part (a) to find $\int \sin^2 x \, dx$. *(2 marks)*
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- 7 (a) Use the identity

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

to express $\tan 2x$ in terms of $\tan x$.

(2 marks)

- (b) Show that

$$2 - 2 \tan x - \frac{2 \tan x}{\tan 2x} = (1 - \tan x)^2$$

for all values of x , $\tan 2x \neq 0$.

(4 marks)

- 3 (a) Express $4 \cos x + 3 \sin x$ in the form $R \cos(x - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 360^\circ$, giving your value for α to the nearest 0.1° . (3 marks)
- (b) Hence solve the equation $4 \cos x + 3 \sin x = 2$ in the interval $0^\circ < x < 360^\circ$, giving all solutions to the nearest 0.1° . (4 marks)
- (c) Write down the minimum value of $4 \cos x + 3 \sin x$ and find the value of x in the interval $0^\circ < x < 360^\circ$ at which this minimum value occurs. (3 marks)
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Trigonometry Answers

3(a)	$R = \sqrt{13}$ Or 3.6	B1	1	
(b)	$\frac{\sin \alpha}{\cos \alpha} = \tan \alpha = \frac{2}{3} \quad \alpha \approx 33.7$	M1A1	2	Allow M1 for $\tan \alpha = \frac{-2}{3}$ or $\pm \frac{3}{2}$ AG convincingly obtained
(c)	maximum value $= \sqrt{13}$ $\cos(\theta + 33.7) = 1 \quad (\theta = -33.7)$ $\theta = 326.3$	B1F M1 A1	3	AWRT 326
Total			6	

6(a)	$\cos 2x = 2 \cos^2 x - 1$	B1B1	2	
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4(a)(i)	$\sin 2x = 2 \sin x \cos x$	B1	1	
(ii)	$\cos 2x = 2 \cos^2 x - 1$	B1	1	
(b)	$\sin 2x - \tan x = 2 \sin x \cos x - \frac{\sin x}{\cos x}$	M1		Use of their $\cos 2x$ or $\sin 2x$
	$= \sin x \left(2 \cos x - \frac{1}{\cos x} \right)$	M1		Use of $\tan x = \frac{\sin x}{\cos x}$ and the other double angle identity
	$= \sin x \left(\frac{2 \cos^2 x - 1}{\cos x} \right) = \tan x \cos 2x$	A1	3	AG convincingly obtained
(c)	$\tan x \cos 2x = 0 \quad x = 180$	B1		Ignore $x = 0$, $x = 360^\circ$ & any others outside range
	$\cos 2x = 0$ or $\cos^2 x = \frac{1}{2} \left(\text{or } \sin^2 x = \frac{1}{2} \right)$	M1		
	$x = 45$	A1		
	$x = 135, 225, 315$	A1	4	CAO max 3/4 for answers in radians
Total			9	

3(a)	$\cos 2x = 1 - 2 \sin^2 x$	B1	1	
(b)(i)	$3 \sin x - \cos 2x = 3 \sin x - (1 - 2 \sin^2 x)$ $= 3 \sin x - 1 + 2 \sin^2 x$	M1 A1	2	Candidate's $\cos 2x$ or $\sin^2 x$ AG
(ii)	$2 \sin^2 x + 3 \sin x - 2 = 0$ $(2 \sin x - 1)(\sin x + 2) = 0$ $\sin x = \frac{1}{2} \quad x = 30 \quad x = 150$ Allow misread for $2 \sin^2 x + 3 \sin x - 1 = 0$ $\sin x = \frac{-3 \pm \sqrt{17}}{4}$ $x = 16.3^\circ, 163.7^\circ$	M1 M1 M1 A1 (M1) (M1) (A1)	4	Soluble quadratic form Attempt to solve (allow one error in formula, allow sign errors) \sin^{-1} and two solutions ($0^\circ < x < 360^\circ$) A0 if radians Soluble quadratic form Use of formula (allow one error) Max 3/4
(c)	$\int \frac{1}{2}(1 - \cos 2x) = \frac{x}{2} - \frac{\sin 2x}{4} (+c)$	M1A1	2	M1 – solve integral, must have 2 terms for $\sin^2 x$ from (a)
			9	

7(a)	$\tan(x+x) = \frac{\tan x + \tan x}{1 - \tan x \tan x} \left(= \frac{2 \tan x}{1 - \tan^2 x} \right)$	M1 A1	2	$A = B = x$ used
(b)	$2 - 2 \tan x - \frac{2 \tan x(1 - \tan^2 x)}{2 \tan x}$ $2 - 2 \tan x - (1 - \tan x)(1 + \tan x)$ $(1 - \tan x)(2 - (1 + \tan x))$ $(1 - \tan x)^2$	M1 M1 M1 A1	4	Substitute from (a) Simplification $2 - 2 \tan x - (1 - \tan^2 x)$ $2 - 2 \tan x - 1 + \tan^2 x$ AG (convincingly obtained) $=(\tan x - 1)^2 = (1 - \tan x)^2$ Any equivalent method
	Total		6	

3(a)	$R = 5$ $\tan \alpha = \frac{3}{4}$ (OE) $\alpha = 36.9^\circ$ (ISW 216.9)	B1 M1A1	3	SC1 $\tan \alpha = \frac{4}{3}$, $\alpha = 53.1^\circ$ R, α PI in (b)
(b)	$\cos(x - \alpha) = \frac{2}{R}$ $x - \alpha = 66.4^\circ$ $x = 103.3^\circ$ $x = 330.4^\circ$	M1 A1 A1F A1F	4	accept 330.5° , -1 each extra ft on acute α
(c)	minimum value = -5 $\cos(x - 36.9) = -1$ $x = 216.9^\circ$	B1F M1 A1	3	ft on R SC $\cos(x + 36.9)$ treat as miscopy 216.9 or better accept graphics calculator solution to this accuracy SC Find max: max = 5 at $(x + 36.9)$ stated 1/3
Total			10	Max 8/10 for work in radians