Vectors

Questions

Q1.

The plane Π_1 has vector equation

$$\mathbf{r}.(3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}) = 5$$

(a) Find the perpendicular distance from the point (6, 2, 12) to the plane Π_1

(3)

The plane Π_2 has vector equation

$$\mathbf{r} = \lambda\,(2\mathbf{i} + \mathbf{j} + 5\mathbf{k}) + \mu(\mathbf{i} - \mathbf{j} - 2\mathbf{k})$$

where λ and μ are scalar parameters.

(b) Show that the vector $-\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ is perpendicular to Π_2

(2)

(c) Show that the acute angle between Π_1 and Π_2 is 52° to the nearest degree.

(3)

(Total for question = 8 marks)

Q2.

The line l_1 has equation $\frac{x-2}{4} = \frac{y-4}{-2} = \frac{z+6}{1}$

The plane Π has equation $x - 2y + z = 6$

The line l_2 is the reflection of the line l_1 in the plane Π.

Find a vector equation of the line l_2

(7)

(Total for question = 7 marks)

Q3.

A mining company has identified a mineral layer below ground.

The mining company wishes to drill down to reach the mineral layer and models the situation as follows.

With respect to a fixed origin *O*,

- the ground is modelled as a horizontal plane with equation $z = 0$
- the mineral layer is modelled as part of the plane containing the points *A*(10, 5, –50), *B*(15, 30, –45) and *C*(–5, 20, –60), where the units are in metres

(a) Determine an equation for the plane containing *A*, *B* and *C*, giving your answer in the form $\mathbf{r} \cdot \mathbf{n} = d$

(5)

(b) Determine, according to the model, the acute angle between the ground and the plane containing the mineral layer. Give your answer to the nearest degree.

(3)

The mining company plans to drill vertically downwards from the point (5, 12, 0) on the ground to reach the mineral layer.

(c) Using the model, determine, in metres to 1 decimal place, the distance the mining company will need to drill in order to reach the mineral layer.

(2)

(d) State a limitation of the assumption that the mineral layer can be modelled as a plane.

(1)

(Total for question = 11 marks)

Q4.

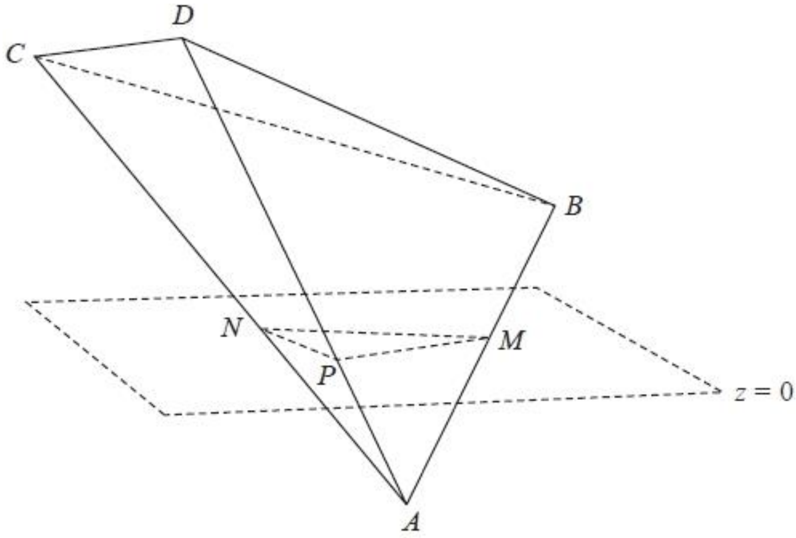


Figure 1

The points $A(3, 2, -4)$, $B(9, -4, 2)$, $C(-6, -10, 8)$ and $D(-4, -5, 10)$ are the vertices of a tetrahedron.

The plane with equation $z = 0$ cuts the tetrahedron into two pieces, one on each side of the plane.

The edges AB, AC and AD of the tetrahedron intersect the plane at the points M, N and P respectively, as shown in Figure 1.

Determine

(a) the coordinates of the points M, N and P,

(3)

(b) the area of triangle MNP,

(2)

(c) the exact volume of the solid $BCDPNM$.

(6)

(Total for question = 11 marks)

Q5.

Two birds are flying towards their nest, which is in a tree.

Relative to a fixed origin, the flight path of each bird is modelled by a straight line.

In the model, the equation for the flight path of the first bird is

$$\mathbf{r}_1 = \begin{pmatrix} -1 \\ 5 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ a \\ 0 \end{pmatrix}$$

and the equation for the flight path of the second bird is

$$\mathbf{r}_2 = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

where λ and μ are scalar parameters and a is a constant.

In the model, the angle between the birds' flight paths is 120°

(a) Determine the value of a.

(4)

(b) Verify that, according to the model, there is a common point on the flight paths of the two birds and find the coordinates of this common point.

(5)

The position of the nest is modelled as being at this common point.

The tree containing the nest is in a park.

The ground level of the park is modelled by the plane with equation

$$2x - 3y + z = 2$$

(c) Hence determine the shortest distance from the nest to the ground level of the park.

(3)

(d) By considering the model, comment on whether your answer to part (c) is reliable, giving a reason for your answer.

(1)

(Total for question = 13 marks)

Q6.

An octopus is able to catch any fish that swim within a distance of 2 m from the octopus's position.

A fish *F* swims from a point *A* to a point *B*.

The octopus is modelled as a fixed particle at the origin *O*.

Fish *F* is modelled as a particle moving in a straight line from *A* to *B*.

Relative to *O*, the coordinates of *A* are $(-3, 1, -7)$ and the coordinates of *B* are $(9, 4, 11)$, where the unit of distance is metres.

(a) Use the model to determine whether or not the octopus is able to catch fish *F*.

(7)

(b) Criticise the model in relation to fish *F*.

(1)

(c) Criticise the model in relation to the octopus.

(1)

(Total for question = 9 marks)

Q7.

Part of the mains water system for a housing estate consists of water pipes buried beneath the ground surface. The water pipes are modelled as straight line segments. One water pipe, *W*, is buried beneath a particular road. With respect to a fixed origin *O*, the road surface is modelled as a plane with equation $3x - 5y - 18z = 7$, and *W* passes through the points $A(-1, -1, -3)$ and $B(1, 2, -3)$. The units are in metres.

(a) Use the model to calculate the acute angle between *W* and the road surface.

(5)

A point $C(-1, -2, 0)$ lies on the road. A section of water pipe needs to be connected to *W* from *C*.

(b) Using the model, find, to the nearest cm, the shortest length of pipe needed to connect *C* to *W*.

(6)

(Total for question = 11 marks)

Q8.

The plane Π has equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$$

where λ and μ are scalar parameters.

(a) Show that vector $2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ is perpendicular to Π.

(2)

(b) Hence find a Cartesian equation of Π.

(2)

The line l has equation

$$\mathbf{r} = \begin{pmatrix} 4 \\ -5 \\ 2 \end{pmatrix} + t \begin{pmatrix} 1 \\ 6 \\ -3 \end{pmatrix}$$

where t is a scalar parameter.

The point A lies on l.

Given that the shortest distance between A and Π is $2\sqrt{29}$

(c) determine the possible coordinates of A.

(4)

(Total for question = 8 marks)

Q9.

The line l_1 has equation

$$\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-4}{3}$$

The line l_2 has equation

$$\mathbf{r} = \mathbf{i} + 3\mathbf{k} + t(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$$

where t is a scalar parameter.

(a) Show that l_1 and l_2 lie in the same plane.

(3)

(b) Write down a vector equation for the plane containing l_1 and l_2

(1)

(c) Find, to the nearest degree, the acute angle between l_1 and l_2

(3)

(Total for question = 7 marks)

Q10.

A gas company maintains a straight pipeline that passes under a mountain.

The pipeline is modelled as a straight line and one side of the mountain is modelled as a plane.

There are accessways from a control centre to two access points on the pipeline.

Modelling the control centre as the origin O, the two access points on the pipeline have coordinates $P(-300, 400, -150)$ and $Q(300, 300, -50)$, where the units are metres.

(a) Find a vector equation for the line PQ, giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$,

where λ is a scalar parameter.

(2)

The equation of the plane modelling the side of the mountain is $2x + 3y - 5z = 300$

The company wants to create a new accessway from this side of the mountain to the pipeline.

The accessway will consist of a tunnel of shortest possible length between the pipeline and the point $M(100, k, 100)$ on this side of the mountain, where k is a constant.

(b) Using the model, find

(i) the coordinates of the point at which this tunnel will meet the pipeline,
(ii) the length of this tunnel.

(7)

It is only practical to construct the new accessway if it will be significantly shorter than both of the existing accessways, OP and OQ.

(c) Determine whether the company should build the new accessway.

(2)

(d) Suggest one limitation of the model.

(1)

(Total for question = 12 marks)

Q11.

With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = \begin{pmatrix} 4 \\ 28 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix}, \qquad l_2 : \mathbf{r} = \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$$

where λ and μ are scalar parameters.

The lines l_1 and l_2 intersect at the point X.

(a) Find the coordinates of the point X.

(3)

(b) Find the size of the acute angle between l_1 and l_2, giving your answer in degrees to 2 decimal places.

(3)

The point A lies on l_1 and has position vector $\begin{pmatrix} 2 \\ 18 \\ 6 \end{pmatrix}$

(c) Find the distance AX, giving your answer as a surd in its simplest form.

(2)

The point Y lies on l_2. Given that the vector $\overrightarrow{YA}$ is perpendicular to the line l_1

(d) find the distance YA, giving your answer to one decimal place.

(2)

The point B lies on l_1 where $|\overrightarrow{AX}| = 2|\overrightarrow{AB}|$.

(e) Find the two possible position vectors of B.

(3)

(Total for question = 13 marks)

Mark Scheme – Vectors

Q1.

Question	Scheme	Marks	AOs
(a)	$\begin{pmatrix}3\\-4\\2\end{pmatrix}\cdot\begin{pmatrix}6\\2\\12\end{pmatrix}=18-8+24$	M1	3.1a
	$d=\dfrac{18-8+24-5}{\sqrt{3^2+4^2+2^2}}$	M1	1.1b
	$=\sqrt{29}$	A1	1.1b
		(3)	
(b)	$\begin{pmatrix}-1\\-3\\1\end{pmatrix}\cdot\begin{pmatrix}2\\1\\5\end{pmatrix}=\ldots$ and $\begin{pmatrix}-1\\-3\\1\end{pmatrix}\cdot\begin{pmatrix}1\\-1\\-2\end{pmatrix}=\ldots$	M1	2.1
	$\begin{pmatrix}-1\\-3\\1\end{pmatrix}\cdot\begin{pmatrix}2\\1\\5\end{pmatrix}=0$ and $\begin{pmatrix}-1\\-3\\1\end{pmatrix}\cdot\begin{pmatrix}1\\-1\\-2\end{pmatrix}=0$ $\therefore\ -\mathbf{i}-3\mathbf{j}+\mathbf{k}$ is perpendicular to Π_2	A1	2.2a
		(2)	
(c)	$\begin{pmatrix}-1\\-3\\1\end{pmatrix}\cdot\begin{pmatrix}3\\-4\\2\end{pmatrix}=-3+12+2$	M1	1.1b
	$\sqrt{(-1)^2+(-3)^2+1^2}\sqrt{(3)^2+(-4)^2+2^2}\cos\theta=11$ $\Rightarrow\cos\theta=\dfrac{11}{\sqrt{(-1)^2+(-3)^2+1^2}\sqrt{(3)^2+(-4)^2+2^2}}$	M1	2.1
	So angle between planes $\theta=52°$ *	A1*	2.4
		(3)	
			(8 marks)

Notes

(a)

M1: Realises the need to and so attempts the scalar product between the normal and the position vector

M1: Correct method for the perpendicular distance

A1: Correct distance

(b)

M1: Recognises the need to calculate the scalar product between the given vector and both direction vectors

A1: Obtains zero both times and makes a conclusion

(c)

M1: Calculates the scalar product between the two normal vectors

M1: Applies the scalar product formula with their 11 to find a value for $\cos\theta$

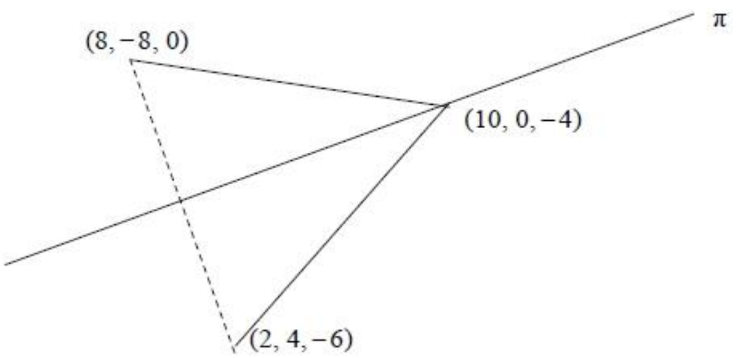
A1*: Identifies the correct angle by linking the angle between the normal and the angle between the planes.

Q2.

Question	Scheme	Marks	AOs
	$2+4\lambda-2(4-2\lambda)-6+\lambda=6 \Rightarrow \lambda=...$	M1	1.1b
	$\lambda=2 \Rightarrow$ Required point is $(2+2(4),\ 4+2(-2),\ -6+2(1))$ $(10,\ 0,\ -4)$	A1	1.1b
	$2+t-2(4-2t)-6+t=6 \Rightarrow t=...$	M1	3.1a
	$t=3$ so reflection of $(2,4,-6)$ is $(2+6(1), 4+6(-2), -6+6(1))$	M1	3.1a
	$(8,\ -8,\ 0)$	A1	1.1b
	$\begin{pmatrix}10\\0\\-4\end{pmatrix}-\begin{pmatrix}8\\-8\\0\end{pmatrix}=\begin{pmatrix}2\\8\\-4\end{pmatrix}$	M1	3.1a
	$\mathbf{r}=\begin{pmatrix}10\\0\\-4\end{pmatrix}+k\begin{pmatrix}1\\4\\-2\end{pmatrix}$ or equivalent e.g. $\left(\mathbf{r}-\begin{pmatrix}10\\0\\-4\end{pmatrix}\right)\times\begin{pmatrix}1\\4\\-2\end{pmatrix}=\mathbf{0}$	A1	2.5
		(7)	
			(7 marks)

Notes:

M1: Substitutes the parametric equation of the line into the equation of the plane and solves for λ
A1: Obtains the correct coordinates of the intersection of the line and the plane
M1: Substitutes the parametric form of the line perpendicular to the plane passing through $(2, 4, -6)$ into the equation of the plane to find t
M1: Find the reflection of $(2, 4, -6)$ in the plane
A1: Correct coordinates
M1: Determines the direction of l by subtracting the appropriate vectors
A1: Correct vector equation using the correct notation.



Q3.

Question	Scheme	Marks	AOs
(a)	Any two of: $\begin{cases} \pm k\overrightarrow{AB} = \pm k(5\mathbf{i}+25\mathbf{j}+5\mathbf{k}), \\ \pm k\overrightarrow{AC} = \pm k(-15\mathbf{i}+15\mathbf{j}-10\mathbf{k}), \\ \pm k\overrightarrow{BC} = \pm k(-20\mathbf{i}-10\mathbf{j}-15\mathbf{k}) \end{cases}$	M1	3.3
	Let normal vector be $a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$ $(a\mathbf{i}+b\mathbf{j}+c\mathbf{k})\bullet(\mathbf{i}+5\mathbf{j}+\mathbf{k})=0,\ (a\mathbf{i}+b\mathbf{j}+c\mathbf{k})\bullet(-3\mathbf{i}+3\mathbf{j}-2\mathbf{k})=0$ $\Rightarrow a+5b+c=0,\ -3a+3b-2c=0 \Rightarrow a=...,\ b=...,\ c=...$ Alternative: cross product $\begin{vmatrix} 1 & 5 & 1 \\ -3 & 3 & -2 \end{vmatrix} = (-10-3)\mathbf{i}-(-2+3)\mathbf{j}+(3+15)\mathbf{k}$	M1	1.1b
	$\mathbf{n}=k(-13\mathbf{i}-\mathbf{j}+18\mathbf{k})$	A1	1.1b
	$(-13\mathbf{i}-\mathbf{j}+18\mathbf{k})\bullet(10\mathbf{i}+5\mathbf{j}-50\mathbf{k})=...$	M1	1.1b
	$\mathbf{r}\bullet(13\mathbf{i}+\mathbf{j}-18\mathbf{k})=1035$ o.e. $\mathbf{r}\bullet(-13\mathbf{i}-\mathbf{j}+18\mathbf{k})=-1035$ $\mathbf{r}\bullet(325\mathbf{i}+25\mathbf{j}-450\mathbf{k})=25875$	A1	2.5
		(5)	
(b)	Attempts the scalar product between their normal vector and the vector **k** and uses trigonometry to find an angle	M1	3.1b
	$(-13\mathbf{i}-\mathbf{j}+18\mathbf{k})\bullet\mathbf{k}=-18=\sqrt{13^2+1^2+18^2}\cos\alpha$	M1	1.1b
	$\cos\alpha=\frac{-18}{\sqrt{494}} \Rightarrow \alpha=144.08... \Rightarrow \theta=36°$	A1	3.2a
		(3)	
(c)	Distance required is $\lvert\lambda\rvert$ where $\begin{pmatrix} 13 \\ 1 \\ -18 \end{pmatrix}\bullet\begin{pmatrix} 5 \\ 12 \\ \lambda \end{pmatrix}=1035$	M1	3.4
	$\lvert\lambda\rvert=53.2\text{m}$	A1	1.1b
		(2)	

(d)	E.g. • The mineral layer will **not** be perfectly **flat**/**smooth** and will not form a plane • The mineral layer will have a **depth** and this should be taken into account	B1	3.5b
		(1)	
			(11 marks)

Notes

(a)
M1: Attempts to find at least 2 vectors in the plane that can be used to set up the model. Two correct value implies the correct method if not explicitly seen.
M1: Attempts a normal vector using an appropriate method. E.g. as in main scheme or may use vector product
A1: A correct normal vector
M1: Applies **r.n** = d with their normal vector and a point in the plane to find a value for d
A1: Correct equation (allow any multiple)
(b)
M1: Realises the scalar product between their from part (a) and a vector parallel to **k** and so applies it and uses trigonometry to find an angle
M1: Forms the scalar product between their from part (a) and a vector parallel to **k**
A1: Correct angle
(c)
M1: Uses the model and a correct strategy to establish the distance from (5, 12, 0) to the plane vertically downwards
A1: Correct distance
(d)
B1: Any reasonable limitation – see scheme

Q4.

Question	Scheme	Marks	AOs
(a)	A correct method to find one coordinates of M, N or P For example $\overrightarrow{AB}=\begin{pmatrix}6\\-6\\6\end{pmatrix}$ so $\overrightarrow{OM}=\begin{pmatrix}3\\2\\-4\end{pmatrix}+\frac{4}{6}\begin{pmatrix}6\\-6\\6\end{pmatrix}=\ldots$ $\overrightarrow{AC}=\begin{pmatrix}-9\\-12\\12\end{pmatrix}$ so $\overrightarrow{ON}=\begin{pmatrix}3\\2\\-4\end{pmatrix}+\frac{4}{12}\begin{pmatrix}-9\\-12\\12\end{pmatrix}=\ldots$ $\overrightarrow{AD}=\begin{pmatrix}-7\\-7\\14\end{pmatrix}$ so $\overrightarrow{OP}=\begin{pmatrix}3\\2\\-4\end{pmatrix}+\frac{4}{14}\begin{pmatrix}-7\\-7\\14\end{pmatrix}=\ldots$	M1	3.1a
	One of $(M=)(7,-2,0)$, $(N=)(0,-2,0)$ or $(P=)(1,0,0)$	A1	1.1b
	All of $(M=)(7,-2,0)$, $(N=)(0,-2,0)$ and $(P=)(1,0,0)$	A1	1.1b
		(3)	
(b)	Correct method, e.g. realises MN is parallel to x axis, so base is 7 and height 2, hence area of intersection is $\frac{1}{2}\times 7\times 2=\ldots$ **Alternatively using** $\frac{1}{2}\lvert a\times b\rvert$ $\overrightarrow{PM}=\pm\begin{pmatrix}6\\-2\\0\end{pmatrix}$ $\overrightarrow{PN}=\pm\begin{pmatrix}-1\\-2\\0\end{pmatrix}$ $\overrightarrow{NM}=\pm\begin{pmatrix}7\\0\\0\end{pmatrix}$ For example $\frac{1}{2}\lvert\overrightarrow{MP}\times\overrightarrow{PN}\rvert=\frac{1}{2}\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\6&-2&0\\-1&-2&0\end{vmatrix}=\frac{1}{2}\left\lvert\begin{pmatrix}0\\0\\-14\end{pmatrix}\right\rvert=\ldots$	M1	1.1b
	$=7$ cso	A1	1.1b
		(2)	

(c)	Vol $NMPA=\frac{1}{3}A_bh=\frac{1}{3}\times 7\times 4=\frac{28}{3}$ Or using triple scalar product $NMPA=\frac{1}{6}\lvert\overrightarrow{AM}\cdot(\overrightarrow{AN}\times\overrightarrow{AP})\rvert=\frac{1}{6}\left\lvert\begin{pmatrix}4\\-4\\4\end{pmatrix}\cdot\left(\begin{pmatrix}-3\\-4\\4\end{pmatrix}\times\begin{pmatrix}-2\\-2\\4\end{pmatrix}\right)\right\rvert$ $=\frac{1}{6}\left\lvert\begin{pmatrix}4\\-4\\4\end{pmatrix}\cdot\begin{pmatrix}-8\\4\\-2\end{pmatrix}\right\rvert=\frac{28}{3}$	M1 A1	3.1a 1.1b

	Vol $ABCD = \frac{1}{6}\left\|\overrightarrow{AB}\cdot\left(\overrightarrow{AC}\times\overrightarrow{AD}\right)\right\| = \ldots$ $= \frac{1}{6}\left\|\begin{pmatrix}6\\-6\\6\end{pmatrix}\cdot\left(\begin{pmatrix}-9\\-12\\12\end{pmatrix}\times\begin{pmatrix}-7\\-7\\14\end{pmatrix}\right)\right\| = \frac{1}{6}\left\|\begin{pmatrix}6\\-6\\6\end{pmatrix}\cdot\begin{pmatrix}-84\\42\\-21\end{pmatrix}\right\| = \ldots$	M1	1.1b
	$= 147$	A1	1.1b
	So volume required is $'147' - \frac{'28'}{3} = \ldots$	M1	3.1a
	$= \frac{413}{3}$	A1	1.1b
		(6)	
			(11 marks)

Notes:

(a)

M1: Correct method for finding at least one of the three points. Allow one slip in coordinates but should have correct fraction to make the value of z to be 0.

A1: Any one of the three points correct, ignoring the labelling.

A1: All three points correct, ignoring the labelling

(b)

M1: Correct method for finding the area of the triangle, e.g realises that MN is parallel to the x-axis so uses $\frac{1}{2}bh$ with $b = MN$ and h is distance of MN from axis.

Alternative using $\frac{1}{2}|a \times b|$ with vectors $\overrightarrow{PM} = \pm\begin{pmatrix}6\\-2\\0\end{pmatrix}$ $\overrightarrow{PN} = \pm\begin{pmatrix}-1\\-2\\0\end{pmatrix}$ $\overrightarrow{NM} = \pm\begin{pmatrix}7\\0\\0\end{pmatrix}$ follow through on their answers in part (a). Condone sign slips except they must be using $-\mathbf{j}$ in the cross product

For example $\frac{1}{2}\left|\overrightarrow{MP}\times\overrightarrow{PN}\right| = \frac{1}{2}\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\6 & -2 & 0\\-1 & -2 & 0\end{vmatrix} = \frac{1}{2}\left|\begin{pmatrix}0\\0\\-14\end{pmatrix}\right| = \ldots$

$$\frac{1}{2}\left|\overrightarrow{PN}\times\overrightarrow{NM}\right| = \frac{1}{2}\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\-1 & -2 & 0\\7 & 0 & 0\end{vmatrix} = \frac{1}{2}\left|\begin{pmatrix}0\\0\\14\end{pmatrix}\right| = \ldots$$

$$\frac{1}{2}\left|\overrightarrow{PM}\times\overrightarrow{NM}\right| = \frac{1}{2}\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\6 & -2 & 0\\7 & 0 & 0\end{vmatrix} = \frac{1}{2}\left|\begin{pmatrix}0\\0\\14\end{pmatrix}\right| = \ldots$$

or attempting to find an angle using dot product or cosine rule followed by $\frac{1}{2}ab\sin C$.

A1: Correct area of 7 from correct vectors

(c) On ePen this is M1 A1 M1 M1 M1 A1

M1: Formulates a correct method to find the volume of *NMPA*. May use method shown, or e.g. $\frac{1}{6}\left|\overrightarrow{AM}\cdot\left(\overrightarrow{AN}\times\overrightarrow{AP}\right)\right|$ or equivalent method.

A1: For $\frac{28}{3}$.

Note there are many ways to find the required volume of *AMNP* applying the triple scalar product to a combination of the following vectors

$$\overrightarrow{AM}=\begin{pmatrix}4\\-4\\4\end{pmatrix}\ \overrightarrow{AN}=\begin{pmatrix}-3\\-4\\4\end{pmatrix}\ \overrightarrow{AP}=\begin{pmatrix}-2\\-2\\4\end{pmatrix}\ \overrightarrow{NA}=\begin{pmatrix}3\\4\\-4\end{pmatrix}\ \overrightarrow{NM}=\begin{pmatrix}7\\0\\0\end{pmatrix}\ \overrightarrow{NP}=\begin{pmatrix}1\\2\\0\end{pmatrix}$$

$$\overrightarrow{MA}=\begin{pmatrix}-4\\4\\-4\end{pmatrix}\ \overrightarrow{MN}=\begin{pmatrix}-7\\0\\0\end{pmatrix}\ \overrightarrow{MP}=\begin{pmatrix}-6\\2\\0\end{pmatrix}\ \overrightarrow{PA}=\begin{pmatrix}2\\2\\-4\end{pmatrix}\ \overrightarrow{PM}=\begin{pmatrix}6\\-2\\0\end{pmatrix}\ \overrightarrow{PN}=\begin{pmatrix}-1\\-2\\0\end{pmatrix}$$

For example

$$\frac{1}{6}\left|\overrightarrow{AM}\cdot\left(\overrightarrow{AN}\times\overrightarrow{AP}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}4\\-4\\4\end{pmatrix}\cdot\left(\begin{pmatrix}-3\\-4\\4\end{pmatrix}\times\begin{pmatrix}-2\\-2\\4\end{pmatrix}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}4\\-4\\4\end{pmatrix}\cdot\begin{pmatrix}-8\\4\\-2\end{pmatrix}\right|=\frac{1}{6}\times 56$$

$$\frac{1}{6}\left|\overrightarrow{NA}\cdot\left(\overrightarrow{NM}\times\overrightarrow{NP}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}3\\4\\-4\end{pmatrix}\cdot\left(\begin{pmatrix}7\\0\\0\end{pmatrix}\times\begin{pmatrix}1\\2\\0\end{pmatrix}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}3\\4\\-4\end{pmatrix}\cdot\begin{pmatrix}0\\0\\14\end{pmatrix}\right|=\frac{1}{6}\times 56$$

$$\frac{1}{6}\left|\overrightarrow{MA}\cdot\left(\overrightarrow{MN}\times\overrightarrow{MP}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}-4\\4\\-4\end{pmatrix}\cdot\left(\begin{pmatrix}-7\\0\\0\end{pmatrix}\times\begin{pmatrix}-6\\2\\0\end{pmatrix}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}-4\\4\\-4\end{pmatrix}\cdot\begin{pmatrix}0\\0\\14\end{pmatrix}\right|=\frac{1}{6}\times 56$$

$$\frac{1}{6}\left|\overrightarrow{PA}\cdot\left(\overrightarrow{PM}\times\overrightarrow{PN}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}2\\2\\-4\end{pmatrix}\cdot\left(\begin{pmatrix}6\\2\\0\end{pmatrix}\times\begin{pmatrix}-1\\-2\\0\end{pmatrix}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}2\\2\\-4\end{pmatrix}\cdot\begin{pmatrix}0\\0\\-14\end{pmatrix}\right|=\frac{1}{6}\times 56$$

Note candidates may write as $\frac{1}{6}\begin{vmatrix}4&-4&4\\-3&-4&4\\-2&-2&4\end{vmatrix}=\frac{1}{6}\left|4(-16+8)+4(-12+8)+4(6-8)\right|=\frac{1}{6}\left|-56\right|=\frac{28}{3}$

M1: A complete attempt at the volume of *ABCD*, with correct method for cross product (oe in other methods). Condone sign slips except they must be using $-\mathbf{j}$ in the cross product

A1 (M1 on ePen): 147

M1: Finds difference of the two volumes must have used a correct method to find the volumes.

A1: $\frac{413}{3}$

Note there are many ways to find the required volume of *ABCD* applying the triple scalar product to a combination of the following vectors

$$\overrightarrow{AB}=\begin{pmatrix}6\\-6\\6\end{pmatrix}\ \overrightarrow{AC}=\begin{pmatrix}-9\\-12\\12\end{pmatrix}\ \overrightarrow{AD}=\begin{pmatrix}-7\\-7\\14\end{pmatrix}\ \overrightarrow{BA}=\begin{pmatrix}-6\\6\\-6\end{pmatrix}\ \overrightarrow{BC}=\begin{pmatrix}15\\-6\\6\end{pmatrix}\ \overrightarrow{BD}=\begin{pmatrix}-13\\-1\\8\end{pmatrix}$$

$$\overrightarrow{CA}=\begin{pmatrix}9\\12\\-12\end{pmatrix}\ \overrightarrow{CB}=\begin{pmatrix}-15\\6\\-6\end{pmatrix}\ \overrightarrow{CD}=\begin{pmatrix}2\\5\\2\end{pmatrix}\ \overrightarrow{DA}=\begin{pmatrix}7\\7\\-14\end{pmatrix}\ \overrightarrow{DB}=\begin{pmatrix}13\\1\\-8\end{pmatrix}\ \overrightarrow{DC}=\begin{pmatrix}-2\\-5\\2\end{pmatrix}$$

For example

$$\frac{1}{6}\left|\overrightarrow{AB}\cdot\left(\overrightarrow{AC}\times\overrightarrow{AD}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}6\\-6\\6\end{pmatrix}\cdot\left(\begin{pmatrix}-9\\-12\\12\end{pmatrix}\times\begin{pmatrix}-7\\-7\\14\end{pmatrix}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}6\\-6\\6\end{pmatrix}\cdot\begin{pmatrix}-84\\42\\-21\end{pmatrix}\right|=\frac{1}{6}\times 882=147$$

$$\frac{1}{6}\left|\overrightarrow{BA}\cdot\left(\overrightarrow{BC}\times\overrightarrow{BD}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}-6\\6\\-6\end{pmatrix}\cdot\left(\begin{pmatrix}15\\-6\\6\end{pmatrix}\times\begin{pmatrix}-13\\-1\\8\end{pmatrix}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}-6\\6\\-6\end{pmatrix}\cdot\begin{pmatrix}-42\\42\\-63\end{pmatrix}\right|=\frac{1}{6}\times 882=147$$

$$\frac{1}{6}\left|\overrightarrow{CA}\cdot\left(\overrightarrow{CD}\times\overrightarrow{CB}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}9\\12\\-12\end{pmatrix}\cdot\left(\begin{pmatrix}2\\5\\2\end{pmatrix}\times\begin{pmatrix}15\\6\\-6\end{pmatrix}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}9\\12\\-12\end{pmatrix}\cdot\begin{pmatrix}-42\\42\\-63\end{pmatrix}\right|=\frac{1}{6}\times 882=147$$

$$\frac{1}{6}\left|\overrightarrow{DA}\cdot\left(\overrightarrow{DB}\times\overrightarrow{DC}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}7\\7\\-14\end{pmatrix}\cdot\left(\begin{pmatrix}13\\1\\-8\end{pmatrix}\times\begin{pmatrix}-2\\-5\\2\end{pmatrix}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}7\\7\\-14\end{pmatrix}\cdot\begin{pmatrix}42\\-42\\63\end{pmatrix}\right|=\frac{1}{6}\times 882=147$$

Note candidates may write as

$$\frac{1}{6}\begin{vmatrix}6&-6&6\\-9&-12&12\\-7&-7&14\end{vmatrix}=\frac{1}{6}\left|6(-168+84)+6(-126+84)+6(63-84)\right|=\frac{1}{6}\left|-882\right|=147$$

Q5.

Question	Scheme	Marks	AOs
(a)	A complete method to use the scalar product of the direction vectors and the angle 120° to form an equation in a $\frac{\begin{pmatrix}2\\a\\0\end{pmatrix}\bullet\begin{pmatrix}0\\1\\-1\end{pmatrix}}{\sqrt{2^2+a^2}\sqrt{1^2+(-1)^2}} = cos\ 1\ 20$	M1	3.1b
	$\frac{a}{\sqrt{4+a^2}\sqrt{2}} = -\frac{1}{2}$	A1	1.1b
	$2a = -\sqrt{4+a^2}\sqrt{2} \Rightarrow 4a^2 = 8 + 2a^2 \Rightarrow a^2 = 4 \Rightarrow a = ...$	M1	1.1b
	$a = -2$	A1	2.2a
		(4)	
(b)	Any two of **i**: $-1 + 2\lambda = 4$ (1) **j**: $5 + \text{'their} - 2\text{'}\lambda = -1 + \mu$ (2) **k**: $2 = 3 - \mu$ (3)	M1	3.4
	Solves the equations to find a value of $\lambda \left\{= \frac{5}{2}\right\}$ **and** $\mu\{= 1\}$	M1	1.1b
	$r_1 = \begin{pmatrix}-1\\5\\2\end{pmatrix} + \frac{5}{2}\begin{pmatrix}2\\\text{'their} - 2\text{'}\\0\end{pmatrix}$ or $r_2 = \begin{pmatrix}4\\-1\\3\end{pmatrix} + 1\begin{pmatrix}0\\1\\-1\end{pmatrix}$	dM1	1.1b
	(4,0,2) or $\begin{pmatrix}4\\0\\2\end{pmatrix}$	A1	1.1b
	Checks the third equation e.g. $\lambda = \frac{5}{2}$: **L HS** $= 5 - 2\lambda = 5 - 5 = 0$ $\mu = 1$: **R HS** $= -1 + \mu = -1 + 1 = 0$ therefore **common point/intersect/consistent/tick** or substitutes the values of λ and μ into the relevant lines and achieves the same coordinate	B1	2.1
		(5)	

(c)	Full attempt to find the minimum distance from the point of intersection (nest) to the plane (ground) E.g. Minimum distance $= \frac{\|2\times\text{'4'}+(-3)\times\text{'0'}+1\times\text{'2'}-2\|}{\sqrt{2^2+(-3)^2+1)^2}} = ...$ **Alternatively** $\mathbf{r} = \begin{pmatrix}\text{'4'}\\\text{'0'}\\\text{'2'}\end{pmatrix} + \lambda\begin{pmatrix}2\\-3\\1\end{pmatrix}$ $2(\text{'4'} + 2\lambda) - 3(\text{'0'} - 3\lambda) + (\text{'2'} + \lambda) = 2 \Rightarrow$ $\lambda = ...\left\{-\frac{4}{7}\right\}$	M1 A1ft	3.1b 3.4

	$\mathbf{r} = \begin{pmatrix} '4' \\ '0' \\ '2' \end{pmatrix} + ' -\frac{4}{7}' \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{20}{7} \\ \frac{12}{7} \\ \frac{10}{7} \end{pmatrix}$ Minimum distance $= \sqrt{\left(2 \times -\frac{4}{7}'\right)^2 + \left(-3 \times -\frac{4}{7}'\right)^2 + \left(1 \times -\frac{4}{7}\right)^2} = \ldots$ $= \sqrt{\left('4' - '\frac{20}{7}'\right)^2 + \left('0' - '\frac{12}{7}'\right)^2 + \left('2' - '\frac{10}{7}'\right)^2} = \ldots$		
	$\frac{8}{\sqrt{14}}$ or $\frac{4\sqrt{14}}{7}$ or awrt 2.1	A1	2.2b
		(3)	
	Alternative Find perpendicular distance from plane to the origin $2x - 3y + z = 2$ $\lvert n \rvert = \sqrt{2^2 + (-3)^2 + 1^2} = \sqrt{14}$ shortest distance $= \frac{2}{\sqrt{14}}$ Find perpendicular distance from the plane containing the point of intersection to the origin $2x - 3y + z = \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = 10$ shortest distance $= \frac{10}{\sqrt{14}}$ Minimum distance $= \frac{10}{\sqrt{14}} - \frac{2}{\sqrt{14}}$	M1 A1ft	3.1b 3.4
	$\frac{8}{\sqrt{14}}$ or $\frac{4\sqrt{14}}{7}$ or awrt 2.1	A1	2.2b
		(3)	
(d)	For example Not reliable as the birds will not fly in a straight line Not reliable as angle between flights paths will not always be 120° Not reliable/reliable as the ground will not be flat/smooth Not reliable as bird's nest is not a point	B1	3.2b
		(1)	
			(13 marks)

Notes:
(a) **M1:** See scheme, allow a sign slip and cos 60 **A1:** Correct simplified equation in a, cos 120 must be evaluated to – ½ and dot product calculated **Note:** If the candidate states either $\left\lvert \frac{a \square b}{\lvert a \rvert \lvert b \rvert} \right\rvert = \cos\theta$ or $\left\lvert \frac{a}{\sqrt{4+a^2}\sqrt{2}} \right\rvert = \cos 60$ then has the equation $\frac{a}{\sqrt{4+a^2}\sqrt{2}} = \frac{1}{2}$ award this mark. If the module of the dot product is not seen then award A0 for this equation.

dM1: Solve a quadratic equation for a, by squaring and solving an equation of the form $a^2 = K$ where $K > 0$

A1: Deduces the correct value of a from a correct equation. Must be seen in part (a) using the angle between the lines.

Alternative cross product method

M1: $\begin{vmatrix} 2 & a & 0 \\ 0 & 1 & -1 \end{vmatrix} = \sqrt{2^2 + a^2}\sqrt{1^2 + (-1)^2}\, sin\, 1\,20$

A1: $\sqrt{a^2 + 8} = \sqrt{4 + a^2}\sqrt{2}\frac{\sqrt{3}}{2}$

Then as above

Note If they use the point of intersection to find a value for a this scores no marks

(b)

M1: Uses the model to write down any two correct equations

M1: Solve two equations simultaneously to find a value for μ and λ

dM1: Dependent on previous method mark. Substitutes μ and λ into a relevant equation. If no method shown two correct ordinates implies this mark.

A1: Correct coordinates. May be seen in part (c)

B1: Shows that the values of μ and λ give the same third coordinate or point of intersection and draws the conclusion that the **lines intersect/common point/consistent** or tick.

Note: If an incorrect value for a is found in part (a) but in part (b) they find that $a = -2$ this scores **B0** but all other marks are available

(c) This is M1M1A1 on ePen marking as M1 A1ft A1

M1: Full attempt to find the minimum distance from a point to a plane. Condone a sign slip with the value of d.

A1ft: Following through on their point of intersection. Uses the model to find a correct expression for minimum distance from the nest to the ground

A1: Correct distance

Alternative

M1: Find the shortest distance from a point to plane by finding the perpendicular distance from the given plane to the origin and the perpendicular distance from the plane contacting their point of intersection to the origin and subtracts

A1ft: Following through on their point of intersection. Uses the model to find a correct expression for minimum distance from the nest to the ground

A1: Correct distance

(d)

B1: Comments on one of the models

- Flight path of the birds modelled as a straight line
- Angle between flight paths modelled as 120°
- The bird's nest is modelled as a point
- Ground modelled as a plane

Then states unreliabl

Any correct answer seen, ignore any other incorrect answers

Q6.

Question	Scheme	Marks	AOs
(a)	$\overrightarrow{AB} = \begin{pmatrix} 9 \\ 4 \\ 11 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} \left\{ = \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} \right\}$ or $\mathbf{d} = \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix}$	M1	3.1a
	$\left\{ \overrightarrow{OF} = \mathbf{r} = \right\} \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}$	M1	1.1b
	$\left\{ \overrightarrow{OF} \bullet \overrightarrow{AB} = 0 \Rightarrow \right\} \begin{pmatrix} -3+12\lambda \\ 1+3\lambda \\ -7+18\lambda \end{pmatrix} \bullet \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} = 0$	M1	1.1b
	$\Rightarrow -36 + 144\lambda + 3 + 9\lambda - 126 + 324\lambda = 0 \Rightarrow 477\lambda - 159 = 0$		
	$\Rightarrow \lambda = \frac{1}{3}$	A1	1.1b
	$\left\{ \overrightarrow{OF} = \right\} \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} + \frac{1}{3} \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and minimum distance $= \sqrt{(1)^2 + (2)^2 + (-1)^2}$	M1	3.1a
	$= \sqrt{6}$ or $2.449\ldots$	A1	1.1b
	> 2, so the octopus is not able to catch the fish *F*.	A1ft	3.2a
		(7)	
(b)	E.g. Fish *F* may not swim in an exact straight line from *A* to *B*. Fish *F* may hit an obstacle whilst swimming from *A* to *B*. Fish *F* may deviate his path to avoid being caught by the octopus.	B1	3.5b
		(1)	
(c)	E.g. Octopus is effectively modelled as a particle – so we may need to look at where the octopus's mass is distributed. Octopus may during the fish *F*'s motion move away from its fixed location at *O*.	B1	3.5b
		(1)	
	(9 marks)		

Question	Scheme	Marks	AOs
(a) ALT 1	$\overrightarrow{AB} = \begin{pmatrix} 9 \\ 4 \\ 11 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} \left\{= \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}\right\}$ or $\mathbf{d} = \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix}$	M1	3.1a
	$\left\{\overrightarrow{OA} = \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} \text{ and } \overrightarrow{AB} = \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix} \Rightarrow\right\} \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} \bullet \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}$	M1	1.1b
	$\cos\theta \left\{= \dfrac{\overrightarrow{OA} \bullet \overrightarrow{AB}}{\lvert\overrightarrow{OA}\rvert . \lvert\overrightarrow{AB}\rvert}\right\} = \dfrac{\pm\left(\begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} \bullet \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}\right)}{\sqrt{(-3)^2 + (1)^2 + (-7)^2} . \sqrt{(12)^2 + (3)^2 + (18)^2}}$	dM1	1.1b
	$\left\{\cos\theta = \dfrac{-36 + 3 - 126}{\sqrt{59}.\sqrt{477}} = \dfrac{-159}{\sqrt{59}.\sqrt{477}}\right\}$		
	$\theta = 161.4038029...$ or $18.59619709...$ or $\sin\theta = 0.3188964021...$	A1	1.1b
	minimum distance $= \sqrt{(-3)^2 + (1)^2 + (-7)^2}\sin(18.59619709...)$	dM1	3.1a
	$= \sqrt{6}$ or $2.449...$	A1	1.1b
	> 2, so the octopus is not able to catch the fish F.	A1ft	3.2a
		(7)	

Question	Scheme	Marks	AOs
(a) ALT 2	$\overrightarrow{AB} = \begin{pmatrix} 9 \\ 4 \\ 11 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} \left\{= \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}\right\}$ or $\mathbf{d} = \begin{pmatrix} 4 \\ 1 \\ 6 \end{pmatrix}$	M1	3.1a
	$\left\{\overrightarrow{OF} = \mathbf{r} = \right\} \begin{pmatrix} -3 \\ 1 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 12 \\ 3 \\ 18 \end{pmatrix}$	M1	1.1b
	$\lvert\overrightarrow{OF}\rvert^2 = (-3 + 12\lambda)^2 + (1 + 3\lambda)^2 + (-7 + 18\lambda)^2$	dM1	1.1b
	$= 9 - 72\lambda + 144\lambda^2 + 1 + 6\lambda + 9\lambda^2 + 49 - 252\lambda + 324\lambda^2$		
	$= 477\lambda^2 - 318\lambda + 59$	A1	1.1b
	$= 53(3\lambda - 1)^2 + 6$	dM1	3.1a
	minimum distance $= \sqrt{6}$ or $2.449...$	A1	1.1b
	> 2, so the octopus is not able to catch the fish F.	A1ft	3.2a
		(7)	

		Question Notes
(a)	M1	Attempts to find $\overrightarrow{OB}-\overrightarrow{OA}$ or $\overrightarrow{OA}-\overrightarrow{OB}$ or the direction vector $\mathbf{d}$.
	M1	Applies $\overrightarrow{OA}+\lambda$(their $\overrightarrow{AB}$ or their $\overrightarrow{BA}$ or their $\mathbf{d}$) or equivalent.
	M1	Depends on previous M mark. Writes down (their $\overrightarrow{OF}$ which is in terms of λ)$\bullet$(their $\overrightarrow{AB}$) $=0$. Can be implied.
	A1	Lambda is correct. e.g. $\lambda=\frac{1}{3}$ for $\overrightarrow{AB}=\begin{pmatrix}12\\3\\18\end{pmatrix}$ or $\lambda=1$ for $\mathbf{d}=\begin{pmatrix}4\\1\\6\end{pmatrix}$
	M1	Depends on previous M mark. Complete method for finding $\lvert\overrightarrow{OF}\rvert$.
	A1	$\sqrt{6}$ or awrt 2.4
	A1ft	Correct follow through conclusion, which is in context with the question.
(a) **ALT 1**	M1	Attempts to find $\overrightarrow{OB}-\overrightarrow{OA}$ or $\overrightarrow{OA}-\overrightarrow{OB}$ or the direction vector $\mathbf{d}$.
	M1	Realisation that the dot product is required between $\overrightarrow{OA}$ and their $\overrightarrow{AB}$. (o.e.)
	M1	Depends on previous M mark. Applies dot product formula between $\overrightarrow{OA}$ and their $\overrightarrow{AB}$. (o.e.)
	A1	$\theta=$ awrt 161.4 or awrt 18.6 or $\sin\theta=$ awrt 0.319
	M1	Depends on previous M mark. (their OA)$\sin$(their θ)
	A1	$\sqrt{6}$ or awrt 2.4
	A1ft	Correct follow through conclusion, which is in context with the question.
(a) **ALT 2**	M1	Attempts to find $\overrightarrow{OB}-\overrightarrow{OA}$ or $\overrightarrow{OA}-\overrightarrow{OB}$ or the direction vector $\mathbf{d}$.
	M1	Applies $\overrightarrow{OA}+\lambda$(their $\overrightarrow{AB}$ or their $\overrightarrow{BA}$ or their $\mathbf{d}$) or equivalent.
	M1	Depends on previous M mark. Applies Pythagoras by finding $\lvert\overrightarrow{OF}\rvert^2$, o.e.
	A1	$\lvert\overrightarrow{OF}\rvert^2=477\lambda^2-318\lambda+59$
	M1	Depends on previous M mark. Method of completing the square or differentiating their $\lvert\overrightarrow{OF}\rvert^2$ w.r.t. λ.
	A1	$\sqrt{6}$ or awrt 2.4
	A1ft	Correct follow through conclusion, which is in context with the question.
(b)	B1	An acceptable criticism for fish F, which is in context with the question.
(c)	B1	An acceptable criticism for the octopus, which is in context with the question.

Q7.

Question	Scheme	Marks	AOs
(a)	Attempts the scalar product between the direction of W and the normal to the road and uses trigonometry to find an angle.	M1	3.1a
	$\left(\begin{pmatrix}1\\2\\-3\end{pmatrix}-\begin{pmatrix}-1\\-1\\-3\end{pmatrix}\right)\bullet\begin{pmatrix}3\\-5\\-18\end{pmatrix}=-9$ or $\left(\begin{pmatrix}-1\\-1\\-3\end{pmatrix}-\begin{pmatrix}1\\2\\-3\end{pmatrix}\right)\bullet\begin{pmatrix}3\\-5\\-18\end{pmatrix}=9$	M1 A1	1.1b 1.1b
	$\sqrt{(2)^2+(3)^3+(0)^2}\sqrt{(3)^2+(-5)^3+(-18)^2}\cos\alpha=\text{"}-9\text{"}$ $\theta=90-\arccos\left(\frac{9}{\sqrt{13}\sqrt{358}}\right)$ or $\theta=\arcsin\left(\frac{9}{\sqrt{13}\sqrt{358}}\right)$ Angle between pipe and road $=7.58°$ (3sf) or 0.132 radians (3sf) (Allow $-7.58°$ or -0.132 radians)	M1 A1	1.1b 3.2a
		(5)	

Question	Scheme	Marks	AOs
(b)	$W:\begin{pmatrix}-1\\-1\\-3\end{pmatrix}+t\begin{pmatrix}2\\3\\0\end{pmatrix}$ or $\begin{pmatrix}1\\2\\-3\end{pmatrix}+\lambda\begin{pmatrix}2\\3\\0\end{pmatrix}$	B1ft	1.1b
	$C\text{ to }W:\left\{\begin{pmatrix}-1\\-1\\-3\end{pmatrix}+t\begin{pmatrix}2\\3\\0\end{pmatrix}-\begin{pmatrix}-1\\-2\\0\end{pmatrix}\right\}$ or $\left\{\begin{pmatrix}1\\2\\-3\end{pmatrix}+\lambda\begin{pmatrix}2\\3\\0\end{pmatrix}-\begin{pmatrix}-1\\-2\\0\end{pmatrix}\right\}$	M1	3.4
	$\begin{pmatrix}2t\\3t+1\\-3\end{pmatrix}\bullet\begin{pmatrix}2\\3\\0\end{pmatrix}=0\Rightarrow t=\ldots$ or $\begin{pmatrix}2+2\lambda\\4+3\lambda\\-3\end{pmatrix}\bullet\begin{pmatrix}2\\3\\0\end{pmatrix}=0\Rightarrow\lambda=\ldots$ or $(2t)^2+(3t+1)^2+(-3)^2=\ldots$ or $(2+2t)^2+(4+3t)^2+(-3)^2=\ldots$	M1	3.1b
	$t=-\frac{3}{13}$ or $\lambda=-\frac{16}{13}\Rightarrow(C\text{ to }W)_{\min}$ is $-\frac{6}{13}\mathbf{i}+\frac{4}{13}\mathbf{j}-3\mathbf{k}$ or $(2t)^2+(3t+1)^2+(-3)^2=13\left(t+\frac{3}{13}\right)^2+\frac{121}{13}$ or $(2+2t)^2+(4+3t)^2+(-3)^2=13\left(\lambda+\frac{16}{13}\right)^2+\frac{121}{13}$ or $\frac{\mathrm{d}\left((2t)^2+(3t+1)^2+(-3)^2\right)}{\mathrm{d}t}=0\Rightarrow t=-\frac{3}{13}\Rightarrow C\text{ to }W\text{ is }-\frac{6}{13}\mathbf{i}+\frac{4}{13}\mathbf{j}-3\mathbf{k}$ Or $\frac{\mathrm{d}\left((2+2t)^2+(4+3t)^2+(-3)^2\right)}{\mathrm{d}t}=0\Rightarrow t=-\frac{16}{13}\Rightarrow(C\text{ to }W)_{\min}\text{ is }-\frac{6}{13}\mathbf{i}+\frac{4}{13}\mathbf{j}-3\mathbf{k}$	A1	1.1b
	$d=\sqrt{\left(-\frac{6}{13}\right)^2+\left(\frac{4}{13}\right)^2+(-3)^2}$ or $d=\sqrt{\frac{121}{13}}$	ddM1	1.1b

	Shortest length of pipe needed is 305 or 305 cm or 3.05 m	A1	3.2a
		(6)	
			(11 marks)

Notes

(a)

M1: Realises the scalar product between the direction of W and the normal to the road is needed and so applies it and uses trigonometry to find an angle

M1: Calculates the scalar product between $\pm\left(\begin{pmatrix}1\\2\\-3\end{pmatrix}-\begin{pmatrix}-1\\-1\\-3\end{pmatrix}\right)$ and $\pm\begin{pmatrix}3\\-5\\-18\end{pmatrix}$ (Allow sign slips as long as the intention is clear)

A1: $\begin{pmatrix}2\\3\\0\end{pmatrix}\bullet\begin{pmatrix}3\\-5\\-18\end{pmatrix}=-9$ or $\begin{pmatrix}-2\\-3\\0\end{pmatrix}\bullet\begin{pmatrix}3\\-5\\-18\end{pmatrix}=9$ or $\begin{pmatrix}2\\3\\0\end{pmatrix}\bullet\begin{pmatrix}-3\\5\\18\end{pmatrix}=9$ or $\begin{pmatrix}-2\\-3\\0\end{pmatrix}\bullet\begin{pmatrix}-3\\5\\18\end{pmatrix}=-9$

M1: A fully complete and correct method for obtaining the acute angle

A1: Awrt 7.58° or awrt 0.132 radians (**must see units**). Do not isw and withhold this mark if extra answers are given.

(b)

B1ft: Forms the correct parametric form for the pipe W. Follow through their direction vector for W from part (a).

M1: Identifies the need to and forms a vector connecting C to W using a parametric form for W

M1: Uses the model to form the scalar product of C to W and the direction of W to find the value of their parameter or finds the distance C to W or $(C \text{ to } W)^2$ in terms of their parameter

A1: Correct vector or correct completion of the square

ddM1: Correct use of Pythagoras on their vector CW or appropriate method to find the shortest distance between the point and the pipe. **Dependent on both previous method marks.**

A1: Correct length for the required section of pipe is 305 or 305 cm or 3.05 m

Alternatives for part (b):

(b) Way 2	$\mathbf{AC}=\begin{pmatrix}0\\1\\-3\end{pmatrix},\quad \mathbf{AB}=\begin{pmatrix}2\\3\\0\end{pmatrix}$	B1ft	1.1b
	$\mathbf{AC.AB}=\begin{pmatrix}0\\1\\-3\end{pmatrix}.\begin{pmatrix}2\\3\\0\end{pmatrix}=3$	M1	3.4
	$\Rightarrow \cos CAB=\dfrac{3}{\sqrt{10}\sqrt{13}} \Rightarrow CAB=...$	M1	3.1b
	$CAB=74.74....°$	A1	1.1b
	$d=\sqrt{10}\sin 74.74....°$	ddM1	1.1b
	Shortest length of pipe needed is 305 or 305 cm or 3.05 m	A1	3.2a
		(6)	

	Notes		
	(b) B1ft: Forms the correct vectors. Follow through their direction vector for *W* from part (a). M1: Identifies the need to and forms the scalar product between **AC** and **AB** M1: Uses the model to form the scalar product and uses this to find the angle *CAB* A1: Correct angle ddM1: Correct method using their values or appropriate method to find the shortest distance between the point and the pipe. **Dependent on both previous method marks.** A1: Correct length for the required section of pipe is 305 or 305 cm or 3.05 m		

(b) Way 3	$\mathbf{AC}=\begin{pmatrix}0\\1\\-3\end{pmatrix},\quad \mathbf{AB}=\begin{pmatrix}2\\3\\0\end{pmatrix}$	B1ft	1.1b
	$\mathbf{AC}\times\mathbf{AB}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\0&-1&3\\2&3&0\end{vmatrix}=\begin{pmatrix}-9\\6\\2\end{pmatrix}$	M1	3.4
	$\lvert\mathbf{AC}\times\mathbf{AB}\rvert=\sqrt{9^2+6^2+2^2}=\ldots$	M1	3.1b
	$=11$	A1	1.1b
	$d=\dfrac{11}{\lvert\mathbf{AB}\rvert}=\dfrac{11}{\sqrt{2^2+3^2}}=\ldots$	ddM1	1.1b
	Shortest length of pipe needed is 305 or 305 cm or 3.05 m	A1	3.2a
		(6)	
	Notes		
	(b) B1ft: Forms the correct vectors. Follow through their direction vector for *W* from part (a). M1: Identifies the need to and forms the vector product between **AC** and **AB** M1: Uses the model to find the magnitude of their vector product A1: Correct value ddM1: Correct method using their values or appropriate method to find the shortest distance between the point and the pipe. **Dependent on both previous method marks.** A1: Correct length for the required section of pipe is 305 or 305 cm or 3.05 m		

Q8.

Question	Scheme	Marks	AOs
(a)	$\begin{pmatrix}-1\\2\\1\end{pmatrix}.\begin{pmatrix}2\\3\\-4\end{pmatrix}=-2+6-4=0$ and $\begin{pmatrix}2\\0\\1\end{pmatrix}.\begin{pmatrix}2\\3\\-4\end{pmatrix}=4+0-4=0$ Alt: $\begin{pmatrix}-1\\2\\1\end{pmatrix}\times\begin{pmatrix}2\\0\\1\end{pmatrix}=\begin{pmatrix}2\times1-1\times0\\-(-1\times1-1\times2)\\-1\times0-2\times2\end{pmatrix}=...$	M1	1.1b
	As $2\mathbf{i}+3\mathbf{j}-4\mathbf{k}$ is perpendicular to both direction vectors (two non-parallel vectors) of Π then it must be perpendicular to Π	A1	2.2a
		(2)	
(b)	$\begin{pmatrix}x\\y\\z\end{pmatrix}.\begin{pmatrix}2\\3\\-4\end{pmatrix}=\begin{pmatrix}3\\3\\2\end{pmatrix}.\begin{pmatrix}2\\3\\-4\end{pmatrix}\Rightarrow...$	M1	1.1a
	$2x+3y-4z=7$	A1	2.2a
		(2)	
(c)	$\frac{\lvert 2(4+t)+3(-5+6t)-4(2-3t)-7\rvert}{\sqrt{2^2+3^2+(-4)^2}}=2\sqrt{29}\Rightarrow t=...$	M1	3.1a
	$t=-\frac{9}{8}$ and $t=\frac{5}{2}$	A1	1.1b
	$r=\begin{pmatrix}4\\-5\\2\end{pmatrix}-\frac{9}{8}\begin{pmatrix}1\\6\\-3\end{pmatrix}=...$ or $r=\begin{pmatrix}4\\-5\\2\end{pmatrix}+\frac{5}{2}\begin{pmatrix}1\\6\\-3\end{pmatrix}=...$	M1	1.1b
	$\left(\frac{23}{8},-\frac{47}{4},\frac{43}{8}\right)$ and $\left(\frac{13}{2},10,-\frac{11}{2}\right)$	A1	2.2a
		(4)	
			(8 marks)

Notes:

(a)
M1: Attempts the scalar product of each direction vector and the vector $2\mathbf{i}+3\mathbf{j}-4\mathbf{k}$. Some numerical calculation is required, just "= 0" is insufficient. Alternatively, attempts the cross product (allow sign slips) with the two direction vectors.
A1: Shows that both scalar products = 0 (minimum $-2+6-4=0$ and $4-4=0$) and makes a minimal conclusion with no erroneous statements. If using cross product, the calculation must be correct, and a minimal conclusion given with no erroneous statements.

(b) **M1:** Applies $\begin{pmatrix} x \\ y \\ z \end{pmatrix} . \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix} . \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} \Rightarrow ...$ **A1:** $2x + 3y - 4z = 7$
(c) **M1:** A fully correct method for finding a value of t. Other methods are possible, but must be valid and lead to a value of t. Examples of other methods: • $2\sqrt{29} = \pm\left(\frac{2(4+t)+3(-5+6t)-4(2-3t)}{\sqrt{2^2+3^2+(-4)^2}} - \frac{7}{\sqrt{29}}\right)$ using plane parallel to Π through origin and shortest distance from plane to origin. • $2(4+t)+3(-5+6t)-4(2-3t) = 7 \Rightarrow t = t_i$ (t at intersection of line and plane) and $\sin\theta = \frac{(2,3,-4)^T.(1,6,-3)^T}{\sqrt{29}\sqrt{46}}$ (sine of angle between line and plane) followed by $\sin\theta = \frac{2\sqrt{29}}{k\sqrt{46}} \Rightarrow k = ... \Rightarrow t = t_i \pm k$ **A1:** Correct values for t. Both are required. **M1:** Uses a value of t to find a set of coordinates for A. **A1:** Both correct sets of coordinates for A.

Q9.

<table>
<tr><th>Question</th><th colspan="2">Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td rowspan="4">(a)
Way 1</td><td colspan="2">$1+2\lambda=1+t$
$-1-\lambda=-t$
$4+3\lambda=3+2t$
$\Rightarrow t=\ldots \text{or } \lambda=\ldots$</td><td>M1</td><td>3.1a</td></tr>
<tr><td colspan="2">Checks the third equation with $t=2$ and $\lambda=1$
Or shows that the coordinate (3, -2, 7) lies on both lines</td><td>A1</td><td>1.1b</td></tr>
<tr><td colspan="2">As the lines intersect at a point the lines lie in the same plane.</td><td>A1</td><td>2.4</td></tr>
<tr><td colspan="2"></td><td>(3)</td><td></td></tr>
<tr><td rowspan="4">(a)
Way 2</td><td>$1=1+2\lambda+t$
$-1=\quad-\lambda-t$
$4=3+3\lambda+2t$
$\Rightarrow t=\ldots \text{or } \lambda=\ldots$</td><td>$1=1+2\lambda+t$
$0=-1-\lambda-t$
$3=4+3\lambda+2t$
$\Rightarrow t=\ldots \text{or } \lambda=\ldots$</td><td>M1</td><td>3.1a</td></tr>
<tr><td>Checks the third equation with $t=2$ and $\lambda=-1$</td><td>Checks the third equation with $t=-2$ and $\lambda=1$</td><td>A1</td><td>1.1b</td></tr>
<tr><td colspan="2">Second coordinates lie on the plane; therefore, the lines lie on the same plane</td><td>A1</td><td>2.4</td></tr>
<tr><td colspan="2"></td><td>(3)</td><td></td></tr>
<tr><td rowspan="4">(a)
Way 3</td><td colspan="2">$x=1+t,\quad y=-t,\quad z=3+2t$
$\frac{1+t-1}{2}=\frac{-t+1}{-1}=\frac{3+2t-4}{3}$
Solves a pair of equations $t=\ldots$</td><td>M1</td><td>3.1a</td></tr>
<tr><td colspan="2">Solve two pairs of equations to find $t=2$</td><td>A1</td><td>1.1b</td></tr>
<tr><td colspan="2">As the lines intersect at a point the lines lie in the same plane.</td><td>A1</td><td>2.4</td></tr>
<tr><td colspan="2"></td><td>(3)</td><td></td></tr>
</table>

(a) Way 4 (Using Further Pure 2 knowledge)	$\begin{pmatrix}2\\-1\\3\end{pmatrix}.\begin{pmatrix}x\\y\\z\end{pmatrix} \Rightarrow 2x-y+3z=0$ and $\begin{pmatrix}1\\-1\\2\end{pmatrix}.\begin{pmatrix}x\\y\\z\end{pmatrix} \Rightarrow x-y+2z=0$ attempts to solve the equations to find a normal vector OR attempts the cross product $\begin{pmatrix}2\\-1\\3\end{pmatrix}\times\begin{pmatrix}1\\-1\\2\end{pmatrix}=...$ **AND** either finds the equation of one plane **OR** finds dot product between the normal and one coordinate $\mathbf{r}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=\begin{pmatrix}1\\-1\\4\end{pmatrix}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=...$ or $\mathbf{r}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=\begin{pmatrix}1\\0\\3\end{pmatrix}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=...$ **OR** $\begin{pmatrix}1\\-1\\4\end{pmatrix}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=...$ or $\begin{pmatrix}1\\0\\3\end{pmatrix}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=...$	M1	3.1a
	Achieves the correct planes containing **each** line $\mathbf{r}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=-2$ or $x-y-z=-2$ o.e. **OR** Shows that $\begin{pmatrix}1\\-1\\4\end{pmatrix}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=-2$ **and** $\begin{pmatrix}1\\0\\3\end{pmatrix}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=-2$ o.e.	A1	1.1b
	Both planes are the same, therefore the lines lie in the same plane.	A1	2.4
		(3)	

(b)	e.g. $\mathbf{r}=\begin{pmatrix}1\\0\\3\end{pmatrix}+p\begin{pmatrix}2\\-1\\3\end{pmatrix}+q\begin{pmatrix}1\\-1\\2\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}1\\-1\\4\end{pmatrix}+p\begin{pmatrix}2\\-1\\3\end{pmatrix}+q\begin{pmatrix}1\\-1\\2\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}3\\-2\\7\end{pmatrix}+p\begin{pmatrix}2\\-1\\3\end{pmatrix}+q\begin{pmatrix}1\\-1\\2\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}3\\-2\\7\end{pmatrix}+p\begin{pmatrix}0\\-1\\1\end{pmatrix}+q\begin{pmatrix}1\\-1\\2\end{pmatrix}$ or $\mathbf{r}.k\begin{pmatrix}1\\-1\\-1\end{pmatrix}=-2k$	B1	2.5
		(1)	
(c) Way 1	$\begin{pmatrix}2\\-1\\3\end{pmatrix}\bullet\begin{pmatrix}1\\-1\\2\end{pmatrix}=2+1+6$	M1	1.1b
	$\sqrt{2^2+(-1)^2+3^2}\sqrt{1^2+(-1)^2+2^2}\cos\theta=9$ $\Rightarrow\cos\theta=\dfrac{9}{\sqrt{2^2+(-1)^2+3^2}\sqrt{1^2+(-1)^2+2^2}}$	dM1	2.1
	$\theta=11$ cao	A1	1.1b
		(3)	

Way 2 (Using Further Pure 2 knowledge)	$\begin{pmatrix}2\\-1\\3\end{pmatrix}\times\begin{pmatrix}1\\-1\\2\end{pmatrix}=\begin{pmatrix}1\\-1\\-1\end{pmatrix}$	M1	1.1b
	$\sqrt{2^2+(-1)^2+3^2}\sqrt{1^2+(-1)^2+2^2}\sin\theta=\sqrt{1^2+(-1)^2+(-1)^2}$ $\Rightarrow\sin\theta=\dfrac{\sqrt{1^2+(-1)^2+(-1)^2}}{\sqrt{2^2+(-1)^2+3^2}\sqrt{1^2+(-1)^2+2^2}}$	dM1	2.1
	$\theta=11$ cao	A1	1.1b
		(3)	
			(7 marks)

Notes

(a)

Allow using $\begin{pmatrix}1\\3\\0\end{pmatrix}$ instead of $\begin{pmatrix}1\\0\\3\end{pmatrix}$ for the method mark.

Way 1

M1: Starts by attempting to find where the two lines intersect. They must set up a parametric equation for line 1 (allow sign slips and as long as the intention is clear), forms simultaneous equations by equating coordinates and attempts to solve to find a value for $t = \ldots$ or $\lambda = \ldots$.

A1: Shows that there is a unique solution by checking the third equation or shows that the coordinate (3, -2, 7) lies on both lines.

A1: Achieves the correct values $t = 2$ and $\lambda = 1$, checks the third equation and concludes that either

- a common point,
- the lines intersect
- the equations are consistent

therefore, the lines lie in the same plane

Way 2

M1: Finds the vector equation of the plane with the both direction vectors and one coordinate (allow a sign slip), sets equal to the other coordinate, forms simultaneous equations and attempts to solve to find a value for $t = \ldots$ or $\lambda = \ldots$.

A1: Shows that the other coordinate lies on the plane by checking the third equation.

A1: Achieves the correct values $t = -2$ and $\lambda = 1$ or $t = 2$ and $\lambda = -1$ and concludes that the second coordinate lie on the plane; therefore, the lines lie on the same plane

Way 3

M1: Substitutes line 2 into line 1 and solves a pair of equations to find a value for t. Allow slip with the position of 0 and sign slips as long as the intention is clear.

A1: Solve two pairs of equations to achieve $t = 2$ for each.

A1: Achieves the correct value $t = 2$ and concludes that either

- a common point,
- the lines intersect
- the equations are consistent

therefore, the lines lie in the same plane

Way 4 (Using Further Pure 2 knowledge)
M1: A complete method to finds a vector which is normal to both lines and attempts to finds the equation of the plane containing one line.
A1: Achieves the correct equation for the plane containing each line.
A1: Conclusion, planes are the same, therefore the lines lie in the same plane.

(b) **This may be seen in part (a)**
B1: Correct **vector** equation allow any letter for the scalers.

Must start with **r** = … and uses two out of the following direction vectors $\pm\begin{pmatrix}1\\-1\\2\end{pmatrix}$, $\pm\begin{pmatrix}2\\-1\\3\end{pmatrix}$ or $\pm\begin{pmatrix}0\\-1\\1\end{pmatrix}$ and one of the following position vectors $\begin{pmatrix}1\\0\\3\end{pmatrix}$, $\begin{pmatrix}1\\-1\\4\end{pmatrix}$ or $\begin{pmatrix}3\\-2\\7\end{pmatrix}$

(c)
Way 1
M1: Calculates the scalar product between the direction vectors, allow one slip, if the intention is clear
dM1: Dependent on the previous method mark. Applies the scalar product formula with their scalar product to find a value for $\cos\theta$
A1: Correct answer only
Way 2 (Using Further Pure 2 knowledge)
M1: Calculates the vector product between the direction vectors, allow one slip, if the intention is clear
dM1: Dependent on the previous method mark. Applies the vector product formula with their vector product to find a value for $\sin\theta$
A1: Correct answer only

Q10.

Question	Scheme	Marks	AOs
(a)	**Note:** Allow alternative vector forms throughout, e.g row vectors, **i**, **j**, **k** notation $\mathbf{b} = \pm\left[\begin{pmatrix}300\\300\\-50\end{pmatrix} - \begin{pmatrix}-300\\400\\-150\end{pmatrix}\right] = \pm\begin{pmatrix}600\\-100\\100\end{pmatrix}$	M1	1.1b
	So $\mathbf{r} = \begin{pmatrix}-300\\400\\-150\end{pmatrix} + \lambda\begin{pmatrix}600\\-100\\100\end{pmatrix}$ oe $\left(\text{e.g. } \mathbf{r} = \begin{pmatrix}300\\300\\-50\end{pmatrix} + \lambda\begin{pmatrix}6\\-1\\1\end{pmatrix}\right)$	A1	2.5
		(2)	
(b)(i)	$k = 200$	B1	2.2a
	If M is the point on mountain, and X a general point on the line then eg. $\overrightarrow{MX} = \begin{pmatrix}-300\\400\\-150\end{pmatrix} + \lambda\begin{pmatrix}600\\-100\\100\end{pmatrix} - \begin{pmatrix}100\\k\\100\end{pmatrix} = \begin{pmatrix}-400+600\lambda\\400-k-100\lambda\\-250+100\lambda\end{pmatrix} = \begin{pmatrix}-400+600\lambda\\200-100\lambda\\-250+100\lambda\end{pmatrix}$ May be in terms of k or with $k = 200$ used.	M1	3.1b
	e.g. $\begin{pmatrix}-400+600\lambda\\200-100\lambda\\-250+100\lambda\end{pmatrix} \bullet \begin{pmatrix}600\\-100\\100\end{pmatrix} = 0 \Rightarrow \lambda = \ldots$	dM1	1.1b
	So e.g. $\overrightarrow{OX} = \begin{pmatrix}-300\\400\\-150\end{pmatrix} + \frac{3}{4}\begin{pmatrix}600\\-100\\100\end{pmatrix} = \ldots$	M1	3.4
	So coordinates of X are $(150, 325, -75)$ Accept as $\begin{pmatrix}150\\325\\-75\end{pmatrix}$	A1	1.1b
		(5)	
(ii)	Length of tunnel is $\sqrt{(150-100)^2 + (325-200)^2 + (-75-100)^2} = \ldots$	M1	1.1b
	Awrt 221m from correct working, so λ must have been correct. (Must include units)	A1	1.1b
		(2)	

(c)	$\lvert\overrightarrow{OP}\rvert = \sqrt{(-300)^2 + 400^2 + (-150)^2} \approx 522$ $\lvert\overrightarrow{OQ}\rvert = \sqrt{300^2 + 300^2 + 50^2} \approx 427$	M1	1.1b
	New tunnel length is signficantly shorter than these values so it is likely that the company will decide to build the accessway. Reason and conclusion needed.	A1ft	2.2b
		(2)	
(d)	E.g. The mountainside is not likely to be flat so a plane may not be a good model. The tunnel and/or pipeline will not have negligible thickness so modelling as lines may not be appropriate. A shortest length tunnel may not be possible, or most practical, as the strata of the rock in the mountain have not been considered by the model.	B1	3.5b
		(1)	
			(12 marks)

Notes		
(a)	**M1**	Attempts the direction between positions *P* and *Q*. If no method shown, two correct entries imply the method.
	A1	A correct equation in the correct form. Any point on the line may used, and any non-zero multiple of the direction. Must begin r = …
(b)		**Note:** mark part (b) as a whole.
(i)	**B1**	Correct value of k deduced.
	M1	Realises the need to find the distance from the point on the mountain to a general point on the line.
	dM1	Takes the dot product with the direction vector of line and sets to zero and proceeds to find a value of λ. If working with k as well, allow for finding either λ in terms of k or k in terms of λ.
	M1	Substitutes their λ into their line equation. (This may not have come from correct work, but the method is for using the line equation here.) May be implied by two out of three correct coordinates for their λ
		Note: May omit this step and substitute λ into $\overrightarrow{MX}$. This gains M0 here, but can gain M1A1 in (ii) for finding the length of $\overrightarrow{MX}$.
	A1	Correct point.
(b)(ii)	**M1**	Uses the distance formula with their point and *M*, or with their $\overrightarrow{MX}$ from (i). (May be implied by two out of three correct coordinates for their λ)
	A1	Correct distance, including units. Accept awrt 221 m or $25\sqrt{78}$ m
(c)	**M1**	Calculates the two distances *OP* and *OQ*.
	A1ft	Makes an appropriate conclusion for their tunnel length, but distances *OP* and *OQ* must be correct. A reason and a conclusion is needed. Accept for reason e.g "significantly shorter" or "tunnel is more than 100m less than either existing accessway", as these act as a comparative judgement. But do not accept just "shorter" or just inequalities given with no comparative evidence.
(d)	**B1**	Any appropriate criticism of the model given. The model must be referred to in some way – e.g. criticise the straightness/thickness of line, flatness of plane or lack of taking strata etc of mountain into account (as e.g this means line may not be straight). Note: reference to measurements not being correct is **NOT** a limitation of the model.

<u>**For reference**</u> Some of the other common equations/values of λ in (b)(i) are:

$$\overrightarrow{MX} = \begin{pmatrix} -300 \\ 400 \\ -150 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ -1 \\ 1 \end{pmatrix} - \begin{pmatrix} 100 \\ 200 \\ 100 \end{pmatrix} = \begin{pmatrix} -400+6\lambda \\ 200-\lambda \\ -250+\lambda \end{pmatrix} \Rightarrow \lambda = 75$$

$$\overrightarrow{MX} = \begin{pmatrix} 300 \\ 300 \\ -50 \end{pmatrix} + \lambda \begin{pmatrix} 600 \\ -100 \\ 100 \end{pmatrix} - \begin{pmatrix} 100 \\ 200 \\ 100 \end{pmatrix} = \begin{pmatrix} 200+600\lambda \\ 100-100\lambda \\ -150+100\lambda \end{pmatrix} \Rightarrow \lambda = -\frac{1}{4}$$

$$\overrightarrow{MX} = \begin{pmatrix} 300 \\ 300 \\ -50 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ -1 \\ 1 \end{pmatrix} - \begin{pmatrix} 100 \\ 200 \\ 100 \end{pmatrix} = \begin{pmatrix} 200+6\lambda \\ 100-\lambda \\ -150+\lambda \end{pmatrix} \Rightarrow \lambda = -25$$

(If the negative direction vectors are used in any case, the value of λ is just the negative of the above.)

See Appendix for some alternatives to part (b)

Q11.

Question Number	Scheme	Notes	Marks
	$l_1 : \mathbf{r} = \begin{pmatrix}4\\28\\4\end{pmatrix} + \lambda\begin{pmatrix}-1\\-5\\1\end{pmatrix}$, $l_2 : \mathbf{r} = \begin{pmatrix}5\\3\\1\end{pmatrix} + \mu\begin{pmatrix}3\\0\\-4\end{pmatrix}$; $\overrightarrow{OA} = \begin{pmatrix}2\\18\\6\end{pmatrix}$ lies on l_1	Let θ_{Acute} be the acute angle between l_1 and l_2	
(a)	$\{l_1 = l_2 \Rightarrow\}$ $28 - 5\lambda = 3$ $\{\Rightarrow \lambda = 5\}$ or $4 - \lambda = 5 + 3\mu$ and $4 + \lambda = 1 - 4\mu$ $\{\Rightarrow \mu = -2\}$	$28 - 5\lambda = 3$ or $4 - \lambda = 5 + 3\mu$ and $4 + \lambda = 1 - 4\mu$ or $\lambda = 5$ or $\mu = -2$ (Can be implied).	B1
	$\{\overrightarrow{OX} =\}$ $\begin{pmatrix}4\\28\\4\end{pmatrix} + 5\begin{pmatrix}-1\\-5\\1\end{pmatrix}$ or $\begin{pmatrix}5\\3\\1\end{pmatrix} - 2\begin{pmatrix}3\\0\\-4\end{pmatrix}$	Puts $l_1 = l_2$ and solves to find λ and/or μ **and** substitutes their value for λ into l_1 or their value for μ into l_2	M1
	So, $X(-1, 3, 9)$	$(-1, 3, 9)$ or $\begin{pmatrix}-1\\3\\9\end{pmatrix}$ or $-\mathbf{i} + 3\mathbf{j} + 9\mathbf{k}$ or condone $\begin{matrix}-1\\3\\9\end{matrix}$	A1 cao
			[3]
(b) **Way 1**	$\mathbf{d}_1 = \begin{pmatrix}-1\\-5\\1\end{pmatrix}$, $\mathbf{d}_2 = \begin{pmatrix}3\\0\\-4\end{pmatrix} \Rightarrow \begin{pmatrix}-1\\-5\\1\end{pmatrix} \bullet \begin{pmatrix}3\\0\\-4\end{pmatrix}$	Realisation that the dot product is required between $\mathbf{d}_1$ and $\mathbf{d}_2$ or a multiple of $\mathbf{d}_1$ and $\mathbf{d}_2$	M1
	$\cos\theta = \dfrac{\pm\begin{pmatrix}-1\\-5\\1\end{pmatrix} \bullet \begin{pmatrix}3\\0\\-4\end{pmatrix}}{\sqrt{(-1)^2 + (-5)^2 + (1)^2}\,.\sqrt{(3)^2 + (0)^2 + (-4)^2}}$ $\left\{= \dfrac{-7}{\sqrt{27}.\sqrt{25}}\right\}$	**dependent on the 1st M mark.** Applies dot product formula between $\mathbf{d}_1$ and $\mathbf{d}_2$ or a multiple of $\mathbf{d}_1$ and $\mathbf{d}_2$	dM1
	$\{\theta = 105.6303588... \Rightarrow\}$ $\theta_{\text{Acute}} = 74.36964117... = 74.37$ (2 dp)	awrt 74.37 seen in (b) only	A1
			[3]
(c)	$\overrightarrow{AX} = "\overrightarrow{OX}" - \overrightarrow{OA} = \begin{pmatrix}-1\\3\\9\end{pmatrix} - \begin{pmatrix}2\\18\\6\end{pmatrix} = \begin{pmatrix}-3\\-15\\3\end{pmatrix}$ or $A_{\lambda=2}, X_{\lambda=5} \Rightarrow AX = 3\lvert\mathbf{d}_1\rvert$, $\left\{\lvert\mathbf{d}_1\rvert = \sqrt{27}\right\}$		
	$AX = \sqrt{(-3)^2 + (-15)^2 + (3)^2}$ or $3\sqrt{27}$ $\left\{= \sqrt{243}\right\} = 9\sqrt{3}$	Full method for finding AX or XA	M1
		$9\sqrt{3}$ seen in (c) only	A1 cao
	Note: You cannot recover work for part (c) in either part (d) or part (e).		**[2]**
(d) **Way 1**	$\dfrac{YA}{"9\sqrt{3}"} = \tan("74.36964...")$	$\dfrac{YA}{\text{their } \lvert\overrightarrow{AX}\rvert} = \tan\theta$ or $YA = \left(\text{their } \lvert\overrightarrow{AX}\rvert\right)\tan\theta$, where θ is their acute or obtuse angle between l_1 and l_2	M1
	$YA = 55.71758... = 55.7$ (1 dp)	anything that rounds to 55.7	A1
			[2]
(e) **Way 1**	$\{A_{\lambda=2}, X_{\lambda=5} \Rightarrow$ So $AX = 2AB \Rightarrow$ So at B, $\lambda = 3.5$ or $\lambda = 0.5)\}$		
	$\overrightarrow{OB} = \begin{pmatrix}4\\28\\4\end{pmatrix} + 3.5\begin{pmatrix}-1\\-5\\1\end{pmatrix}; = \begin{pmatrix}0.5\\10.5\\7.5\end{pmatrix}$	Substitutes **either** $\lambda = \dfrac{(\text{their } \lambda_X \text{ found in } (a)) + 2}{2}$ **or** $\lambda_\beta = 3 - \dfrac{(\text{their } \lambda_X \text{ found in } (a))}{2}$ into l_1	M1;
	$\overrightarrow{OB} = \begin{pmatrix}4\\28\\4\end{pmatrix} + 0.5\begin{pmatrix}-1\\-5\\1\end{pmatrix}; = \begin{pmatrix}3.5\\25.5\\4.5\end{pmatrix}$	At least one position vector is correct. (Also allow coordinates).	A1
		Both position vectors are correct. (Also allow coordinates).	A1
			[3]
			13

Question Number	Scheme	Notes	Marks
(e) **Way 2**	$\left\{AX = 2AB \Rightarrow AB = \frac{1}{2}AX.\ \text{So, } \overrightarrow{OB} = \overrightarrow{OA} \pm \overrightarrow{AB} \Rightarrow \overrightarrow{OB} = \overrightarrow{OA} \pm \frac{1}{2}\overrightarrow{AX}\right\}$		
	$\overrightarrow{OB} = \begin{pmatrix} 2 \\ 18 \\ 6 \end{pmatrix} + 0.5\begin{pmatrix} -3 \\ -15 \\ 3 \end{pmatrix};= \begin{pmatrix} 0.5 \\ 10.5 \\ 7.5 \end{pmatrix}$	Applies either $\overrightarrow{OA} + 0.5\overrightarrow{AX}$ **or** $\overrightarrow{OA} - 0.5\overrightarrow{AX}$ where (their $\overrightarrow{AX}$) $= \pm\left[(\text{their } \overrightarrow{OX}) - \overrightarrow{OA}\right]$	M1;
	$\overrightarrow{OB} = \begin{pmatrix} 2 \\ 18 \\ 6 \end{pmatrix} - 0.5\begin{pmatrix} -3 \\ -15 \\ 3 \end{pmatrix};= \begin{pmatrix} 3.5 \\ 25.5 \\ 4.5 \end{pmatrix}$	At least one position vector is correct (Also allow coordinates)	A1
		Both position vectors are correct (Also allow coordinates)	A1
			[3]
(e) **Way 3**	$\overrightarrow{AB} = \begin{pmatrix} 4-\lambda \\ 28-5\lambda \\ 4+\lambda \end{pmatrix} - \begin{pmatrix} 2 \\ 18 \\ 6 \end{pmatrix} = \begin{pmatrix} 2-\lambda \\ 10-5\lambda \\ -2+\lambda \end{pmatrix} = \begin{pmatrix} 1(2-\lambda) \\ 5(2-\lambda) \\ -1(2-\lambda) \end{pmatrix};\ \overrightarrow{AX} = \begin{pmatrix} -3 \\ -15 \\ 3 \end{pmatrix}$ $\quad AX^2 = 243 \Rightarrow AB^2 = 27(2-\lambda)^2$ $AX = 2AB \Rightarrow AX^2 = 4AB^2 \Rightarrow 243 = 4(27)(2-\lambda)^2 \Rightarrow (2-\lambda)^2 = \frac{9}{4}$ **or** $27\lambda^2 - 108\lambda + \frac{189}{4} = 0$ **or** $108\lambda^2 - 432\lambda + 189 = 0$ **or** $4\lambda^2 - 16\lambda + 7 = 0 \Rightarrow \lambda = 3.5$ or $\lambda = 0.5$		
	$\overrightarrow{OB} = \begin{pmatrix} 4 \\ 28 \\ 4 \end{pmatrix} + 3.5\begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix};= \begin{pmatrix} 0.5 \\ 10.5 \\ 7.5 \end{pmatrix}$	Full method of solving for λ the equation $AX^2 = 4AB^2$ using (their $\overrightarrow{AX}$) and $\overrightarrow{AB}$ **and** substitutes at least one of their values for λ into l_1	M1;
	$\overrightarrow{OB} = \begin{pmatrix} 4 \\ 28 \\ 4 \end{pmatrix} + 0.5\begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix};= \begin{pmatrix} 3.5 \\ 25.5 \\ 4.5 \end{pmatrix}$	At least one position vector is correct (Also allow coordinates)	A1
		Both position vectors are correct (Also allow coordinates)	A1
	Note: $AX = 2AB \Rightarrow \overrightarrow{AX} = \pm 2\overrightarrow{AB}$. Hence, $\lambda = 3.5$ or $\lambda = 0.5$ can be found from solving either $x: -3 = \pm 2(2-\lambda)$ or $y: -15 = \pm 2(10-5\lambda)$ or $z: -3 = \pm 2(-2+\lambda)$		**[3]**

(e) **Way 4**	$\overrightarrow{OB} = \begin{pmatrix} -1 \\ 3 \\ 9 \end{pmatrix} + 0.5\begin{pmatrix} 3 \\ 15 \\ -3 \end{pmatrix};= \begin{pmatrix} 0.5 \\ 10.5 \\ 7.5 \end{pmatrix}$	Applies **either** (their $\overrightarrow{OX}$) $+ 0.5\overrightarrow{XA}$ **or** (their $\overrightarrow{OX}$) $+ 1.5\overrightarrow{XA}$ where (their $\overrightarrow{XA}$) $= \overrightarrow{OA} -$ (their $\overrightarrow{OX}$)	M1;
	$\overrightarrow{OB} = \begin{pmatrix} -1 \\ 3 \\ 9 \end{pmatrix} + 1.5\begin{pmatrix} 3 \\ 15 \\ -3 \end{pmatrix};= \begin{pmatrix} 3.5 \\ 25.5 \\ 4.5 \end{pmatrix}$	At least one position vector is correct (Also allow coordinates)	A1
		Both position vectors are correct (Also allow coordinates)	A1
			[3]
(e) **Way 5**	$\overrightarrow{OB} = 0.5\left(\begin{pmatrix} -1 \\ 3 \\ 9 \end{pmatrix} + \begin{pmatrix} 2 \\ 18 \\ 6 \end{pmatrix}\right);= \begin{pmatrix} 0.5 \\ 10.5 \\ 7.5 \end{pmatrix}$	Applies $\frac{1}{2}\left[(\text{their } \overrightarrow{OX}) + \overrightarrow{OA}\right]$	M1;
	$\overrightarrow{OB} = \begin{pmatrix} 2 \\ 18 \\ 6 \end{pmatrix} - 0.5\begin{pmatrix} -3 \\ -15 \\ 3 \end{pmatrix};= \begin{pmatrix} 3.5 \\ 25.5 \\ 4.5 \end{pmatrix}$	At least one position vector is correct (Also allow coordinates)	A1
		Both position vectors are correct (Also allow coordinates)	A1
			[3]

Question Number	Scheme	Notes	Marks
(e) **Way 6**	$\left\{\lvert\overrightarrow{AX}\rvert = 9\sqrt{3}, \lvert d_1\rvert = 3\sqrt{3} \Rightarrow K = \dfrac{9\sqrt{3}}{3\sqrt{3}} = 3 \Rightarrow \overrightarrow{AX} = 3\mathbf{d}_1; \text{So, } \overrightarrow{OB} = \overrightarrow{OA} \pm \tfrac{1}{2}\overrightarrow{AX} = \overrightarrow{OA} \pm \tfrac{1}{2}(3\mathbf{d}_1)\right\}$		
	$\overrightarrow{OB} = \begin{pmatrix}2\\18\\6\end{pmatrix} + 0.5\left(3\begin{pmatrix}-1\\-5\\1\end{pmatrix}\right); = \begin{pmatrix}0.5\\10.5\\7.5\end{pmatrix}$	Applies either $\overrightarrow{OA} + 0.5(K\mathbf{d}_1)$ **or** $\overrightarrow{OA} - 0.5(K\mathbf{d}_1)$, where $K = \dfrac{\text{their } \lvert\overrightarrow{AX}\rvert}{3\sqrt{3}}$	M1;
	$\overrightarrow{OB} = \begin{pmatrix}2\\18\\6\end{pmatrix} - 0.5\left(3\begin{pmatrix}-1\\-5\\1\end{pmatrix}\right); = \begin{pmatrix}3.5\\25.5\\4.5\end{pmatrix}$	At least one position vector is correct (Also allow coordinates)	A1
		Both position vectors are correct (Also allow coordinates)	A1
			[3]

		Question Notes
(a)	**Note**	M1 can be implied by at least two correct follow through coordinates from their λ or from their μ
(b)	**Note**	***Evaluating*** the dot product (i.e. $(-1)(3) + (-5)(0) + (1)(-4)$) is not required for the M1, dM1 marks.
	Note	**For M1 dM1:** Allow one slip in writing down their direction vectors, $\mathbf{d}_1$ and $\mathbf{d}_2$
	Note	Allow M1 dM1 for $\left(\sqrt{(-1)^2+(-5)^2+(1)^2}\,.\sqrt{(3)^2+(0)^2+(-4)^2}\right)\cos\theta = \pm\begin{pmatrix}-1\\-5\\1\end{pmatrix}\bullet\begin{pmatrix}3\\0\\-4\end{pmatrix}$
	Note	$\theta = 1.297995...^c$, (without evidence of awrt 74.37) is A0

(b) **Way 2**	**Alternative Method: Vector Cross Product**		
	Only apply this scheme if it is clear that a vector cross product method is being applied.		
	$\mathbf{d}_1 \times \mathbf{d}_2 = \begin{pmatrix}-1\\-5\\1\end{pmatrix} \times \begin{pmatrix}3\\0\\-4\end{pmatrix} = \left\{\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ -1 & -5 & 1\\ 3 & 0 & -4\end{vmatrix} = 20\mathbf{i} - \mathbf{j} + 15\mathbf{k}\right\}$	Realisation that the vector cross product is required between $\mathbf{d}_1$ and $\mathbf{d}_2$ or a multiple of $\mathbf{d}_1$ and $\mathbf{d}_2$	M1
	$\sin\theta = \dfrac{\sqrt{(20)^2+(-1)^2+(15)^2}}{\sqrt{(-1)^2+(-5)^2+(1)^2}\,.\sqrt{(3)^2+(0)^2+(-4)^2}}$	Applies the vector product formula between $\mathbf{d}_1$ and $\mathbf{d}_2$ or a multiple of $\mathbf{d}_1$ and $\mathbf{d}_2$	dM1
	$\sin\theta = \dfrac{\sqrt{626}}{\sqrt{27}.\sqrt{25}} \Rightarrow \theta = 74.36964117... = 74.37$ (2 dp)	awrt 74.37 seen in (b) only	A1
			[3]
(c)	**M1**	Finds the difference between their $\overrightarrow{OX}$ and $\overrightarrow{OA}$ and applies Pythagoras to the result to find AX or XA **OR** applies $\lvert(\text{their } \lambda_X \text{ found in } (a)) - 2\rvert.\sqrt{(-1)^2+(-5)^2+(1)^2}$	
	Note	For M1: Allow one slip in writing down their $\overrightarrow{OX}$ and $\overrightarrow{OA}$	
	Note	Allow M1A1 for $\begin{pmatrix}3\\15\\3\end{pmatrix}$ leading to $AX = \sqrt{(3)^2+(15)^2+(3)^2} = \sqrt{243} = 9\sqrt{3}$	
(e)	**Note**	Imply M1 for no working leading to any two components of one of the $\overrightarrow{OB}$ which are correct.	

Question Number	Scheme	Notes	Marks
(d) Way 2	$\dfrac{"9\sqrt{3}"}{YA} = \tan(90 - "74.36964...")$	$\dfrac{\text{their } \lvert \overrightarrow{AX} \rvert}{YA} = \tan(90-\theta)$ or $AY = \dfrac{\text{their } \lvert \overrightarrow{AX} \rvert}{\tan(90-\theta)}$, where θ is the acute or obtuse angle between l_1 and l_2	M1
	$YA = 55.71758... = 55.7$ (1 dp)	anything that rounds to 55.7	A1
			[2]
(d) Way 3	$\dfrac{YA}{\sin("74.36964...")} = \dfrac{"9\sqrt{3}"}{\sin(90 - "74.36964...")}$	$\dfrac{YA}{\sin\theta} = \dfrac{\text{their } \lvert \overrightarrow{AX} \rvert}{\sin(90-\theta)}$ o.e., where θ is the acute or obtuse angle between l_1 and l_2	M1
	$YA = \dfrac{9\sqrt{3}\sin(74.36964...)}{\sin(15.63036...)} = 55.71758... = 55.7$ (1 dp)	anything that rounds to 55.7	A1
			[2]
(d) Way 4	$\mathbf{d}_1 = \begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix}$, $\overrightarrow{OY} = \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix} = \begin{pmatrix} 5+3\mu \\ 3 \\ 1-4\mu \end{pmatrix}$		
	$\overrightarrow{YA} = \begin{pmatrix} 2 \\ 18 \\ 6 \end{pmatrix} - \begin{pmatrix} 5+3\mu \\ 3 \\ 1-4\mu \end{pmatrix} = \begin{pmatrix} -3-3\mu \\ 15 \\ 5+4\mu \end{pmatrix}$ $\overrightarrow{YA} \bullet \mathbf{d}_1 = 0 \Rightarrow \begin{pmatrix} -3-3\mu \\ 15 \\ 5+4\mu \end{pmatrix} \bullet \begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix} = 0$ $\Rightarrow 3 + 3\mu - 75 + 5 + 4\mu = 0 \Rightarrow \mu = \dfrac{67}{7}$ $YA^2 = \left(-3 - 3\left(\dfrac{67}{7}\right)\right)^2 + (15)^2 + \left(5 + 4\left(\dfrac{67}{7}\right)\right)^2$	(Allow a sign slip in copying $\mathbf{d}_1$) Applies $\overrightarrow{YA} \bullet \mathbf{d}_1 = 0$ or $\overrightarrow{AY} \bullet \mathbf{d}_1 = 0$ or $\overrightarrow{YA} \bullet (K\mathbf{d}_1) = 0$ or $\overrightarrow{AY} \bullet (K\mathbf{d}_1) = 0$ to find μ and applies Pythagoras to find a numerical expression for AY^2 or for the distance AY	M1
	So, $YA = \sqrt{\left(-\dfrac{222}{7}\right)^2 + (15)^2 + \left(\dfrac{303}{7}\right)^2}$		
	$= 55.71758... = 55.7$ (1 dp)	anything that rounds to 55.7	A1
	Note: $\overrightarrow{OY} = \dfrac{236}{7}\mathbf{i} + 3\mathbf{j} - \dfrac{261}{7}\mathbf{k}$, $\overrightarrow{AY} = -\dfrac{222}{7}\mathbf{i} + 15\mathbf{j} + \dfrac{303}{7}\mathbf{k}$		[2]