Dynamics

Questions

Q1.

A light elastic spring has natural length l and modulus of elasticity $4mg$. A particle P of mass m is attached to one end of the spring. The other end of the spring is attached to a fixed point A. The point B is vertically below A with $AB = \frac{7}{4}l$. The particle P is released from rest at B.

(a) Show that P moves with simple harmonic motion with period $\pi\sqrt{\frac{l}{g}}$

(7)

(b) Find, in terms of m, l and g, the maximum kinetic energy of P during the motion.

(3)

(c) Find the time within each complete oscillation for which the length of the spring is less than l.

(5)

(Total for question = 15 marks)

Q2.

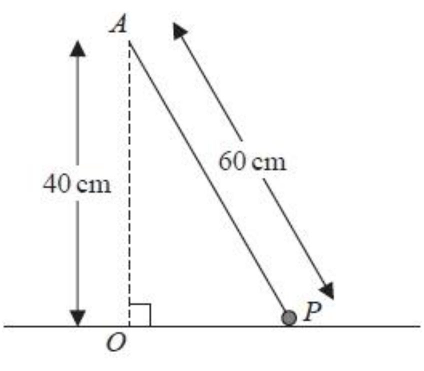


Figure 4

A particle P of mass 0.75 kg is attached to one end of a light inextensible string of length 60 cm. The other end of the string is attached to a fixed point A that is vertically above the point O on a smooth horizontal table, such that OA = 40 cm. The particle remains in contact with the table, with the string taut, and moves in a horizontal circle with centre O, as shown in Figure 4.

The particle is moving with a constant angular speed of 3 radians per second.

(a) Find

(i) the tension in the string,
(ii) the normal reaction between P and the table.

(7)

The angular speed of P is now gradually increased.

(b) Find the angular speed of P at the instant P loses contact with the table.

(4)

(Total for question = 11 marks)

Q3.

A particle P of mass 0.5 kg is moving along the positive x-axis in the direction of x increasing. At time t seconds ($t \geq 0$), P is x metres from the origin O and the speed of P is v m s^{-1}. The resultant force acting on P is directed towards O and has magnitude kv^2 N, where k is a positive constant.

When $x = 1$, $v = 4$ and when $x = 2$, $v = 2$

(a) Show that $v = ab^x$, where a and b are constants to be found.

(6)

The time taken for the speed of P to decrease from 4 m s^{-1} to 2 m s^{-1} is T seconds.

(b) Show that $T = \dfrac{1}{4\ln 2}$

(4)

(Total for question = 10 marks)

Q4.

At time $t = 0$, a small stone P of mass m is released from rest and falls vertically through the air. At time t, the speed of P is v and the resistance to the motion of P from the air is modelled as a force of magnitude kv^2, where k is a constant.

(a) Show that $t = \frac{V}{2g}\ln\left(\frac{V+v}{V-v}\right)$ where $V^2 = \frac{mg}{k}$

(4)

(b) Give an interpretation of the value of V, justifying your answer.

(2)

At time t, P has fallen a distance s.

(c) Show that $s = \frac{V^2}{2g}\ln\left(\frac{V^2}{V^2 - v^2}\right)$

(4)

(Total for question = 10 marks)

Q5.

A cyclist and her cycle have a combined mass of 60 kg. The cyclist is moving along a straight horizontal road and is working at a constant rate of 200 W.

When she has travelled a distance x metres, her speed is v m s^{-1} and the magnitude of the resistance to motion is $3v^2$N.

(a) Show that $\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{200 - 3v^3}{60v^2}$

(4)

The distance travelled by the cyclist as her speed increases from 2 m s^{-1} to 4 m s^{-1} is D metres.

(b) Find the exact value of D

(3)

(Total for question = 7 marks)

Q6.

A particle, P, of mass 0.4 kg is moving along the positive x-axis, in the positive x direction under the action of a single force. At time t seconds, $t > 0$, P is x metres from the origin O and the speed of P is v m s^{-1}. The force is acting in the direction of x increasing and has

magnitude $\frac{k}{v}$ newtons, where k is a constant.

At $x = 3$, $v = 2$ and at $x = 6$, $v = 2.5$

Show that $v^3 = \frac{61x + 9}{24}$

(6)

The time taken for the speed of P to increase from 2 m s^{-1} to 2.5 m s^{-1} is T seconds.

(b) Use algebraic integration to show that $T = \frac{81}{61}$

(4)

(Total for question = 10 marks)

Q7.

A cyclist and her bicycle have a combined mass of 75 kg. The cyclist travels along a straight horizontal road. The cyclist produces a constant driving force of magnitude 150N. At time t seconds, the speed of the cyclist is $v$ms^{-1}, where $v < \sqrt{50}$. As the cyclist moves, the total resistance to motion of the cyclist and her bicycle has magnitude $3v^2$ newtons. The cyclist starts from rest. At time t seconds, she has travelled a distance x metres from her starting point.

Find

(a) v in terms of x,

(7)

(b) t in terms of v.

(5)

(Total for question = 12 marks)

Q8.

Two points A and B are 6 m apart on a smooth horizontal surface.

A light elastic string of natural length 2 m and modulus of elasticity 20 N, has one end attached to the point A.

A second light elastic string of natural length 2 m and modulus of elasticity 50 N, has one end attached to the point B.

A particle P of mass 3.5 kg is attached to the free end of each string.

The particle P is held at the point on AB which is 2 m from B and then released from rest.

In the subsequent motion both strings remain taut.

(a) Show that P moves with simple harmonic motion about its equilibrium position.

(7)

(b) Find the maximum speed of P.

(2)

(c) Find the length of time within each oscillation for which P is closer to A than to B.

(5)

(Total for question = 14 marks)

Q9.

The points A and B lie on a smooth horizontal surface with AB = 4.5 m.

A light elastic string has natural length 1.5 m and modulus of elasticity 15 N. One end of the string is attached to A and the other end of the string is attached to B. A particle, P, of mass 0.2 kg, is attached to the stretched string so that APB is a straight line and AP = 1.5 m. The particle rests in equilibrium on the surface.

The particle is now moved directly towards A and is held on the surface so APB is a straight line with AP = 1 m.

The particle is released from rest.

(a) Prove that P moves with simple harmonic motion.

(5)

(b) Find

(i) the maximum speed of P during the motion,
(ii) the maximum acceleration of P during the motion.

(3)

(c) Find the total time, in each complete oscillation of P, for which the speed of P is greater than 5 m s^{-1}.

(5)

(Total for question = 13 marks)

Q10.

A light elastic string, of natural length *l* and modulus of elasticity 2*mg*, has one end attached to a fixed point *A* and the other end attached to a particle *P* of mass *m*. The particle *P* hangs in equilibrium at the point *O*.

(a) Show that $AO = \frac{3l}{2}$

(2)

The particle *P* is pulled down vertically from *O* to the point *B*, where *OB* = *l*, and released from rest.

Air resistance is modelled as being negligible.

Using the model,

(b) prove that *P* begins to move with simple harmonic motion about *O* with period $\pi\sqrt{\frac{2l}{g}}$

(5)

The particle *P* first comes to instantaneous rest at the point *C*.

Using the model,

(c) find the length *BC* in terms of *l*,

(4)

(d) find, in terms of *l* and *g*, the exact time it takes *P* to move directly from *B* to *C*.

(5)

(Total for question = 16 marks)

Q11.

Throughout this question, use $g = 10$ m s^{-2}

A light elastic string has natural length 1.25 m and modulus of elasticity 25 N.

A particle *P* of mass 0.5 kg is attached to one end of the string. The other end of the string is attached to a fixed point *A*. Particle *P* hangs freely in equilibrium with *P* vertically below *A*

The particle is then pulled vertically down to a point *B* and released from rest.

(a) Show that, while the string is taut, *P* moves with simple harmonic motion with period $\frac{\pi}{\sqrt{10}}$ seconds.

(6)

The maximum kinetic energy of *P* during the subsequent motion is 2.5 J.

(b) Show that *AB* = 2 m

(3)

The particle returns to *B* for the first time *T* seconds after it was released from rest at *B*

(c) Find the value of *T*

(5)

(Total for question = 14 marks)

Q12.

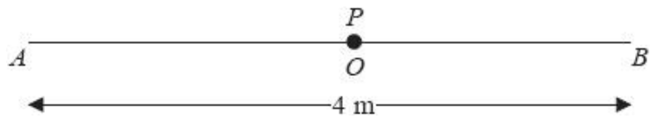


Figure 6

The fixed points *A* and *B* are 4 m apart on a smooth horizontal floor. One end of a light elastic string, of natural length 1.8 m and modulus of elasticity 45 N, is attached to a particle *P* and the other end is attached to *A*. One end of another light elastic string, of natural length 1.2 m and modulus of elasticity 20 N, is attached to *P* and the other end is attached to *B*. The particle *P* rests in equilibrium at the point *O*, where *AOB* is a straight line, as shown in Figure 6.

(a) Show that *AO* = 2.2 m.

(4)

The point *C* lies on the straight line *AOB* with *AC* = 2.7 m. The mass of *P* is 0.6 kg. The particle *P* is held at *C* and then released from rest.

(b) Show that, while both strings are taut, *P* moves with simple harmonic motion with centre *O*.

(5)

The point *D* lies on the straight line *AOB* with *AD* = 1.8 m. When *P* reaches *D* the string *PB* breaks.

(c) Find the time taken by *P* to move directly from *C* to *A*.

(8)

(Total for question = 17 marks)

Q13.

A particle *P* of mass *m* kg is initially held at rest at the point *O* on a smooth plane which is inclined at 30° to the horizontal. The particle is released from rest and slides down the plane against a force of magnitude $\frac{1}{2}mx^2$ newtons acting towards *O*, where *x* metres is the distance of *P* from *O*.

(a) Find the speed of *P* when $x = 3$

(7)

(b) Find the distance *P* has moved when it first comes to instantaneous rest.

(2)

(Total for question = 9 marks)

Mark Scheme - Dynamics

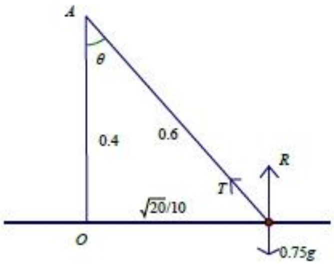
Q1.

Question	Scheme	Marks	AOs
(a)	Equation of motion about equilibrium position:	M1	3.1a
	$$\frac{4mg(x+e)}{l} - mg = -m\ddot{x}$$	A1	1.1b
	Extension e at equilibrium: $\frac{4mge}{l} = mg$, $\left(e = \frac{l}{4}\right)$	B1	1.1b
	$$\Rightarrow \frac{4gx}{l} = -\ddot{x},\ \left(\ddot{x} = -\frac{4g}{l}x\right)$$	M1	3.1a
	This is of the form $\ddot{x} = -\omega^2 x$, so SHM *	A1*	3.2a
	Period $= \frac{2\pi}{\omega}$	M1	3.4
	$= 2\pi\sqrt{\frac{l}{4g}} = \pi\sqrt{\frac{l}{g}}$ *	A1*	2.2a
		(7)	
(a) alt	Equation of motion for extension x:	M1	3.1a
	$$\frac{4mgx}{l} - mg = -m\ddot{x},\ \ddot{x} = -\frac{4g}{l}\left(x - \frac{l}{4}\right)$$	A1	1.1b
	Use substitution $X = x - \frac{l}{4}$	B1	1.1b
	$$\Rightarrow \frac{4gX}{l} = -\ddot{X},\ \left(\ddot{X} = -\frac{4g}{l}X\right)$$	M1	3.1a
	This is of the form $\ddot{X} = -\omega^2 X$, so SHM *	A1*	3.2a
	Period $= \frac{2\pi}{\omega}$	M1	3.4
	$= 2\pi\sqrt{\frac{l}{4g}} = \pi\sqrt{\frac{l}{g}}$ *	A1*	2.2a
		(7)	

Question	Scheme	Marks	AOs
(b)	Max speed $= a\omega \left(= \frac{l}{2}\sqrt{\frac{4g}{l}} \right)$	M1	3.4
	Max KE $= \frac{1}{2}m\left(\frac{l}{2}\sqrt{\frac{4g}{l}}\right)^2$	M1	1.2
	$= \frac{1}{2}m\frac{l^2}{4}\times\frac{4g}{l} = \frac{1}{2}mlg$	A1	1.1b
		(3)	
(c)	$x = a\cos\omega t = \frac{l}{2}\cos\sqrt{\frac{4g}{l}}t$	B1ft	2.2a
	Length of spring $< l \Rightarrow x = -\frac{l}{4}$, $-\frac{l}{4} = \frac{l}{2}\cos\sqrt{\frac{4g}{l}}t$	M1	1.1b
	$\Rightarrow \sqrt{\frac{4g}{l}}t = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$, $t = \frac{\pi}{3}\sqrt{\frac{l}{g}}$ or $t = \frac{2\pi}{3}\sqrt{\frac{l}{g}}$	A1	1.1b
	Correct strategy	M1	3.1a
	Length of time $= \frac{2\pi}{3}\sqrt{\frac{l}{g}} - \frac{\pi}{3}\sqrt{\frac{l}{g}} = \frac{\pi}{3}\sqrt{\frac{l}{g}}$	A1	2.2a
		(5)	
			(15 marks)

Notes:		
(a)	M1	Equation of motion about equilibrium position. Need all terms. Dimensionally correct. Allow with their $e \neq 0$. Condone sign errors.
	A1ft	Correct unsimplified equation with e or their $e \neq 0$
	B1	Correct e
	M1	Complete strategy e.g. use equation of motion and equilibrium position to form equation in x.
	A1*	Reach given conclusion from correct working
	M1	Use the model to find periodic time (their ω)
	A1*	Obtain given answer from correct working
(b)	M1	Use the model to find the max speed. Follow their ω
	M1	Follow their a, ω
	A1	Correct simplified
(c)	B1	Or equivalent. Follow their a, ω
	M1	Follow their e and solve for t
	A1	One correct solution. Accept $t = \frac{2\pi}{3\omega}$, or $t = \frac{4\pi}{3\omega}$
	M1	Complete strategy to find the required interval: select formula for displacement as function of time and use symmetry of motion to find the time interval.
	A1	Correct answer from correct working

Q2.

Question	Scheme	Marks	AOs
(a)			
	Resolve vertically	M1	3.4
	$\updownarrow 0.75g = T\cos\theta + R$ $\left(\frac{3g}{4} = \frac{2}{3}T + R\right)$	A1	1.1b
	Equation of motion	M1	3.4
	$\leftrightarrow 0.75 \times \sin\theta \times 9 = T\sin\theta$ $\left(0.75 \times \frac{\sqrt{20}}{10} \times 9 = T \times \frac{\sqrt{20}}{6}\right)$	A1	1.1b
	Complete strategy to find T and R	M1	3.1a
	$T = \frac{6 \times 0.75 \times 9}{10} = 4.05$ (N)	A1	1.1b
	$R = 0.75g - \frac{2}{3}T = 4.65$ (N) or 4.7 (N)	A1	1.1b
		(7)	

(b)	Use $R = 0$ to form revised equations	M1	3.4
	$T\cos\theta = 0.75g$, $T\sin\theta = 0.75 \times \frac{10\sqrt{20}}{100}\omega^2$ $\left(\text{or } T\sin\theta = 0.75 \times 0.6\sin\theta \times \omega^2\right)$	A1	1.1b
	Complete strategy to find ω e.g. $\Rightarrow \tan\theta = \frac{\sqrt{20}\omega^2}{10g} = \frac{\sqrt{20}}{4}$	M1	1.1b
	$\omega = \sqrt{\frac{5g}{2}} = 4.95$ (rad/s)	A1	1.1b
		(4)	
			(11 marks)

Notes:		
(a)	M1	Correct number of terms
	A1	Correct unsimplified equation
	M1	Circular motion. Condone confusion over units. $\frac{\sqrt{20}}{10}$ might not be seen as r cancels.
	A1	Correct unsimplified equation
	M1	Complete strategy to form sufficient equations to solve for T and R.
	A1	One force correct
	A1	Both correct (Finding value for R involves g)
(b)	M1	Correct interpretation of loss of contact
	A1	Revised equations
	M1	Solve for ω
	A1	Exact, 4.9 or 4.95 (non-exact answer requires substitution for g).

Q3.

Question	Scheme	Marks	AOs
(a)	Form differential equation: $0.5a = 0.5v\frac{\mathrm{d}v}{\mathrm{d}x} = -kv^2$	M1	2.5
	$\Rightarrow \int \frac{1}{2v}\mathrm{d}v = \int -k\mathrm{d}x$	M1	2.1
	$\frac{1}{2}\ln v = -kx + C$	A1	1.1b
	$x=1,\ v=4 \quad \frac{1}{2}\ln 4 = -k + C$ $x=2,\ v=2 \quad \frac{1}{2}\ln 2 = -2k + C$	M1	3.1a
	$\Rightarrow k = \frac{1}{2}(\ln 4 - \ln 2) = \frac{1}{2}\ln 2\ ,\ C = \frac{1}{2}\ln 8\ :\ \ln v = -x\ln 2 + \ln 8$	A1	1.1b
	$\ln v = x\ln\frac{1}{2} + \ln 8,\ \ v = 8\times\left(\frac{1}{2}\right)^x$	A1	2.2a
	$\left(a = 8,\ b = \frac{1}{2}\right)$		
		(6)	

(b)	$0.5\frac{dv}{dt} = -kv^2$ (follow their k)	M1	2.5
	$\int \frac{1}{v^2} dv = \int -\ln 2 dt \quad \Rightarrow -\frac{1}{v} + C' = -t\ln 2$	M1	2.1
	$\Rightarrow \left[-\frac{1}{v}\right]_4^2 = \left[-t\ln 2\right]_0^T$	M1	1.1b
	$-\frac{1}{2} + \frac{1}{4} = -T\ln 2\ , \quad T = \frac{1}{4\ln 2}$ *	A1*	2.2a
		(4)	
(b) alt	$v = \frac{8}{2^x} = \frac{dx}{dt}$ (follow their v)	M1	2.5
	$\int 2^x dx = \int 8 dt \quad \Rightarrow \frac{2^x}{\ln 2} = 8t + C'$	M1	2.1
	$\left[\frac{2^x}{\ln 2}\right]_1^2 = \left[8t\right]_0^T$	M1	1.1b
	$\frac{1}{\ln 2}(4-2) = 8T, \quad T = \frac{1}{4\ln 2}$ *	A1*	2.2a
		(4)	
			(10 marks)

Notes:		
(a)	M1	Form differential equation in v and x. Condone sign error
	M1	Separate and integrate to form equation in v and x. Condone missing constant of integration
	A1	Any equivalent form. Condone missing constant of integration.
	M1	Complete strategy to use the differential equation and boundary conditions to find v
	A1	Correct expression in v and x in any form. Accept $\ln v = -0.693..x + 2.079...$
	A1	Expression in the required form. Do not need to see a separate statement of the values of a and b.
		If mass is omitted from the differential equation can score M0M1A1M1A1A0
(b)	M1	Differential equation in v and t (in x and t for alternative solution)
	M1	Separate and integrate
	M1	Use limits on a definite integral or to find value of C'
	A1*	Obtain given result from correct working
		If mass is omitted from the differential equation can score M0M1M1A0

Q4.

Question	Scheme	Marks	AOs
(a)	$mg - kv^2 = m\frac{dv}{dt}$	M1	2.5
	Separate variables and integrate	M1	2.1
	A correct equation in any form (ignore constant or limits) e.g. $t = \frac{m}{k}\frac{1}{2\sqrt{\frac{mg}{k}}}\ln\left(\frac{\sqrt{\frac{mg}{k}}+v}{\sqrt{\frac{mg}{k}}-v}\right) \quad (+C)$	A1	1.1b
	$t = \frac{V}{2g}\ln\left(\frac{V+v}{V-v}\right)$ where $V^2 = \frac{mg}{k}$ *	A1*	2.2a
		(4)	
(b)	$V^2 = \frac{mg}{k} \Rightarrow kV^2 = mg$ i.e. resistance = weight **OR** using answer to (a): As $t \to \infty,\ v \to V$ from below	B1	1.1b
	Hence V is the terminal velocity of the stone oe	B1	2.4
		(2)	
(c)	$mg - kv^2 = mv\frac{dv}{ds}$	M1	2.5
	Separate variables and integrate	M1	2.1
	$s = -\frac{m}{2k}\ln\left(\frac{mg}{k} - v^2\right) \quad (+D)$	A1	1.1b
	$s = \frac{V^2}{2g}\ln\left(\frac{V^2}{V^2 - v^2}\right)$ *	A1*	2.2a
		(4)	
		(10 marks)	

Notes:		
a	M1	Equation of motion with correct form for the acceleration
	M1	Separate the variables and integrate ('standard integral')
	A1	Correct equation in any form (ignoring constant or limits)
	A1*	Correctly obtain the printed answer including dealing with constant or limits
b	B1	Correctly rearrange and interpret OR correctly argue and interpret
	B1	Correct statement or equivalent
c	M1	Equation of motion with correct form for the acceleration
	M1	Separate the variables and integrate ('standard integral')
	A1	Correct equation in any form (ignoring constant or limits)
	A1*	Correctly obtain the printed answer including dealing with constant or limits

Q5.

Question	Scheme	Marks	AOs
(a)	Use of $P = Fv$	B1	3.3
	Equation of motion $\left(F - 3v^2 = 60a\right)$	M1	2.1
	$\frac{200}{v} - 3v^2 = 60v\frac{dv}{dx}$	A1	2.5
	$\frac{dv}{dx} = \frac{200-3v^3}{60v^2}$ *	A1*	2.2a
		(4)	
2(b)	$\Rightarrow \int \frac{60v^2}{200-3v^3}\,dv = \int 1\,dx \quad \left(-\frac{60}{9}\ln\left(200-3v^3\right) = x(+C)\right)$	M1	1.1b
	$D = \left[-\frac{60}{9}\ln\left(200-3v^3\right)\right]_2^4 = -\frac{60}{9}\ln\left(\frac{200-3\times 64}{200-3\times 8}\right)$	M1	1.1b
	$= \frac{60}{9}\ln\frac{176}{8} = \frac{60}{9}\ln 22$	A1	1.1b
		(3)	
		(7)	
			(7 marks)

Notes:	
(a)	
B1	Seen or implied Not just quoted. Need at least $200 = Fv$ Could be on its own, in an equation or on a diagram
M1	Form equation of motion. Need all terms and dimensionally correct. Condone any correct form for acceleration and sign errors Allow with m not substituted
A1	Correct equation - any equivalent form with correct acceleration
A1*	Obtain given answer from correct working Must be as written in the question but could swap LHS and RHS
(b)	
M1	Separate variables and integrate to obtain $(x =)k\ln(\ldots\ldots)$ (Constant of integration not required) Condone if the x is not explicitly stated but M0 if it is an incorrect function.
M1	Use limits correctly in an expression containing $k\ln\left(200-3v^3\right)$ to find D

	Substitute and subtract in the correct order
A1	Obtain **exact** answer from correct working Any equivalent single term No working seen is Max M1M0A0

Q6.

Question	Scheme	Marks	AOs
(a)	Strategy to find v^3 in terms of x	M1	3.1a
	Differential equation in v and x: $0.4v\dfrac{\mathrm{d}v}{\mathrm{d}x}=\dfrac{k}{v}$	M1	2.1
	$\Rightarrow 0.4v^2\dfrac{\mathrm{d}v}{\mathrm{d}x}=k$, $\dfrac{0.4}{3}v^3=kx+C$	A1	1.1b
	$x=3,\ v=2\quad \dfrac{3.2}{3}=3k+C$ $x=6,\ v=2.5\quad \dfrac{25}{12}=6k+C$	M1	2.1
	$\Rightarrow 3k=\dfrac{25}{12}-\dfrac{3.2}{3}$, $k=\dfrac{61}{180}$, $C=\dfrac{1}{20}$	A1	1.1b
	$v^3=\dfrac{3}{0.4}\left(\dfrac{61x}{180}+\dfrac{1}{20}\right)=\dfrac{61x+9}{24}$ *	A1*	2.2a
		(6)	

(b)	$\frac{5k}{2v}=\frac{\mathrm{d}v}{\mathrm{d}t}=\left(\frac{61}{72v}\right)$	M1	2.5
	$\int 2v\mathrm{d}v=\int 5k\mathrm{d}t \Rightarrow v^2=5kt+C' \quad \left(36v^2=61t+C'\right)$	M1	2.1
	$\left[v^2\right]_2^{2.5}=\left[5kt\right]_0^T$ $\left(61T=36\left(2.5^2-2^2\right)\right)$	M1	1.1b
	$T=\frac{180}{61}\left(\frac{9}{20}\right)=\frac{81}{61}$ *	A1*	2.2a
		(4)	
(b) alt	$\frac{\mathrm{d}x}{\mathrm{d}t}=\sqrt[3]{\frac{61x+9}{24}}$	M1	2.5
	$\int(61x+9)^{-\frac{1}{3}}\mathrm{d}x=\int\frac{1}{\sqrt[3]{24}}\mathrm{d}t, \quad \frac{3}{2\times 61}(61x+9)^{\frac{2}{3}}=\frac{t}{\sqrt[3]{24}}+C''$	M1	2.1
	$T=2\times\sqrt[3]{3}\times\frac{3}{2\times 61}\left(375^{\frac{2}{3}}-192^{\frac{2}{3}}\right)=\frac{3\times\sqrt[3]{3}}{61}\left(\left(5\sqrt[3]{3}\right)^2-\left(4\sqrt[3]{3}\right)^2\right)$	M1	1.1b
	$T=\frac{9}{61}(25-16)=\frac{81}{61}$ *	A1*	2.2a
			(10 marks)

Question	Marks	Marking Guidance
(a)	M1	Complete strategy e.g. use of $F = ma$ with appropriate form for a, and use boundary conditions to confirm given result
	M1	Separate variables and integrate. Usual rules for integration. Condone missing C
	A1	Correct integration. Accept equivalent forms. Condone missing C.
	M1	Use boundary conditions to form 2 equations in 2 unknowns and solve for k or C
	A1	Obtain correct values for the constants
	A1*	Obtain **given answer** from correct working
	(6)	
(b)	M1	Select correct form for derivative and form a correct differential equation in v and t – follow their k
	M1	Separate and integrate. Condone with no $+C'$ – follow their k
	M1	Evaluate definite integral of the form $pv^2 = qt + C'$ or use limits to find value of constant of integration – follow their k
	A1*	Obtain **given answer** from correct working
	(4)	
(b) alt	M1	Select correct form for derivative and form a correct differential equation in x and t
	M1	Separate and integrate. Condone with no $+C''$
	M1	Evaluate definite integral of the form $pt = q\left(61x+9\right)^{\frac{2}{3}} + C''$ or use limits to find value of constant of integration
	A1*	Obtain **given answer** from correct working
		NB: Both parts have given answers, so check very carefully

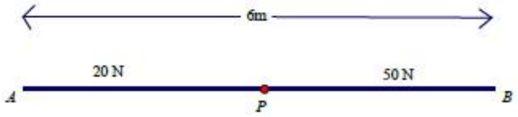
Q7.

a		M1	Differential equation in v and x No additional/missing terms. Condone sign error(s)
	$75v\frac{dv}{dx}=150-3v^2$	A1	
	$\int\frac{75v}{150-3v^2}dv=\int 1dx$	M1	Separate variables
	$x=\left[-\frac{25}{2}\ln\left(50-v^2\right)\right]_0^v$	DM1	Integrate and use limits
	$=-\frac{25}{2}\ln\left(\frac{50-v^2}{50}\right)$	A1	
	$-\frac{2x}{25}=\ln\left(1-\frac{v^2}{50}\right), v^2=50\left(1-e^{\frac{-2x}{25}}\right)$	DM1	Change the subject to v or v^2
	$v=\sqrt{50\left(1-e^{\frac{-2x}{25}}\right)}$	A1	
		(7)	

a alt	$\frac{dv^2}{dx}+\frac{2}{25}v^2=4$	M1A1	
	Integrating factor: $e^{\frac{2}{25}x}$	M1	
	$v^2e^{\frac{2}{25}x}=\int 4e^{\frac{2}{25}x}dx=50e^{\frac{2}{25}x}(+C)$ $v=0, x=0\Rightarrow C=-50$	M1A1	Integrate and use limits
	$v^2=50-50e^{-\frac{2}{25}x}$	DM1	Change the subject to v or v^2
	$v=\sqrt{50\left(1-e^{\frac{-2x}{25}}\right)}$	A1	
		(7)	

b	$75\frac{dv}{dt}=150-3v^2$	M1	Differential equation in v and t No additional/missing terms. Condone sign error(s)
	$\int\frac{75}{150-3v^2}dv=\int 1dt$	M1	Separate variables and integrate
	$t=\int\frac{25}{50-v^2}dv$ $=\frac{25}{2\sqrt{50}}\int\frac{1}{\sqrt{50}+v}+\frac{1}{\sqrt{50}-v}dv$		
	$=\frac{25}{2\sqrt{50}}\left(\ln\left(\sqrt{50}+v\right)-\ln\left(\sqrt{50}-v\right)\right)$	A1	With or without constant of integration Or $\frac{25}{\sqrt{50}}\text{arc}\tanh\frac{v}{\sqrt{50}}$
	$t=\frac{25}{2\sqrt{50}}\ln\left(\frac{\sqrt{50}+v}{\sqrt{50}-v}\right)-\frac{25}{2\sqrt{50}}\ln\left(\frac{\sqrt{50}}{\sqrt{50}}\right)$	DM1	Use limits 0 and v
	$=\frac{25}{2\sqrt{50}}\ln\left(\frac{\sqrt{50}+v}{\sqrt{50}-v}\right)$	A1	$\left(=\frac{5\sqrt{2}}{4}\ln\left(\frac{\sqrt{50}+v}{\sqrt{50}-v}\right)\right)$ or equivalent
		(5)	
		[12]	

Q8.

Question	Scheme	Marks	AOs
(a)			
	$T_A = \frac{20e}{2}$, $T_B = \frac{50(2-e)}{2}$	M1	3.1a
	In equilibrium $T_A = T_B$, $10e = 25(2-e)$	M1	3.1a
	$(35e = 50)$, $e = \frac{10}{7}$	A1	1.1b
	Equation of motion for P when distance x from equilibrium position towards B:	M1	3.1a
	$3.5\ddot{x} = T_B - T_A = \frac{50(2-e-x)}{2} - \frac{20(e+x)}{2}$	A1 A1	1.1b 1.1b
	$= \frac{50\left(\frac{4}{7}-x\right)}{2} - \frac{20\left(\frac{10}{7}+x\right)}{2}$		
	$\Rightarrow 3.5\ddot{x} = -35x$, $\ddot{x} = -10x$ and hence SHM about the equilibrium position	A1	3.2a
		(7)	
(b)	Amplitude $= 2 - \frac{10}{7} = \frac{4}{7}$	B1 ft	2.2a
	Use of max speed $= a\omega$	M1	1.1b
	$= \frac{4}{7}\sqrt{10} = 1.81$ (m s^{-1})	A1 ft	1.1b
		(3)	

Question	Scheme	Marks	AOs
(c)	Nearer to A than to B: $x < -\frac{3}{7}$	B1	3.1a
	Solve for $\sqrt{10}t$: $\cos\sqrt{10}t = -\frac{3}{4}$, $\sqrt{10}t = 2.418$............	M1	3.1a
	Length of time: $\frac{2}{\sqrt{10}}(\pi - 2.418...)$	M1	1.1b
	$= 0.457$ (seconds)	A1	1.1b
	Alternative: $\frac{3.864 - 2.419}{\sqrt{10}} = 0.457$		
	Alternative: $x = \frac{4}{7}\sin\sqrt{10}t = \frac{3}{7} \Rightarrow \sqrt{10}t = 0.8481$ or $\sqrt{10}t = 2.29353$ $t_1 = 0.2682,\ t_2 = 0.72527$ $\Rightarrow$ time $= 0.457$ (seconds)		
		(4)	
		(14 marks)	

Notes:
(a)   **M1:** Use of $T = \frac{\lambda x}{a}$ **M1:** Dependent on the preceding M1. Equate their tensions   **A1:** cao   **M1:** Condone sign error   **A1:** Correct unsimplified equation in e and x A1A1   Equation with one error A1A0   **A1:** Full working to justify conclusion that it is SHM about the equilibrium position.
(b)   **B1ft:** Seen or implied. Follow their e **M1:** correct method for max. speed   **A1ft:** 1.81 or better. Follow their a, ω
(c)   **B1:** Seen or implied   **M1:** Use of $x = a\cos wt$ **M1:** Correct strategy for the required interval   **A1:** 0.457 or better

Q9.

Question	Scheme	Marks	AOs
(a)	Use of Hooke's law seen	B1	2.1
	Let x be the displacement from equilibrium, then $T_B - T_A = -0.2\ddot{x}$.	M1	3.1a
	$\frac{15(2+x)}{1} - \frac{15(1-x)}{0.5} = -0.2\ddot{x}$	A1 A1	1.1b 1.1b
	$\Rightarrow \ddot{x} = -225x = -15^2 x$, which is of the form $\ddot{x} = -\omega^2 x$, so SHM *	A1*	3.2a
		(5)	
(b)	$a = 0.5,\ \ \omega = 15$ or their ω	B1ft	1.1b
	Max speed $= a\omega = 7.5$ (m s^{-1})	B1ft	1.2
	Max acceleration $= a\omega^2 = 112.5$ (m s^{-2})	B1ft	1.2
		(3)	
(c)	$x = 0.5\cos 15t \Rightarrow \dot{x} = -7.5\sin 15t$	B1ft	2.2a
	$\lvert\dot{x}\rvert = 7.5\sin 15t = 5 \Rightarrow \sin 15t = \frac{2}{3} \Rightarrow t = ...$	M1	1.1b
	$t = 0.0486...$ (s)	A1	1.1b
	Complete strategy to find the required time	M1	3.1a
	Speed > 5 for $\frac{2\pi}{15} - 4t \ = 0.22$ (s)	A1	1.1b
		(5)	

Question	Marks	Marking Guidance
(a)	B1	Use of Hooke's law seen at least once (with natural length ≤ 1.5).
	M1	Form equation of motion involving a difference of 2 tensions. Must be dimensionally correct. Need all terms. Condone incorrect lengths. Condone sign errors.
	A1 A1	Unsimplified equation with at most one error Correct unsimplified equation
	A1*	All correct and justification of SHM – comment required
	(5)	
(b)	B1ft	Correct values seen or implied. a must be correct but follow their ω from (a)
	B1ft	Follow their a, ω
	(3)	
(c)	B1ft	Use correct equation for speed $(-a\omega \sin \omega t)$ Follow their a and ω Allow sin or cos form Can also use correct expression for x combined with use of $v^2 = \omega^2 (a^2 - x^2)$
	M1	Find a relevant time e.g. time taken for 0 m s^{-1} to 5 m s^{-1}
	A1	Seen or implied
	M1	Complete strategy for the required time – e.g. find value for t when $v = 5$ and use symmetry and periodic time
	A1	2 s.f. or better (0.22428…)
	(5)	**(13 marks)**

Q10.

Question	Scheme	Marks	AOs
(a)	$mg = \frac{2mge}{l}$	M1	3.1a
	$e = \frac{1}{2}l$ so $AO = \frac{3l}{2}$ *	A1*	1.1b
		(2)	
(b)	Equation of motion vertically: $mg - T = m\ddot{x}$.	M1	2.1
	$mg - \frac{2mg(x+e)}{l} = m\ddot{x}$.	A1	1.1b
	$-\frac{2g}{l}x = \ddot{x}$, so SHM with $\omega^2 = \frac{2g}{l}$	A1	1.1b
	Use of $\frac{2\pi}{\omega}$	M1	3.1a
	$2\pi\sqrt{\frac{l}{2g}} = \pi\sqrt{\frac{2l}{g}}$ *	A1*	2.2a
		(5)	
(c)	Complete method to find h	M1	3.1a
	$mgh = \frac{2mg\left(\frac{3l}{2}\right)^2}{2l}$ OR $v^2 = \frac{2g}{l}(l^2 - (-\frac{1}{2}l)^2)$ and $0 = \frac{3gl}{2} - 2gs$	A1	1.1b
		A1	1.1b
	$h = \frac{9l}{4}$	A1	1.1b
		(4)	
(d)	$-\frac{1}{2}l = l\cos\omega t$	M1	3.1a
	$t = \frac{2\pi}{3}\sqrt{\frac{l}{2g}}$	A1	1.1b
	$v = \sqrt{\frac{2g}{l}(l^2 - (-\frac{1}{2}l)^2}$ OR	M1	2.1
	$0 = \sqrt{\frac{3gl}{2}} - gt_1 \Rightarrow t_1 = \sqrt{\frac{3l}{2g}}$	M1	3.1a

	Total time = $\frac{2\pi}{3}\sqrt{\frac{l}{2g}}+\sqrt{\frac{3l}{2g}}$ oe	A1	1.1b
		(5)	
			(16 marks)

Notes:

a	M1	Use of Hooke's Law and $T = mg$
	A1*	Correct answer fully justified
b	M1	All terms needed but allow a for the acceleration
	A1	Correct unsimplified equation including $\ddot{x}$
	A1	Must mention SHM
	M1	Correct method
	A1*	Correct answer correctly shown
c	M1	Correct no. of terms, dimensionally correct in energy equation **OR** use SHM to find v^2 at unstretched position AND then use motion under gravity Correct no. of terms, dimensionally correct in both equations
	A1 A1	Equation with at most one error Correct equation **OR** correct equations
	A1	cao
d	M1	Complete method to find time to reach unstretched position
	A1	Correct time
	M1	Complete method to find speed at unstretched position
	A1	Correct speed
	M1	Complete method to find time to rest position
	A1	Cao

Q11.

Question	Scheme	Marks	AOs
(a)	At equilibrium: $0.5g=\frac{25e}{1.25}$, $e=\frac{0.5\times10\times1.25}{25}=\frac{1}{4}$	B1	3.3
	For taut string, when distance x from equilibrium, equation of motion	M1	2.1
	Alternative for M1: Conservation of energy using a known point (E or B) and a general position: From E: $\frac{25e^2}{2\times1.25}+KE(\text{constant}\neq0)+0.5gx=\frac{25(e+x)^2}{2\times1.25}+\frac{1}{2}0.5v^2+0(\text{GPE})$ and differentiate wrt x for M1 $\quad\Rightarrow 0.5g=\frac{25(e+x)}{1.25}+\frac{1}{2}v\frac{dv}{dx}$		
	$\frac{25(e+x)}{1.25}-0.5g=-0.5\ddot{x}$	A1ft	1.1b
	$\ddot{x}=-40x \quad$ hence SHM*	A1*	2.2a
	Periodic time:	M1	3.4
	$T=\frac{2\pi}{\sqrt{40}}=\frac{\pi}{\sqrt{10}}$ *	A1*	2.2a
		(6)	
(b)	Max KE $=2.5=\frac{1}{2}\times\frac{1}{2}\times\max v^2 \quad \Rightarrow \max v^2=10$	B1	1.2
	Max speed $=a\omega$: $\sqrt{10}=a\sqrt{40}$	M1	3.4
	$AB=1.25+\frac{1}{4}+\frac{1}{2}=2(\text{m})$ *	A1*	1.1b
		(3)	
(b) alt	Energy : $\frac{25e^2}{2.5}+2.5+0.5ga=\frac{25(e+a)^2}{2.5}$	B1	
	Solve for a	M1	
	$AB=1.25+\frac{1}{4}+\frac{1}{2}=2(\text{m})$ *	A1*	1.1b
		(3)	

(c)	$a=0.5$, $x=0.5\cos\sqrt{40}t$	B1	2.2a
	$-0.25=0.5\cos\sqrt{40}t \quad \Rightarrow t=0.3311...$	M1	3.1a
	$v^2=40(0.5^2-0.25^2)=\frac{15}{2}$	M1	3.4
	Total time $=2\times0.3311...+\frac{2\times\sqrt{7.5}}{10}$	DM1	3.1a
	$=1.2(\text{s})$ or better	A1	2.2a

		(5)	
(14 marks)			

Notes:	
(a)B1	Correct only Award if see $mg = \frac{\lambda e}{l}$ used in their equation of motion
M1	Equation of motion with x measured from the equilibrium position. Need all terms and dimensionally correct. Allow with their $e \neq 0$. Condone sign errors. Allow with a in place of $\ddot{x}$
A1ft	Correct unsimplified equation with their e or $e \neq 0$. Could have the negative of the whole equation or x replaced with $-x$ throughout
A1*	Reach **given conclusion** from correct working. Condone correct conclusion without explanation
M1	Use the model to find the periodic time: $T = \frac{2\pi}{\omega}$ From an equation of the form $\ddot{x} = -\omega^2 x$
A1*	Obtain given answer from correct **working** throughout. Available if only error is not to conclude SHM
(b)B1	Use the KE to find $\max v$ or $\max v^2$
M1	Use the model to find the amplitude of the motion
A1*	Obtain the given answer from correct working.
(b) alt B1	Using correct λ and l and e or their e
M1	Requires an energy equation with all the right terms
A1*	Obtain the given answer from correct working.
(c) B1	Correct equation for SHM seen or implied
M1	Find the time until the string goes slack If working from $x = 0.5\sin\sqrt{40}t$ need $\frac{T}{4} + \frac{1}{\sqrt{40}}\sin^{-1}\frac{1}{2}$
M1	Use the model to find v or v^2 at the instant the string goes slack $(v = 2.738\ldots)$ Using SHM formula or conservation of energy.
M1	Complete method to find the total time until return to B Requires the preceding M marks

	If they use suvat to find the time as a projectile it must be a complete method e.g. $\sqrt{\frac{15}{2}}=-\sqrt{\frac{15}{2}}+gt$ or a combination of $v^2=u^2+2as$ and $s=ut+\frac{1}{2}at^2$
A1	$=1.2(\text{s})$ or better Condone an answer to > 2 s.f. Not scored if they have used 9.8.

Q12.

Question Number	Scheme	Marks
(a)	$T=\frac{45e}{1.8}=\frac{20(1-e)}{1.2}$ or $\frac{45(1-e')}{1.8}=\frac{20e'}{1.2}$ $e=0.4$ or $e'=0.6$ $AO=2.2\text{ m}$ *	M1A1 A1 A1cso (4)
ALT:	$AO=y$ $\frac{45(y-1.8)}{1.8}=\frac{20(2.8-y)}{1.2}$ M1A1A1 $54y-97.2=100.8-36y$ $y=2.2\text{ m}$ * A1cso	
(b)	$0.6\ddot{x}=\frac{20(0.6-x)}{1.2}-\frac{45(0.4+x)}{1.8}$ $0.6\ddot{x}=-\frac{125}{3}x$ $\ddot{x}=-\frac{625}{9}x$ $\therefore$ SHM	M1A1A1 dM1A1cso (5)

(c)	$\omega^2 = \dfrac{625}{9}$ oe	B1
	Time from C to D: $-0.4 = 0.5\cos\dfrac{25}{3}t$	M1A1ft
	$t = \dfrac{3}{25}\cos^{-1}(-0.8)$	A1
	Speed at D: $v^2 = \dfrac{625}{9}(0.5^2 - 0.4^2)$ or use $v = -a\omega\sin\omega t$	M1
	$v = \dfrac{25}{3} \times 0.3 = 2.5$	A1
	Time from D to A $= \dfrac{1.8}{2.5}$	M1
	Total time: $= \dfrac{1.8}{2.5} + \dfrac{3}{25}\cos^{-1}(-0.8) = 1.01977...$ (s) (accept 1.0 or better)	A1 cao (8) [17]

(a)

M1 Use Hooke's law to obtain the tension in each string and equate the 2 tensions. Either extension can be used as the unknown. ALT: Use both extensions and use $e + e' = 1$ later

A1 Correct equation.

A1 Correct result for their choice of unknown.

A1cso Correct completion to the **given** answer with no errors seen.

(b)

M1 Form an equation of motion with the difference of 2 tensions (from applying Hooke's law) which must have different extensions which both include a variable. Acceleration can be a or $\ddot{x}$

If the variable is measured from A or B a substitution is required to obtain the necessary SHM equation. **Do not** award this mark until this substitution is attempted.

A1A1 Deduct 1 for each error. Difference of tensions the wrong way round counts as one error. Acceleration can be a or $\ddot{x}$ but if a is used it must be in the same direction as $\ddot{x}$. The numbers in the extensions must be as shown (as SHM cannot be assumed before it is shown and so the numbers must come from the situation and not just chosen so that the constant terms cancel out).

dM1 Solve the equation to $\pm\ddot{x} = ...$ Acceleration a scores M0 now.

A1cso Correct equation (must be $\ddot{x} = ...$ now) **and** the conclusion.

(c)

B1 Correct value for ω^2 or ω seen explicitly or used

M1 Obtain the time from C to D. $x = a\cos\omega t$ or $x = a\sin\omega t$ can be used as long as the method is complete. Amplitude to be 0.5, $x = \pm 0.4$ with their ω which must have come from an equation of the form $\ddot{x} = \pm\omega^2 x$

Sometimes done in two parts: Time C to O $\left(\dfrac{3\pi}{50} \text{ or } 0.1884...\right)$ and O to D $(0.111...)$

added now or later.

A1ft Correct equation. Follow through their ω

A1 Correct time, as shown or 0.2997...seen now or later

M1 Find the speed of P as it reaches D with their ω.

A1 Correct speed at D

M1 Use their speed at D to find the time from D to A

A1cao Add the 2 (or 3) times to obtain the required time.

Q13.

Question Number	Scheme	Marks
(a)	$mv\frac{dv}{dx} = mg\sin 30 - \frac{1}{2}mx^2$	M1A1A1
	$\frac{1}{2}v^2 = xg\sin 30 - \frac{1}{6}x^3\ (+c)$	dM1A1ft
	$x = 3\ \ \frac{1}{2}v^2 = 3g\sin 30 - \frac{9}{2}$	dM1
	$(v = 4.5166...)$	
	$v = 4.5$ or $4.52\ (\text{ms}^{-1})$	A1cso (7)
(b)	$v = 0 \Rightarrow x^2 = 6g\sin 30\ (x \neq 0)$	
	$x = 5.4$ or 5.42 (m)	M1A1 (2)
		[9]

(a)M1 Attempt NL2 parallel to the plane. Acceleration must be $v\frac{dv}{dx}$ and weight must be resolved. (Variable force not resolved.) m may be cancelled. Integrating a to obtain $\frac{1}{2}v^2$ gains this mark by implication.

A1 A1 Deduct 1 mark for each error in the equation. Both signs incorrect on RHS is one error.

dM1 Attempt the integration (wrt x) of both sides of the equation. Depends on the first M mark.

A1ft Correct integration with or without the constant. Follow through their integrand.

dM1 Substitute $x = 3$ in their integrated equation. Depends on both previous M marks.

A1cso Correct value of v. Must be 2 or 3 sf. **CSO**: Evidence of a constant of integration must be seen. C included and then crossed out or disappearing is sufficient evidence.

Definite integration:

M1A1A1 as above

dM1A1ft For the integration - ignore any limits shown

dM1 Use of correct limits. No sub need be shown for 0.

A1 Correct value of v. Must be 2 or 3 sf. **CSO**: Evidence of a zero lower limit must be seen.

By work-energy:

F is variable, so if no integral seen score 0/7

$\frac{1}{2}v^2(-0) = xg\sin 30 - \int \frac{1}{2}x^2 dx$ .. M1A1A1

$\frac{1}{2}v^2(-0) = xg\sin 30 - \frac{1}{6}x^3$ M1A1

For the final A mark, evidence of initial KE being 0 must be seen.

(b)

M1 Substitute $v = 0$ in their equation for v^2 (from (a)) and obtain a numerical value of x

A1 Correct value of x. Must be 2 or 3 sf. Do not penalise missing constant here.