Binomial Questions

5 (a) (i) Obtain the binomial expansion of $(1-x)^{-1}$ up to and including the term in x^2. *(2 marks)*

(ii) Hence, or otherwise, show that

$$\frac{1}{3-2x} \approx \frac{1}{3} + \frac{2}{9}x + \frac{4}{27}x^2$$

for small values of x. *(3 marks)*

(b) Obtain the binomial expansion of $\dfrac{1}{(1-x)^2}$ up to and including the term in x^2. *(2 marks)*

(c) Given that $\dfrac{2x^2-3}{(3-2x)(1-x)^2}$ can be written in the form $\dfrac{A}{(3-2x)} + \dfrac{B}{(1-x)} + \dfrac{C}{(1-x)^2}$,

find the values of A, B and C. *(5 marks)*

(d) Hence find the binomial expansion of $\dfrac{2x^2-3}{(3-2x)(1-x)^2}$ up to and including the term in x^2. *(3 marks)*

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2 (a) Obtain the binomial expansion of $(1-x)^{-3}$ up to and including the term in x^2. *(2 marks)*

(b) Hence obtain the binomial expansion of $\left(1-\dfrac{5}{2}x\right)^{-3}$ up to and including the term in x^2. *(2 marks)*

(c) Find the range of values of x for which the binomial expansion of $\left(1-\dfrac{5}{2}x\right)^{-3}$ would be valid. *(2 marks)*

(d) Given that x is small, show that $\left(\dfrac{4}{2-5x}\right)^3 \approx a + bx + cx^2$, where a, b and c are integers. *(2 marks)*

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5 (a) Find the binomial expansion of $(1+x)^{\frac{1}{3}}$ up to the term in x^2. *(2 marks)*

(b) (i) Show that $(8+3x)^{\frac{1}{3}} \approx 2+\frac{1}{4}x-\frac{1}{32}x^2$ for small values of x. *(3 marks)*

(ii) Hence show that $\sqrt[3]{9} \approx \frac{599}{288}$. *(2 marks)*

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2 (a) (i) Find the binomial expansion of $(1+x)^{-1}$ up to the term in x^3. *(2 marks)*

(ii) Hence, or otherwise, obtain the binomial expansion of $\frac{1}{1+3x}$ up to the term in x^3. *(2 marks)*

(b) Express $\frac{1+4x}{(1+x)(1+3x)}$ in partial fractions. *(3 marks)*

(c) (i) Find the binomial expansion of $\frac{1+4x}{(1+x)(1+3x)}$ up to the term in x^3. *(3 marks)*

(ii) Find the range of values of x for which the binomial expansion of $\frac{1+4x}{(1+x)(1+3x)}$ is valid. *(2 marks)*

Binomial Answers

5(a)(i)	$(1-x)^{-1}=1+(-1)(-x)+\frac{(-1)(-2)}{2}(-x)^2$	M1		First two terms + kx^2
	$=1+x+x^2$	A1	2	
(ii)	$\frac{1}{(3-2x)}=\frac{1}{3}\left(1-\frac{2}{3}x\right)^{-1}$	B1		**Or** directly substitute into formula;
	$\approx *\left(1+\frac{2}{3}x+\left(\frac{2}{3}x\right)^2\right)$	M1		M1 power of 3 M1 other coefficients (allow one error) A1 CAO
	$\approx\frac{1}{3}+\frac{2}{9}x+\frac{4}{27}x^2$	A1	3	AG convincingly obtained
(b)	$(1-x)^{-2}=1+(-2)(-x)+\frac{(-2)(-3)(-x)^2}{2}$	M1		First two terms + kx^2
	$=1+2x+3x^2$	A1	2	

5(c)	$2x^2-3=$ $A(1-x)^2+B(3-2x)(1-x)+C(3-2x)$	M1		**Or** by equating coefficients
	$x=1 \quad -1=C\times1 \quad x=\frac{3}{2} \quad \frac{3}{2}=A\times\frac{1}{4}$	M1		M1 same A1 collect terms M1 equate coefficients A1 2 correct A1 3 correct
	$C=-1 \qquad A=6$	A1		Follow on A and C
	$x=0 \quad (-3=6+3B-3)$ or other value $\Rightarrow$ equation in A, B, C	m1		
	$B=-2$	A1	5	
(d)	$\frac{6}{3-2x}-\frac{2}{1-x}-\frac{1}{(1-x)^2}$			
	$\approx\frac{6}{3}\left(1+\frac{2}{3}x+\frac{4}{9}x^2\right)-2\left(1+x+x^2\right)$	M1A1F		Follow on A B C and expansions
	$-\left(1+2x+3x^2\right) \quad \approx -1-\frac{8}{3}x-\frac{37}{9}x^2$	A1	3	CAO
	Total		**15**	

2(a)	$(1-x)^{-3}=1+(-3)(-x)+\frac{(-3)(-4)(-x)^2}{2}$	M1		$1\pm 3x+x^2$ term
	$=1+3x+6x^2$	A1	2	
(b)	$\left(1-\frac{5}{2}x\right)^{-3}=1+3\left(\frac{5}{2}x\right)+6\left(\frac{5}{2}x\right)^2$	M1		$x\to\frac{5}{2}x$, incl. $\left(\frac{5}{2}x\right)^2$ seen or implied (or start again)
	$=1+\frac{15}{2}x+\frac{75}{2}x^2$	A1	2	CAO OE
(c)	$\left\lvert\frac{5}{2}x\right\rvert<1 \qquad \lvert x\rvert<\frac{2}{5}$	M1A1	2	Sight of $\frac{\pm 5}{2}$ or $\frac{\pm 2}{5}$
(d)	$=8(1+\frac{15}{2}x+\frac{75}{2}x^2)=8+60x+300x^2$	M1		$k\times$ their $\left(1-\frac{5}{2}x\right)^{-3}$
		A1F	2	ft only on $8\left(1-\frac{5}{2}x\right)^{-3}$
	Alternatively, start again: $8\times$ expression or $k\times\left(1-3\left(\pm\frac{5}{2}x\right)\right)$	(M1)		
	CAO	(A1)		
	Total		**8**	

5(a)	$(1+x)^{\frac{1}{3}}=1+\frac{1}{3}x+\frac{1}{3}\left(-\frac{2}{3}\right)\frac{1}{2}x^2$	M1		$1+\frac{1}{3}x+kx^2$
		A1	2	
(b)(i)	$\sqrt[3]{8}\left(1+\frac{3}{8}x\right)^{\frac{1}{3}}$	B1		$8^{\frac{1}{3}}(1+kx)^{\frac{1}{3}}$
	$=2\left(1+\frac{1}{3}\left(\frac{3}{8}x\right)-\frac{1}{9}\left(\frac{3}{8}x\right)^2\right)$	M1		Replacing x with kx in answer to (a)
	$=2+\frac{1}{4}x-\frac{1}{32}x^2$	A1	3	For numerical expression which would evaluate to answer given
	Alternative: B1 – all powers of 8 correct: $8^{\frac{1}{3}}\ 8^{-\frac{2}{3}}\ 8^{-\frac{5}{3}}$ M1 – powers of $3x$ (condone $3x^2$) $2+\frac{1}{8^{\frac{2}{3}}}x-\frac{1}{9}\frac{1}{8^{\frac{5}{3}}}9x^2$ A1 – see some arithmetic processing must see 9s in last term			
(ii)	$x=\frac{1}{3}$: $\sqrt[3]{8+1}=2+\frac{1}{4}\times\frac{1}{3}-\frac{1}{32}\times\left(\frac{1}{3}\right)^2$	M1		Using $x=\frac{1}{3}$ in given answer
	$\sqrt[3]{9}=\frac{576+24-1}{288}=\frac{599}{288}$	A1	2	Any correct numerical expression $=\frac{599}{288}$
	Total		**7**	

2(a)(i)	$(1+x)^{-1}=1+(-1)x+px^2+qx^3$	M1		$p\neq 0,\ q\neq 0$
	$=1-x+x^2-x^3$	A1	2	SC 1/2 for $=1-x+px^2$
(ii)	$(1+3x)^{-1}=1-3x+(3x)^2-(3x)^3$	M1		x replaced by $3x$ in candidate's (a)(i);condone missing brackets
	$=1-3x+9x^2-27x^3$	A1	2	CAO SC x^3-term : $1-3x+\frac{3}{9}x^2$ 1/2
	Alt (starting again) $(1+3x)^{-1}=1-(3x)+\frac{(-1)(-2)(3x)^2}{2!}+\frac{(-1)(-2)(-3)(3x)^3}{3!}$	(M1)		condone missing brackets accept 2 for 2!, 3.2 for 3!
	$=1-3x+9x^2-27x^3$	(A1)	(2)	CAO
(b)	$\frac{1+4x}{(1+x)(1+3x)}=\frac{A}{1+x}+\frac{B}{1+3x}$ $1+4x=A(1+3x)+B(1+x)$	M1		correct partial fractions form, and multiplication by denominator
	$x=-1,\ x=-\frac{1}{3}$	m1		Use (any) two values of x to find A and B
	$A=\frac{3}{2},\ B=-\frac{1}{2}$	A1	3	A and B both correct
	Alt: $\frac{1+4x}{(1+x)(1+3x)}=\frac{A}{1+x}+\frac{B}{1+3x}$ $1+4x=A(1+3x)+B(1+x)$	(M1)		correct partial fractions form, and multiplication by denominator
	$A+B=1,\ 3A+B=4$	(m1)		Set up and solve
	$A=\frac{3}{2},\ B=-\frac{1}{2}$	(A1)	(3)	A and B both correct
(c)(i)	$\frac{1+4x}{(1+x)(1+3x)}=\frac{3}{2(1+x)}-\frac{1}{2(1+3x)}$	M1		
	$=\frac{3}{2}(1-x+x^2-x^3)-\frac{1}{2}(1-3x+9x^2-27x^3)$	m1		multiply candidate's expansions by A and B, and expand and simplify
	$=1-3x^2+12x^3$	A1	3	CAO SC A and B interchanged, treat as miscopy. $(1-4x+13x^2-40x^3)$
	Alt: $=\frac{1+4x}{(1+x)(1+3x)}=(1+4x)(1+x)^{-1}(1+3x)^{-1}$ $=(1+4x)(1-x+x^2-x^3)(1-3x+9x^2-27x^3)$	(M1)		write as product, using expansions condone missing brackets on $(1+4x)$ only
	$=1-4x+13x^2-40x^3+4x-16x^2+52x^3$	(m1)		attempt to multiply the three expansions up to terms in x^3
	$=1-3x^2+12x^3$	(A1)	(3)	CAO
(ii)	$\lvert x\rvert<1$ and $\lvert 3x\rvert<1$	M1		OE and nothing else incorrect
	$\lvert x\rvert<\frac{1}{3}$ (0.33)	A1	2	OE Condone $\leq$
	Total		**12**	