Normal Distribution Questions

7 (a) The weight, X grams, of soup in a carton may be modelled by a normal random variable with mean 406 and standard deviation 4.2.

Find the probability that the weight of soup in a carton:

(i) is less than 400 grams; *(3 marks)*

(ii) is between 402.5 grams and 407.5 grams. *(4 marks)*

(b) The weight, Y grams, of chopped tomatoes in a tin is a normal random variable with mean μ and standard deviation σ.

(i) Given that $P(Y < 310) = 0.975$, explain why:

$$310 - \mu = 1.96\sigma$$ *(3 marks)*

(ii) Given that $P(Y < 307.5) = 0.86$, find, to two decimal places, values for μ and σ.

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2 The heights of sunflowers may be assumed to be normally distributed with a mean of 185 cm and a standard deviation of 10 cm.

(a) Determine the probability that the height of a randomly selected sunflower:

(i) is less than 200 cm; *(3 marks)*

(ii) is more than 175 cm; *(3 marks)*

(iii) is between 175 cm and 200 cm. *(2 marks)*

(b) Determine the probability that the mean height of a random sample of 4 sunflowers is more than 190 cm. *(4 marks)*

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6 When Monica walks to work from home, she uses either route A or route B.

(a) Her journey time, X minutes, by route A may be assumed to be normally distributed with a mean of 37 and a standard deviation of 8.

Determine:

(i) $P(X < 45)$; *(3 marks)*

(ii) $P(30 < X < 45)$. *(3 marks)*

(b) Her journey time, Y minutes, by route B may be assumed to be normally distributed with a mean of 40 and a standard deviation of σ.

Given that $P(Y > 45) = 0.12$, calculate the value of σ. *(4 marks)*

(c) If Monica leaves home at 8.15 am to walk to work hoping to arrive by 9.00 am, state, with a reason, which route she should take. *(2 marks)*

(d) When Monica travels to work from home by car, her journey time, W minutes, has a mean of 18 and a standard deviation of 12.

Estimate the probability that, for a random sample of 36 journeys to work from home by car, Monica's mean time is more than 20 minutes. *(4 marks)*

(e) Indicate where, if anywhere, in this question you needed to make use of the Central Limit Theorem. *(1 mark)*

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7 (a) Electra is employed by E & G Ltd to install electricity meters in new houses on an estate. Her time, X minutes, to install a meter may be assumed to be normally distributed with a mean of 48 and a standard deviation of 20.

Determine:

(i) $P(X < 60)$; *(2 marks)*

(ii) $P(30 < X < 60)$; *(3 marks)*

(iii) the time, k minutes, such that $P(X < k) = 0.9$. *(4 marks)*

(b) Gazali is employed by E & G Ltd to install gas meters in the same new houses. His time, Y minutes, to install a meter has a mean of 37 and a standard deviation of 25.

(i) Explain why Y is unlikely to be normally distributed. *(2 marks)*

(ii) State why $\overline{Y}$, the mean of a random sample of 35 gas meter installations, is likely to be approximately normally distributed. *(1 mark)*

(iii) Determine $P(\overline{Y} > 40)$. *(4 marks)*

Normal Distribution Answers

7 (a)	Weight, $X \sim N(406, 4.2^2)$			
(i)	$P(X < 400) = P\left(Z < \frac{400-406}{4.2}\right)$	M1		Standardising (399.5, 400 or 400.5) with 406 and ($\sqrt{4.2}$, 4.2 or 4.2^2) and/or $(406 - x)$
	$= P(Z < -1.428 \text{ to } -1.43)$ $= 1 - P(Z < 1.428 \text{ to } 1.43)$	m1		$\Phi(-z) = 1 - \Phi(z)$
	$= 0.076$ to 0.077	A1	3	AWRT 0.07636
(ii)	$P(402.5 < X < 407.5) =$ $P(X < 407.5) - P(X < 402.5) =$	M1		Difference OE
	$P(Z < 0.36) - P(Z < -0.83)$	B2,1		AWRT; ignoring signs
	$= 0.64058 - (1 - 0.79673) = 0.433$ to 0.44	A1	4	AWFW 0.43731
(b)(i)	$0.975 \Rightarrow z = 1.96$	M1		Accept explanation in words
	$P(Y < 310) = P\left(Z < \frac{310-\mu}{\sigma}\right)$ **or** $x = \mu +/\pm z\sigma$	M1		Standardising 310 using μ and σ Accept in words
	Thus $\frac{310-\mu}{\sigma} = 1.96 \Rightarrow$ result **or** $310 = \mu + 1.96\sigma \Rightarrow$ result NB: Working backwards from given equation $\Rightarrow$ at most M1 M0 mo	m1	3	Equating AG Substitution
(ii)	$0.86 \Rightarrow z = 1.08$	B1		AWRT 1.0803
	$310 - \mu = 1.96\sigma$ $307.5 - \mu = 1.08\sigma$			
	$2.5 = 0.88\sigma$	M1		Attempt at solving 2 equations each of form $x - \mu = z\sigma$
	$\sigma = 2.84$ to 2.842	A1		AWFW 2.841
	$\mu = 304.4$ to 304.5	A1	4	AWFW 304.43
	Total		**14**	

2(a)	Height, $X \sim \text{N}(185, 10^2)$			
(i)	$\text{P}(X < 200) = \text{P}\left(Z < \frac{200-185}{10}\right)$	M1		standardising (199.5, 200 or 200.5) with 185 and ($\sqrt{10}$, 10 or 10^2) and/or $(185 - x)$
	$= \text{P}(Z < 1.5)$	A1		CAO; ignore sign
	$= \Phi(1.5) = 0.933$	A1	3	AWRT (0.93319)
(ii)	$\text{P}(X > 175) = \text{P}\left(Z > \frac{175-185}{10}\right)$	M1		standardising (174.5, 175 or 175.5) with 185 and ($\sqrt{10}$, 10 or 10^2) and/or $(185 - x)$
	$= \text{P}(Z > -1) = \text{P}(Z < 1)$	m1		area change
	$= 0.841$	A1	3	AWRT (0.84134)
(iii)	$\text{P}(175 < X < 200) = \text{(i)} - [1 - \text{(ii)}]$	M1		or equivalent
	$= 0.93319 - [1 - 0.84134]$			
	$= 0.774$ to 0.775	A1√	2	AWFW (0.77453) √ on (i) and (ii) providing > 0
(b)	Mean of $\bar{X} = 185$	B1		CAO; may be implied by use in standardising
	Variance of $\bar{X} = \frac{10^2}{4} = 25$	B1		CAO; or equivalent
	$\text{P}(\bar{X} > 190) = \text{P}\left(Z > \frac{190-185}{5}\right)$	M1		standardising 190 with 185 and 5 and/or (185 – 190)
	$= \text{P}(Z > 1) = 1 - \Phi(1)$			
	$= 0.159$	A1√	4	AWRT (0.15866) √ on (a)(ii) if used
		Total	**12**	

6(a)(i)	$\text{P}(X < 45) = \text{P}\left(Z < \frac{45-37}{8}\right)$	M1		Standardising (44.5, 45 or 45.5) with 37 and ($\sqrt{8}$, 8 or 8^2) and/or $(37 - x)$
	$= \text{P}(Z < 1)$	A1		CAO; ignore sign
	$= 0.841$	A1	3	AWRT (0.84134)
(ii)	$\text{P}(30 < X < 45) = \text{(i)} - \text{P}(X < 30)$	M1		Used; OE
	$= \text{(i)} - \text{P}(Z < -0.875)$			
	$= \text{(i)} - [1 - (0.80785$ to $0.81057)]$	m1		Area change
	$= 0.648$ to 0.652	A1	3	AWFW (0.65056)
(b)	$0.12 \Rightarrow z = 1.17$ to 1.18	B1		AWFW; ignore sign (1.1750)
	$z = \frac{45-40}{\sigma}$	M1		Standardising 45 with 40 and σ
	$= 1.175$	m1		Equating z-term to z-value but not using 0.12, 0.88 or $\lvert 1-z \rvert$
	$\sigma = 4.23$ to 4.28	A1	4	AWFW

(c)	**Route A:** $P(X > 45) = 1 - $(a)(i) **Route B:** $P(Y > 45) = 0.12$ so Monica should use **Route B** (smaller prob)	B1 ↑Dep↑ B1√	2	OE; must use 45 √ on (a)(i); allow Route Y
(d)	Mean of $\overline{W} = 18$	B1		CAO; can be implied by **use** in standardising
	Variance of $\overline{W} = \frac{12^2}{36} = 4$	B1		CAO; OE
	$P\left(\overline{W} > 20\right) = P\left(Z > \frac{20-18}{2}\right)$	M1		Standardising 20 with 18 and 2 and/or (18 – 20)
	$= P(Z > 1) = 0.159$	A1√	4	AWRT (0.15866); √ on (a)(i) if used
(e)	In part (d)	B1	1	CAO; OE
	Total		**17**	

7(a)	Time, $X \sim N(48, 20^2)$			
(i)	$P(X < 60) = P\left(Z < \frac{60-48}{20}\right) =$	M1		Standardising (59.5, 60 or 60.5) with 48 and ($\sqrt{20}$, 20 or 20^2) and/or $(48 - x)$
	$P(Z < 0.6) = 0.725$ to 0.73	A1	2	AWFW (0.72575)
(ii)	$P(30 < X < 60) =$ $P(X < 60) - P(X < 30) =$ (i) $- P(X < 30) =$ (i) $- P(Z < -0.9) =$	M1		Difference or equivalent Standardising other than 60 and 30 ⇒ max of M1 m1 A0
	(i) $- \{1 - P(Z < +0.9)\} =$ $0.72575 - \{1 - 0.81594\} =$	m1		Area change
	0.54 to 0.542	A1	3	AWFW (0.54169)
(iii)	$0.9 \Rightarrow z = 1.28$ to 1.282	B1		AWFW (1.2816)
	$z = \frac{k-48}{20}$	M1		Standardising k with 48 and 20
	$= 1.2816$	m1		Equating z-term to z-value; not using 0.9, 0.1, $\lvert 1 - z \rvert$ or $\Phi(0.9) = 0.81594$
	$k = 73.6$ to 74	A1	4	AWFW

(b)	Time, $Y \sim N(37, 25^2)$			
(i)	Use of $\mu - (2 \text{ or } 3) \times \sigma$ = 37 – (50 or 75)	M1		Or equivalent justification
	$< 0 \Rightarrow$ likely negative times	B1	2	for (likely) negative times
(ii)	Central Limit Theorem or n large / > 30	B1	1	
(iii)	Variance of $\overline{Y}$ = $\frac{25^2}{35}$	B1		OE; stated or used
	$P(\overline{Y} > 40) = P\left(Z > \frac{40-37}{25/\sqrt{35}}\right) =$	M1		Standardising 40 with 37 and $25/\sqrt{35}$ and/or (37 – 40)
	$P(Z > 0.71) = 1 - P(Z < 0.71) =$	m1		Area change
	0.238 to 0.24	A1	4	AWFW (1 – 0.76115)
	Total		**16**	