Integration Questions

2 (a) Use the trapezium rule with five ordinates (four strips) to find an approximate value for

$$\int_0^4 \frac{1}{x^2+1}\,\mathrm{d}x$$

giving your answer to four significant figures. *(4 marks)*

(b) State how you could obtain a better approximation to the value of the integral using the trapezium rule. *(1 mark)*

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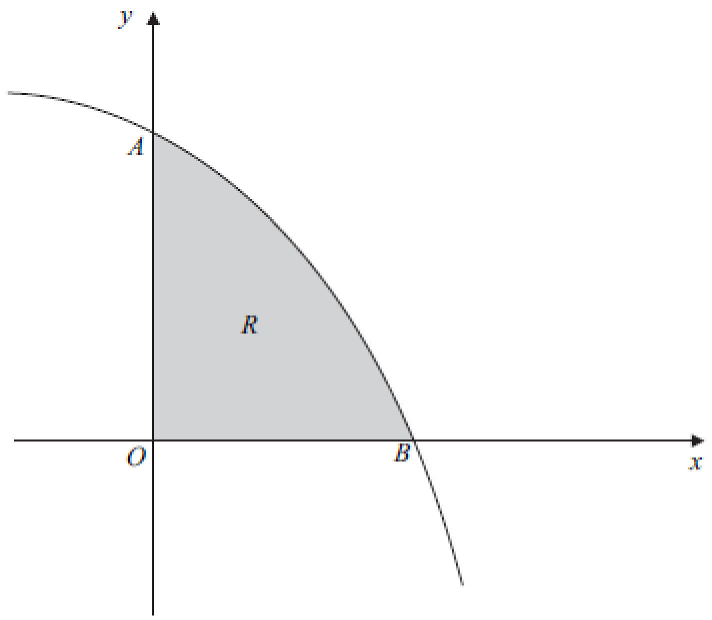
6 The diagram shows a sketch of the curve with equation $y = 27 - 3^x$.



The curve $y = 27 - 3^x$ intersects the y-axis at the point A and the x-axis at the point B.

(a) (i) Find the y-coordinate of point A. *(2 marks)*

(ii) Verify that the x-coordinate of point B is 3. *(1 mark)*

(b) The region, R, bounded by the curve $y = 27 - 3^x$ and the coordinate axes is shaded. Use the trapezium rule with four ordinates (three strips) to find an approximate value for the area of R. *(4 marks)*

(c) (i) Use logarithms to solve the equation $3^x = 13$, giving your answer to four decimal places. *(3 marks)*

(ii) The line $y = k$ intersects the curve $y = 27 - 3^x$ at the point where $3^x = 13$.
Find the value of k. *(1 mark)*

(d) (i) Describe the single geometrical transformation by which the curve with equation $y = -3^x$ can be obtained **from** the curve $y = 27 - 3^x$. *(2 marks)*

(ii) Sketch the curve $y = -3^x$. *(2 marks)*

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2 Use the trapezium rule with four ordinates (three strips) to find an approximate value for

$$\int_0^3 \sqrt{2^x}\,\mathrm{d}x$$

giving your answer to three decimal places. *(4 marks)*

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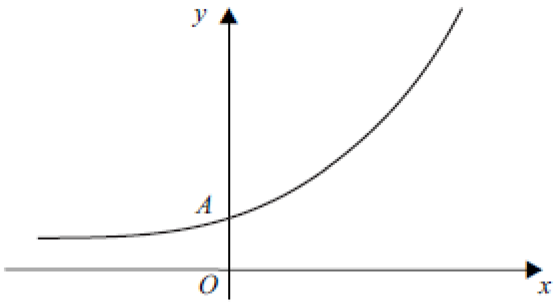
6 The diagram shows a sketch of the curve with equation $y = 3(2^x + 1)$.



The curve $y = 3(2^x + 1)$ intersects the y-axis at the point A.

(a) Find the y-coordinate of the point A. *(2 marks)*

(b) Use the trapezium rule with four ordinates (three strips) to find an approximate value for $\int_0^6 3(2^x + 1)\,\mathrm{d}x$. *(4 marks)*

(c) The line $y = 21$ intersects the curve $y = 3(2^x + 1)$ at the point P.

(i) Show that the x-coordinate of P satisfies the equation

$$2^x = 6$$ *(1 mark)*

(ii) Use logarithms to find the x-coordinate of P, giving your answer to three significant figures. *(3 marks)*

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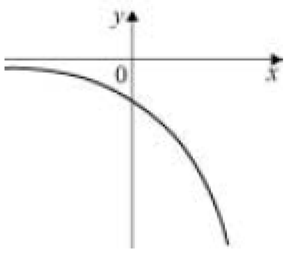
integration Answers

2(a)	$h=1$	B1		PI
	Integral $=\frac{h}{2}\{\ldots\}$ $\{\ldots\}=\text{f}(0)+\text{f}(4)+2[\text{f}(1)+\text{f}(2)+\text{f}(3)]$	M1		OE summing of areas of the four trapezia. [0.75+0.35+0.15+0.079…]
	$=\left[1+\frac{1}{17}+2\left(\frac{1}{2}+\frac{1}{5}+\frac{1}{10}\right)\right]$	A1		Exact or to 3dp values Condone one numerical slip
	Integral = 1.329	A1	4	**CSO**. Must be 1.329
(b)	Increase the number of ordinates	E1	1	OE
	Total		**5**	

6(a)(i)	y-coordinate of A is $27-3^0$; $=26$	M1A1	2	
(ii)	When $x=3$, $y=27-3^3=0 \Rightarrow B(3,0)$	B1	1	AG; be convinced
(b)	$h=1$	B1		PI
	Area $\approx h/2\{\ldots\}$ $\{\ldots\}=$ f(0)+f(3)+2[f(1)+f(2)] $\{\ldots\}=$ "26" + 0 + 2(24 + 18)	 M1 A1√		 OE summing of areas of the 'trapezia'.. on (a)(i) (∑trap= "25"+21+9)
	(Area ≈) 55	A1√	4	on [42 + 0.5× "(a)(i)"]
(c)(i)	$\log_{10} 3^x = \log_{10} 13$	M1		Takes ln or $\log_{10}$ on both or $x = \log_3 13$
	$x\log_{10} 3 = \log_{10} 13$	m1		Use of $\log 3^x = x\log 3$ or $\log_3 13 = \frac{\lg 13}{\lg 3}$ OE (PI by $\log_3 13 = 2.335$ or better)
	$x = \frac{\lg 13}{\lg 3} = 2.334717\ldots$ $= 2.3347$ to 4dp	A1	3	Must show that logarithms have been used
(ii)	$\{k=\}$ 14	B1	1	Condone $y = 14$; Accept final answer 14 with only zeros after decimal point eg 14.000
(d)(i)	Translation;	B1;		'Translation'/'translate(d)' B0 if more than one transformation
	$\begin{bmatrix} 0 \\ -27 \end{bmatrix}$	B1	2	Accept full equivalent in words provided linked to 'translation/move/shift' and negative y-direction (Note: B0 B1 is possible)
(ii)		B1 B1	2	Correct shape (translation of given curve vertically downwards) Only point of intersection with coord axes is on negative y-axis and curve is asymptotic to the negative x-axis
	Total		**15**	

2	$h=1$	B1		PI
	$f(x) = \sqrt{2^x}$			
	Area $\approx h/2\{\ldots\}$ $\{\ldots\}=$ f(0)+f(3)+2[f(1)+f(2)]	 M1		OE summing of areas of the 'trapezia'..
	$\{\ldots\}= 1 + \sqrt{8} + 2(\sqrt{2} + 2)$	A1		OE
	(Area ≈) 5.3284… = 5.328 (to 3dp)	A1	4	CAO Must be 5.328
	Total		**4**	

6(a)	$y_A = 3\left(2^0 + 1\right)$	M1		Substituting $x = 0$ PI
	$= 6$	A1	2	
(b)	$h = 2$	B1		PI
	Integral = $h/2$ {……}			
	{….} = $\text{f}(0) + 2[\text{f}(2) + \text{f}(4)] + \text{f}(6)$	M1		OE summing of areas of the three traps.
	{} = $6 + 2[3\times5 + 3\times17] + 3\times65$ $= 6 + 2[15 + 51] + 195$	A1		Condone 1 numerical slip {ft on (a) for f(0) if not recovered} [Sum of 3 traps. = 21 + 66 + 246]
	Integral = 333	A1	4	CAO
(c)(i)	$21 = 3\left(2^x + 1\right) \Rightarrow 2^x = 6$	B1	1	AG (be convinced)
(ii)	$\log_{10} 2^x = \log_{10} 6$	M1		Take ln or $\log_{10}$ of both sides of $a^x = b$ or other relevant base if clear. The equation $a^x = b$ used must be correct.
	$x\log_{10} 2 = \log_{10} 6$	m1		Use of $\log 2^x = x\log 2$ OE
	$x = \dfrac{\lg 6}{\lg 2} = 2.5849\ldots = 2.58$ to 3sf	A1	3	Both method marks must have been awarded.
	Total		**10**	