Differentiation Questions

1 Given that $y = 16x + x^{-1}$, find the two values of x for which $\frac{dy}{dx} = 0$. *(5 marks)*

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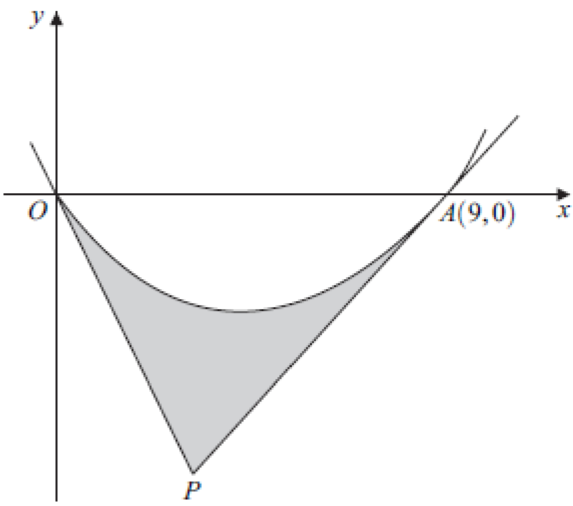
8 A curve, drawn from the origin O, crosses the x-axis at the point $A(9, 0)$. Tangents to the curve at O and A meet at the point P, as shown in the diagram.



The curve, defined for $x \geqslant 0$, has equation

$$y = x^{\frac{3}{2}} - 3x$$

(a) Find $\frac{dy}{dx}$. *(2 marks)*

(b) (i) Find the value of $\frac{dy}{dx}$ at the point O and hence write down an equation of the tangent at O. *(2 marks)*

(ii) Show that the equation of the tangent at $A(9, 0)$ is $2y = 3x - 27$. *(3 marks)*

(iii) Hence find the coordinates of the point P where the two tangents meet. *(3 marks)*

(c) Find $\int \left(x^{\frac{3}{2}} - 3x \right) dx$. *(3 marks)*

(d) Calculate the area of the shaded region bounded by the curve and the tangents OP and AP. *(5 marks)*

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7 At the point (x, y), where $x > 0$, the gradient of a curve is given by

$$\frac{dy}{dx} = 3x^{\frac{1}{2}} + \frac{16}{x^2} - 7$$

(a) (i) Verify that $\frac{dy}{dx} = 0$ when $x = 4$. *(1 mark)*

(ii) Write $\frac{16}{x^2}$ in the form $16x^k$, where k is an integer. *(1 mark)*

(iii) Find $\frac{d^2y}{dx^2}$. *(3 marks)*

(iv) Hence determine whether the point where $x = 4$ is a maximum or a minimum, giving a reason for your answer. *(2 marks)*

(b) The point $P(1, 8)$ lies on the curve.

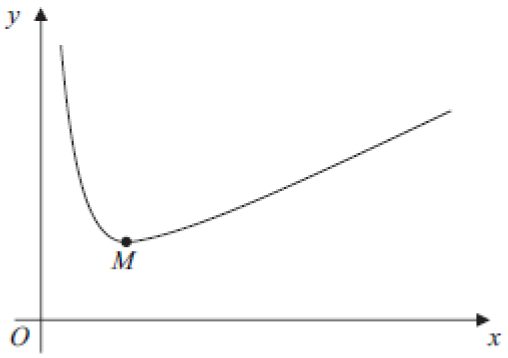
(i) Show that the gradient of the curve at the point P is 12. *(1 mark)*

(ii) Find an equation of the normal to the curve at P. *(3 marks)*

(c) (i) Find $\int (3x^{\frac{1}{2}} + \frac{16}{x^2} - 7)\,dx$. *(3 marks)*

(ii) Hence find the equation of the curve which passes through the point $P(1, 8)$. *(3 marks)*

6 A curve C is defined for $x > 0$ by the equation $y = x + 1 + \frac{4}{x^2}$ and is sketched below.



(a) (i) Given that $y = x + 1 + \frac{4}{x^2}$, find $\frac{dy}{dx}$. *(3 marks)*

(ii) The curve C has a minimum point M. Find the coordinates of M. *(4 marks)*

(iii) Find an equation of the normal to C at the point $(1, 6)$. *(4 marks)*

(b) (i) Find $\int \left(x + 1 + \frac{4}{x^2}\right) dx$. *(3 marks)*

(ii) Hence find the area of the region bounded by the curve C, the lines $x = 1$ and $x = 4$ and the x-axis. *(2 marks)*

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5 A curve is defined for $x > 0$ by the equation

$$y = \left(1 + \frac{2}{x}\right)^2$$

The point P lies on the curve where $x = 2$.

(a) Find the y-coordinate of P. *(1 mark)*

(b) Expand $\left(1 + \frac{2}{x}\right)^2$. *(2 marks)*

(c) Find $\frac{dy}{dx}$. *(3 marks)*

(d) Hence show that the gradient of the curve at P is -2. *(2 marks)*

(e) Find the equation of the normal to the curve at P, giving your answer in the form $x + by + c = 0$, where b and c are integers. *(4 marks)*

Differentiation Answers

	Solution	Marks	Total	Comments
1	$y'(x) = 16 - x^{-2}$	M1 A1		One term correct Both correct
	$y'(x) = 16 - \frac{1}{x^2}$	B1		$x^{-2} = \frac{1}{x^2}$ OE PI
	$y'(x) = 0 \Rightarrow 16x^2 = 1$;	M1		c's $y'(x)$=0 and one relevant further step
	$\Rightarrow x = \pm\frac{1}{4}$	A1	5	Both answers required.
	Total		**5**	

8(a)	$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} - 3$	M1 A1	2	One term correct Both correct
(b)(i)	When $x = 0$, $\frac{dy}{dx} = -3$	B1F√		Ft provided answer < 0.
	Eqn of tangent at O is $y = -3x$	B1F√	2	OE Ft on $y'(0)$
(ii)	At (9,0) $\frac{dy}{dx} = \frac{3}{2}(9)^{\frac{1}{2}} - 3$	M1		Attempt to find $y'(9)$
	Eqn tangent at A is $y - 0 = y'(9)[x - 9]$	m1		OE
	$\Rightarrow y = \frac{3}{2}(x-9) \Rightarrow 2y = 3x - 27$	A1	3	CSO. AG
(iii)	Eliminating $y \Rightarrow -6x = 3x - 27$	M1		OE method to one variable (eg $2y = -y - 27$)
	$9x = 27 \Rightarrow x = 3$	A1F		[A1F for each coordinate; only ft on $y = kx$ tangent in (b)(i) for $k < 0$]
	When $x = 3$, $y = -9$. $\{P(3, -9)\}$	A1F	3	
(c)	$\int \left(x^{\frac{3}{2}} - 3x\right) dx = \frac{2}{5}x^{\frac{5}{2}} - \frac{3x^2}{2}$ $(+c)$	M1 A2,1,0	3	One power correct Condone absence of "+c" and unsimplified forms
(d)	$\int_0^9 \left(x^{\frac{3}{2}} - 3x\right) dx =$	B1		PI
	$= \frac{2}{5}\times 9^{\frac{5}{2}} - \frac{3}{2}\times 9^2 - 0$ $= -24.3$	M1		Correct use of limits following integration
	Area of triangle $OPA = \frac{1}{2}\times 9\times \lvert y_P \rvert$	M1		
	Sh.Area $= \frac{1}{2}\times 9\times \lvert y_P \rvert - \left\lvert \int_0^9 \left(x^{\frac{3}{2}} - 3x\right) dx \right\rvert$	M1		OE
	$= 40.5 - 24.3 = 16.2$	A1	5	
	Total		**18**	

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7(a)(i)	When $x=4$, $\frac{dy}{dx}=3(2)+\frac{16}{16}-7=0$	B1	1	AG Be convinced
(ii)	$\frac{16}{x^2}=16x^{-2}$	B1	1	Accept $k=-2$
(iii)	$\frac{d^2y}{dx^2}=3\times\frac{1}{2}x^{-\frac{1}{2}}+16\times(-2)x^{-3}-0$	M1		A power decreased by 1
	$\frac{d^2y}{dx^2}=\frac{3}{2}x^{-\frac{1}{2}}$; $\quad -32x^{-3}$	A1; A1√	3	candidate's negative integer k [−1 for >2 term(s)]
(iv)	When $x=4$, $\frac{d^2y}{dx^2}=\frac{3}{4}-\frac{32}{64}=\frac{1}{4}$	M1		Attempt to find $y''(4)$ reaching as far as two simplified terms
	Minimum since $y''(4)>0$	E1√	2	candidate's sign of $y''(4)$
	[**Alternative:** Finds the sign of $y'(x)$ either side of the point where x=4, need evidence rather than just a statement: (M1) Correct ft conclusion with valid reason E1√] [In both, condone absent statement $y'(4)=0$]			
(b)(i)	At $P(1,8)$, $\frac{dy}{dx}=3(1)^{\frac{1}{2}}+\frac{16}{1^2}-7=12$	B1	1	AG Be convinced
(ii)	Gradient of normal $=-\frac{1}{12}$	M1		Use of or stating $m\times m'=-1$
(ii)	Gradient of normal $=-\frac{1}{12}$	M1		Use of or stating $m\times m'=-1$
	Equation of normal is $y-8=m[x-1]$	M1		Can be awarded even if m=12
	$y-8=-\frac{1}{12}(x-1)\Rightarrow 12y-96=-x+1$ $\Rightarrow 12y+x=97$	A1	3	Any correct form of the equation
(c)(i)	$\int 3x^{\frac{1}{2}}+\frac{16}{x^2}-7\,dx=$			
	.... $=3\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+16\frac{x^{-1}}{-1}-7x+c$	M1 A2,1,0 √	3	One power correct. A1 if 2 of 3 terms correct candidate's negative integer k Condone absence of "+ c"
(ii)	$y=2x^{\frac{3}{2}}-16x^{-1}-7x+c \qquad (*)$	B1√		y = candidate's answer to (c)(i) with tidied coefficients and with '+c'. ('y =' PI by next line)
	When $x=1$, $y=8\Rightarrow 8=2-16-7+c$	M1		Substitute. (1,8) in attempt to find constant of integration
	$y=2x^{\frac{3}{2}}-16x^{-1}-7x+29$	A1	3	Accept $c=29$ after (*), including y =, stated
	Total		**17**	

Q	Solution	Marks	Total	Comments
6(a)(i)	$y = x + 1 + 4x^{-2} \Rightarrow \frac{dy}{dx} = 1 - 8x^{-3}$	M1 A2,1,0	3	Power $p \to p-1$ (A1 if $1 + ax^n$ with $a = -8$ or $n = -3$)
(ii)	$1 - 8x^{-3} = 0$	M1		Puts c's $\frac{dy}{dx} = 0$
	$x^3 = 8$	m1		Using $x^{-k} = \frac{1}{x^k}$ to reach $x^a = b$, $a>0$ or correct use of logs.
	$x = 2$ When $x = 2$, $y = 4$	A1 A1ft	4	
(iii)	At (1, 6), $\frac{dy}{dx} = 1 - 8 = -7$	M1		Attempt to find $y'(1)$
	Gradient of normal = $\frac{1}{7}$	M1		Use of or stating $m \times m' = -1$
	Equation of normal is $y - 6 = m[x - 1]$	M 1		m numerical
	$y - 6 = \frac{1}{7}(x - 1)$ $\{\frac{y-6}{x-1} = \frac{1}{7};\ 7y = x + 41\}$	A1ft	4	OE ft on c's answer for (a)(i) provided at least A1 given in (a)(i) and previous 3M marks awarded
(b)(i)	$\int x\left(+1 + \frac{4}{x^2}\right) dx =$			
	$\ldots\ldots = \frac{x^2}{2} + x - 4x^{-1}\ \{+c\}$	M1 A2,1,0	3	One of three terms correct. For A2 need all <u>three</u> terms as printed or better (A1 if 2 of 3 terms correct)
(ii)	$\{\text{Area}=\}\ \int_1^4 x + 1 + \frac{4}{x^2} dx =$			
	$\left[\frac{x^2}{2} + x - \frac{4}{x}\right]_1^4 = (8 + 4 - 1) - \left(\frac{1}{2} + 1 - 4\right)$	M1		Dealing correctly with limits; F(4)–F(1) (must have integrated)
	$= 13.5$	A1	2	
	Total		**16**	

5(a)	$y_P = 4$	B1	1	
(b)	$y = 1 + \frac{2}{x} + \frac{2}{x} + \frac{4}{x^2}$ $y = 1 + 4x^{-1} + 4x^{-2}$	B2,1,0	2	(B1 if only one error in the expansion) For B2 the last line of the candidate's solution must be correct
(c)	$\frac{dy}{dx} = -4x^{-2} - 8x^{-3}$	M1 A1ft A1	 3	Index reduced by 1 after differentiating x to a negative power At least 1 term in x correct ft on expn CSO Full correct solution. ACF
(d)	When $x = 2$, $\frac{dy}{dx} = -4 \times 2^{-2} - 8 \times 2^{-3}$ Gradient $= -1 \ -1 = -2$	M1 A1	 2	Attempt to find $y'(2)$. AG (be convinced-no errors seen)
(e)	$-2 \times m' = -1$ $y - 4 = m(x - 2)$ $y - 4 = \frac{1}{2}(x - 2)$ $x - 2y + 6 = 0$	M1 M1 A1ft A1	 4	$m_1 \times m_2 = -1$ OE stated or used. PI C's y_P from part (a) if not recovered; m must be numerical. Ft on candidate's y_P from part (a) if not recovered. CAO Must be this or $0 = x - 2y + 6$
	Total		**12**	