Methods in Calculus

Questions

Q1.

(a) Given that

$$y = \arcsin x \qquad -1 \leq x \leq 1$$

show that

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}}$$

(3)

(b) $\mathrm{f}(x) = \arcsin(e^x) \quad x \leq 0$

Prove that f(x) has no stationary points.

(3)

(Total for question = 6 marks)

Q2.

Show that

$$\int_0^{\infty} \frac{8x - 12}{(2x^2 + 3)(x + 1)} dx = \ln k$$

where k is a rational number to be found.

(Total for question = 7 marks)

Q3.

Given that y = arsinh(tanh x), show that

$$\frac{dy}{dx} = \frac{\operatorname{sech}^2 x}{\sqrt{1 + \tanh^2 x}}$$

(5)

(Total for question = 5 marks)

Q4.

The curve C has equation

$$y = \arccos\left(\frac{1}{2}x\right) \qquad -2 \leqslant x \leqslant 2$$

(a) Show that C has no stationary points.

(3)

The normal to C, at the point where x = 1, crosses the x-axis at the point A and crosses the y-axis at the point B.

Given that O is the origin,

(b) show that the area of the triangle OAB is $\frac{1}{54}\left(p\sqrt{3} + q\pi + r\sqrt{3}\,\pi^2\right)$ where p, q and r are integers to be determined.

(5)

(Total for question = 8 marks)

Q5.

(a) Explain why $\int_1^{\infty} \frac{1}{x(2x+5)}\,\mathrm{d}x$ is an improper integral.

(1)

(b) Prove that

$$\int_1^{\infty} \frac{1}{x(2x+5)}\,\mathrm{d}x = a\ln b$$

where *a* and *b* are rational numbers to be determined.

(6)

(Total for question = 7 marks)

Q6.

$$\mathrm{f}(x) = \frac{x+2}{x^2+9}$$

(a) Show that

$$\int \mathrm{f}(x)\,\mathrm{d}x = A\ln\left(x^2+9\right) + B\arctan\left(\frac{x}{3}\right) + c$$

where *c* is an arbitrary constant and *A* and *B* are constants to be found.

(4)

(b) Hence show that the mean value of f(*x*) over the interval [0, 3] is

$$\frac{1}{6}\ln 2 + \frac{1}{18}\pi$$

(3)

(c) Use the answer to part (b) to find the mean value, over the interval [0, 3], of

$$\mathrm{f}(x) + \ln k$$

where *k* is a positive constant, giving your answer in the form $p + \frac{1}{6}\ln q$, *p* and *q* are constants and *q* is in terms of *k*.

(2)

(Total for question = 9 marks)

Q7.

(a) Express as partial fractions

$$\frac{2x^2 + 3x + 6}{(x+1)(x^2+4)}$$

(3)

(b) Hence, show that

$$\int_0^2 \frac{2x^2 + 3x + 6}{(x+1)(x^2+4)}\,\mathrm{d}x = \ln\left(a\sqrt{2}\right) + b\pi$$

where *a* and *b* are constants to be determined.

(4)

(Total for question = 7 marks)

Mark Scheme – Methods in Calculus

Q1.

Question	Scheme		Marks	AOs
(a)	$\sin y = x \Rightarrow \cos y \dfrac{dy}{dx} = 1$	$\sin y = x \Rightarrow \dfrac{dx}{dy} = \cos y$	M1	1.1b
	Uses $\sin^2 y + \cos^2 y = 1 \Rightarrow \cos y = \sqrt{1-\sin^2 y} \Rightarrow \sqrt{1-x^2}$		M1	2.1
	$\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}}$ * cso		A1*	1.1b
			(3)	
(b)	Using the answer to (a) $f'(x) = \dfrac{1}{\sqrt{1-e^{2x}}} \times \ldots$	Restart $\sin y = e^x \Rightarrow \cos y \dfrac{dy}{dx} = e^x$	M1	3.1a
	$f'(x) = \dfrac{1}{\sqrt{1-e^{2x}}} \times e^x$	$f'(x) = \dfrac{e^x}{\cos y}$	A1	1.1b
	$e^x \neq 0$ (or $e^x > 0$) therefore, there are no stationary points Alternatively, $e^x = 0$ leading to $x = \ln 0$ which is impossible/undefined therefore there are no stationary points.		A1	2.4
			(3)	
			(6 marks)	

Notes:

(a)

M1: Finds x in terms of y and differentiates

M1: Uses the trig identity $\sin^2 y + \cos^2 y = 1$ to express $\cos y$ in terms of x. This may be seen in their derivative or stated on the side

A1*: Correctly achieves the printed answer $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$. cso

(b)

M1: Differentiates using the chain rule to achieve the correct form, condone $f'(x) = \frac{1}{\sqrt{1-e^{2x}}}$

Note $f'(x) = \frac{1}{\sqrt{1-e^x}}$ is B0 for incorrect form

Alternatively restart, finds x in terms of y and differentiates

A1: Correct differentiation

A1: Follows correct differentiation. States that as $e^x \neq 0$ (or $e^x > 0$) or no solutions to $e^x = 0$ therefore there are no stationary points.

Alternatively, $e^x = 0$ leading to $x = \ln 0$ which is impossible/undefined/error therefore there are no stationary points. Ignore any reference to the denominator = 0

Q2.

Question	Scheme	Marks	AOs
	$\frac{8x-12}{(2x^2+3)(x+1)}=\frac{Ax+B}{2x^2+3}+\frac{C}{x+1}$	M1	3.1a
	$8x-12=(Ax+B)(x+1)+C(2x^2+3)$ E.g. $x=-1\Rightarrow C=-4, x=0\Rightarrow B=0, x=1\Rightarrow A=8$ Or Compares coefficients and solves $(A+2C=0\quad A+B=8\quad B+3C=-12)$ $\Rightarrow A=..., B=..., C=...$	dM1	1.1b
	$A=8\quad B=0\quad C=-4$	A1	1.1b
	$\int\left(\frac{8x}{2x^2+3}-\frac{4}{x+1}\right)dx=2\ln(2x^2+3)-4\ln(x+1)$	A1ft	1.1b
	$2\ln(2x^2+3)-4\ln(x+1)=\ln\left(\frac{(2x^2+3)^2}{(x+1)^4}\right)$ or $2\ln(2x^2+3)-4\ln(x+1)=2\ln\left(\frac{(2x^2+3)}{(x+1)^2}\right)$	M1	2.1
	$\lim_{x\to\infty}\left\{\ln\frac{(2x^2+3)^2}{(x+1)^4}\right\}=\ln 4$ or $\lim_{x\to\infty}\left\{2\ln\frac{(2x^2+3)}{(x+1)^2}\right\}=2\ln 2$	B1	2.2a
	$\Rightarrow\int_0^\infty\frac{8x-12}{(2x^2+3)(x+1)}dx=\ln\frac{4}{9}$ cao	A1	1.1b
		(7)	
		(7 marks)	

Notes
M1: Selects the correct form for partial fractions. dM1: Full method for finding values for all three constants. Dependent on having the correct form for the partial fractions. Allow slips as long as the intention is clear. A1: Correct constants or partial fractions. A1ft: Integrates $\int\frac{px}{2x^2+3}-\frac{q}{x+1}dx=\frac{p}{4}\ln(2x^2+3)-q\ln(x+1)$ and no extra terms M1: Combines two algebraic log terms correctly. B1: Correct upper limit for $x\to\infty$ by recognising the dominant terms. (Simply replacing x with ∞ scores B0). This can be implied. A1: Deduces the correct value for the improper integral in the correct form, cao A0 for $2\ln\frac{2}{3}$ Correct answer with no working seen is no marks. **Note:** Incorrect partial fraction form, $\frac{A}{2x^2+3}+\frac{B}{x+1}$ or $\frac{Ax}{2x^2+3}+\frac{B}{x+1}$ the maximum it can score is M0M0A0A0M1B1A0

Q3.

Question Number	Scheme	Notes	Marks
	$y = \text{arsinh}(\tanh x)$		
Way 1	$\sinh y = \tanh x$		B1
	$\cosh y \dfrac{dy}{dx} = \text{sech}^2 x$ or $\cosh y = \text{sech}^2 x \dfrac{dx}{dy}$	M1: $\pm\cosh y$ or $\pm\text{sech}^2 x$ A1: All correct	M1A1
	$\dfrac{dy}{dx} = \dfrac{\text{sech}^2 x}{\cosh y}$		
	$\dfrac{dy}{dx} = \dfrac{\text{sech}^2 x}{\sqrt{1+\sinh^2 y}} = f(x)$	Uses a **correct identity** to express $\dfrac{dy}{dx}$ in terms of x only	M1
	$= \dfrac{\text{sech}^2 x}{\sqrt{1+\tanh^2 x}}$ *	cso. There must be no errors such as incorrect or missing or inconsistent variables and no missing h's.	A1*
			Total 5
Way 2	$t = \tanh x \Rightarrow y = \text{arsinh} t$	Replaces tanh$x$ by e.g. t	B1
	$\dfrac{dt}{dx} = \text{sech}^2 x, \dfrac{dy}{dt} = \dfrac{1}{\sqrt{1+t^2}}$	M1: $\dfrac{dt}{dx} = \pm\text{sech}^2 x, \dfrac{dy}{dt} = \pm\dfrac{1}{\sqrt{1+t^2}}$ A1: Correct $\dfrac{dt}{dx}$ and $\dfrac{dy}{dt}$ and correctly labelled	M1A1
	$\dfrac{dy}{dx} = \dfrac{dy}{dt}\dfrac{dt}{dx} = \dfrac{\text{sech}^2 x}{\sqrt{1+t^2}} = f(x)$	Uses **correct form of the chain rule** for their variables to express $\dfrac{dy}{dx}$ in terms of x only	M1
	$= \dfrac{\text{sech}^2 x}{\sqrt{1+\tanh^2 x}}$ *	Cso. There must be no errors such as incorrect or missing or inconsistent variables and no missing h's.	A1*
			Total 5
Way 3	$u = \tanh x \Rightarrow \dfrac{du}{dx} = \text{sech}^2 x$	Correct derivative	B1
	$\displaystyle\int \dfrac{\text{sech}^2 x}{\sqrt{1+\tanh^2 x}}dx = \int \dfrac{\text{sech}^2 x}{\sqrt{1+u^2}}\dfrac{1}{\text{sech}^2 x}du$	M1: Complete substitution including the "dx" A1: Fully correct substitution	M1A1
	$= \displaystyle\int \dfrac{1}{\sqrt{1+u^2}}du = \text{arsinh}u(+c)$	Reaches arsinh$u$	M1
	$y = \text{arsinh}(\tanh x)(+c)$	Reaches $y = \text{arsinh}(\tanh x)$ with or without + c and no errors such as incorrect or missing or inconsistent variables or missing h's.	A1*
			Total 5

	Special Case:	
	$y = \text{arsinh}(\tanh x) \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{\sqrt{1+\tanh^2 x}}(\times)\text{sech}^2 x$ $= \dfrac{\text{sech}^2 x}{\sqrt{1+\tanh^2 x}}$ **Note that the sech^2x needs to appear separate from the fraction as above <u>and not just the printed answer written down</u>.** **<u>To score more than 2 marks using a chain rule method, a third variable must be introduced</u>**	M1A1

Q4.

Question	Scheme	Marks	AOs
(a)	$\frac{dy}{dx}=\frac{-\lambda}{\sqrt{1-\beta x^2}}$ where $\lambda>0$ and $\beta>0$ and $\beta\neq 1$ Alternatively $2\cos y = x \Rightarrow \frac{dx}{dy}=\alpha\sin y \Rightarrow \frac{dy}{dx}=\frac{1}{\alpha\sin y}$	M1	1.1b
	$\frac{dy}{dx}=\frac{-\frac{1}{2}}{\sqrt{1-\frac{1}{4}x^2}}$ or $\frac{dy}{dx}=\frac{-1}{2\sqrt{1-\frac{1}{4}x^2}}$ o.e. or $\frac{dy}{dx}=-\frac{1}{2\sin y}$ or	A1	1.1b
	States that $\frac{dy}{dx}\neq 0$ therefore C has no stationary points. Tries to solve $\frac{dy}{dx}=0$ and ends up with a contradiction e.g. -1 = 0 therefore C has no stationary points. As cosec $y>1$ therefore C has no stationary points.	A1	2.4
		(3)	
(b)	$\frac{dy}{dx}=\frac{-1}{2\sqrt{1-\frac{1}{4}\times 1^2}}=\left\{-\frac{1}{\sqrt{3}}\right\}$	M1	1.1b
	Normal gradient $=-\frac{1}{m}$ and $y-\frac{\pi}{3}=m_n(x-1)$ Alternatively $\frac{\pi}{3}=m_n(1)+c\Rightarrow c=...\left\{\frac{\pi}{3}-\sqrt{3}\right\}$ and then $y=m_nx+c$	M1	1.1b
	$y=0\Rightarrow 0-\frac{\pi}{3}=\sqrt{3}(x_A-1)\Rightarrow x_A=...\left\{1-\frac{\pi}{3\sqrt{3}}\text{ or }1-\frac{\pi\sqrt{3}}{9}\right\}$ and $x=0\Rightarrow y_B-\frac{\pi}{3}=\sqrt{3}(0-1)\Rightarrow y_B=...\left\{\frac{\pi}{3}-\sqrt{3}\right\}$	M1	3.1a
	Area $=\frac{1}{2}\times x_A\times -y_B=\frac{1}{2}\left(1-\frac{\pi}{3\sqrt{3}}\right)\left(\sqrt{3}-\frac{\pi}{3}\right)$	M1	1.1b
	Area $\frac{1}{54}\left(27\sqrt{3}-18\pi+\sqrt{3}\pi^2\right)$ $(p=27, q=-18, r=1)$	A1	2.1
		(5)	
			(8 marks)

Notes:

(a)

M1: Finds the correct form for $\frac{dy}{dx}$

A1: Correct $\frac{dy}{dx}$ **A1:** States or shows that $\frac{dy}{dx} \neq 0$ and draws the required conclusion. This mark can be scored as long as the M mark has been awarded.
(b) **M1:** Substitutes $x=1$ into their $\frac{dy}{dx}$ **M1:** Finds the normal gradient and finds the equation of the normal using $y-\frac{\pi}{3}=m_n(x-1)$ **M1:** Finds where their normal cuts the x-axis and the y-axis. **M1:** Finds the area of the triangle OAB $=\frac{1}{2}\times x_A \times -y_B$. **A1:** Correct area Special case: If finds the tangent to the curve, the x and y intercepts and the area of the triangle max score M1 M0 M1 M0 A0 Note common error $\frac{dy}{dx}=\frac{-1}{\sqrt{1-\frac{1}{4}x^2}}$ In part (b) this leads to $\frac{dy}{dx}=\frac{-2}{\sqrt{3}}$ leading to normal gradient $\frac{\sqrt{3}}{2}$ and $y=\frac{\sqrt{3}}{2}x-\frac{\sqrt{3}}{2}+\frac{\pi}{3}$ and $\left(0,\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right)$ and $\left(1-\frac{2\pi}{3\sqrt{3}},0\right)$ therefore area $=\frac{1}{2}\left(\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right)\left(\frac{2\pi}{3\sqrt{3}}-1\right)$ This can score M1 M1 M1 M1 A0

Q5.

Question	Scheme	Marks	AOs
(a)	E.g. • Because the interval being integrated over is unbounded • Accept because the upper limit is infinity • Accept because a limit is required to evaluate it	B1	2.4
		(1)	
(b)	$\frac{1}{x(2x+5)} = \frac{A}{x} + \frac{B}{2x+5} \Rightarrow A = \ldots, B = \ldots$	M1	3.1a
	$\frac{1}{x(2x+5)} = \frac{1}{5x} - \frac{2}{5(2x+5)}$	A1	1.1b
	$\int \frac{1}{5x} - \frac{2}{5(2x+5)} \, dx = \frac{1}{5}\ln x - \frac{1}{5}\ln(2x+5)$	A1ft	1.1b
	$\frac{1}{5}\ln x - \frac{1}{5}\ln(2x+5) = \frac{1}{5}\ln\frac{x}{(2x+5)}$	M1	2.1
	$\lim_{x\to\infty}\left\{\frac{1}{5}\ln\frac{x}{2x+5}\right\} = \frac{1}{5}\ln\frac{1}{2}$	B1	2.2a
	$\Rightarrow \int_1^\infty \frac{1}{x(2x+5)} \, dx = \frac{1}{5}\ln\frac{1}{2} - \frac{1}{5}\ln\frac{1}{7} = \frac{1}{5}\ln\frac{7}{2}$	A1	1.1b
		(6)	
			(7 marks)

Notes
(a) B1: For a suitable explanation with no contrary reasoning. Technically this should refer to the interval being unbounded, but this is unlikely to be seen. Accept "Because the upper limit is infinity". Do not award if there are erroneous statements e.g. referring to as $x = 0$ the integrand is not defined. Do not accept "because one of the limits is undefined" unless they state they mean ∞. Do not accept "it is undefined when $x = \infty$" without reference to "it" being the upper limit. (b) M1: Selects the correct form for partial fractions and proceeds to find values for A and B A1: Correct constants or partial fractions A1ft: $\int \frac{p}{x} + \frac{q}{2x+5} \, dx = p\ln x + \frac{q}{2}\ln(2x+5)$ Note that $\frac{1}{5}\ln 5x - \frac{1}{5}\ln(10x+25)$ is correct. M1: Combines logs correctly. May see $-\frac{1}{5}\ln\left(\frac{2x+5}{x}\right) = -\frac{1}{5}\ln\left(2+\frac{5}{x}\right)$ B1: Correct upper limit for $x \to \infty$ by recognising the dominant terms. (Simply replacing x with ∞ scores B0) A1: Deduces the correct value for the improper integral in the correct form

Question	Scheme	Marks	AOs
(b) Way 2	$\frac{1}{x(2x+5)}=\frac{1}{2\left(x^2+\frac{5}{2}x\right)}=\frac{1}{2}\times\frac{1}{\left(x+\frac{5}{4}\right)^2-\frac{25}{16}}$	M1 A1	3.1a 1.1b
	$\int\frac{1}{x(2x+5)}\mathrm{d}x=\frac{1}{2}\times\frac{2}{5}\ln\left\lvert\frac{x+\frac{5}{4}-\frac{5}{4}}{x+\frac{5}{4}+\frac{5}{4}}\right\rvert=\frac{1}{5}\ln\left\lvert\frac{2x}{2x+5}\right\rvert$	M1 A1ft	2.1 1.1b
	$\lim_{x\to\infty}\left\{\frac{1}{5}\ln\frac{2x}{2x+5}\right\}=\frac{1}{5}\ln\frac{2}{2}=0$	B1	2.2a
	$\Rightarrow\int_1^{\infty}\frac{1}{x(2x+5)}\mathrm{d}x=0-\frac{1}{5}\ln\frac{2}{7}=\frac{1}{5}\ln\frac{7}{2}$	A1	1.1b
		(6)	

Notes

Note the method marks as MAMABA, and should be entered in this order on ePEN.

M1: Expands the denominator and completes the square.

A1: Correct expression

M1: For $\frac{1}{(x+p)^2-a^2}\to k\ln\left\lvert\frac{x+p-a}{x+p+a}\right\rvert$

A1ft: $\frac{1}{2}\frac{1}{(x+a)^2-a^2}\to\frac{1}{2a}\ln\left\lvert\frac{x}{x+2a}\right\rvert$ with their a (may be simplified as in scheme).

B1: Correct upper limit for $x\to\infty$ by recognising the dominant terms. (Simply replacing x with ∞ scores B0) Note in this method the upper limit evaluates to zero.

A1: Deduces the correct value for the improper integral in the correct form. Accept $-\frac{1}{5}\ln\frac{2}{7}$

Q6.

Question	Scheme	Marks	AOs
(a)	$f(x)=\frac{x+2}{x^2+9}=\frac{x}{x^2+9}+\frac{2}{x^2+9}$	B1	3.1a
	$\int\frac{x}{x^2+9}\,dx=k\ln(x^2+9)(+c)$	M1	1.1b
	$\int\frac{2}{x^2+9}\,dx=k\arctan\left(\frac{x}{3}\right)(+c)$	M1	1.1b
	$\int\frac{x+2}{x^2+9}\,dx=\frac{1}{2}\ln(x^2+9)+\frac{2}{3}\arctan\left(\frac{x}{3}\right)+c$	A1	1.1b
		(4)	
(b)	$\int_0^3 f(x)\,dx=\left[\frac{1}{2}\ln(x^2+9)+\frac{2}{3}\arctan\left(\frac{x}{3}\right)\right]_0^3$ $=\frac{1}{2}\ln 18+\frac{2}{3}\arctan\left(\frac{3}{3}\right)-\left(\frac{1}{2}\ln 9+\frac{2}{3}\arctan(0)\right)$ $=\frac{1}{2}\ln\frac{18}{9}+\frac{2}{3}\arctan\left(\frac{3}{3}\right)$	M1	1.1b
	Mean value $=\frac{1}{3-0}\left(\frac{1}{2}\ln 2+\frac{\pi}{6}\right)$	M1	2.1
	$\frac{1}{6}\ln 2+\frac{1}{18}\pi$ *	A1*	2.2a
		(3)	
(c)	$\frac{1}{6}\ln 2+\frac{1}{18}\pi+\ln k$	M1	2.2a
	$\frac{1}{6}\ln 2k^6+\frac{1}{18}\pi$	A1	1.1b
		(2)	
			(9 marks)

Notes:

(a)
B1: Splits the fraction into two correct separate expressions
M1: Recognises the required form for the first integration
M1: Recognises the required form for the second integration
A1: Both expressions integrated correctly and added together with constant of integration included

(b)
M1: Uses limits correctly and combines logarithmic terms
M1: Correctly applies the method for the mean value for their integration
A1*: Correct work leading to the given answer

(c)
M1: Realises that the effect of the transformation is to increase the mean value by $\ln k$
A1: Combines ln's correctly to obtain the correct expression

Q7.

Question	Scheme	Marks	AOs
(a)	$\dfrac{2x^2+3x+6}{(x+1)(x^2+4)} = \dfrac{A}{x+1} + \dfrac{Bx+C}{x^2+4} \Rightarrow 2x^2+3x+6$ $= A(x^2+4) + (Bx+C)(x+1)$	M1	1.1b
	e.g. $x=-1 \Rightarrow A = \ldots,\ x = 0 \Rightarrow C = \ldots,$ coeff $x^2 \Rightarrow B = \ldots$ or Compares coefficients and solves to find values for A, B and C $2 = A + B,\ 3 = B + C,\ 6 = 4A + C$	dM1	1.1b
	$A = 1,\quad B = 1,\quad C = 2$	A1	1.1b
		(3)	
(b)	$\displaystyle\int_0^2 \frac{1}{x+1} + \frac{x+2}{x^2+4}\,dx = \int_0^2 \frac{1}{x+1} + \frac{x}{x^2+4} + \frac{2}{x^2+4}\,dx$ $= \left[\alpha\ln(x+1) + \beta\ln(x^2+4) + \lambda\arctan\left(\frac{x}{2}\right)\right]_0^2$	M1	3.1a
	$= \left[ln(x+1) + \frac{1}{2}ln(x^2+4) + arctan\left(\frac{x}{2}\right)\right]_0^2$	A1	2.1
	$= \left[ln(3) + \frac{1}{2}ln(8) + arctan\ 1\right] - \left[ln(1) + \frac{1}{2}ln(4) + arctan(0)\right]$ $=$ $= \left[ln(3) + \frac{1}{2}ln(8) + arctan(1)\right] - \left[\frac{1}{2}ln\ 4\right] = \underline{ln\left(\frac{3\sqrt{8}}{2}\right)} + \frac{\pi}{4}$	dM1	2.1
	$ln(3\sqrt{2}) + \frac{\pi}{4}$	A1	2.2a
		(4)	
		(7 marks)	

Notes:
(a) **M1:** Selects the correct form for partial fractions and multiplies through to form suitable identity or uses a method to find at least one value (e.g. cover up rule). **dM1:** Full method for finding values for all three constants. Dependent on first M. Allow slips as long as the intention is clear. **A1:** Correct constants or partial fractions.
(b) **M1:** Splits the integral into an integrable form and integrates at least two terms to the correct form. They may use a substitution on the arctan term **A1:** Fully correct Integration. **dM1:** Uses the limits of 0 and 2 (or appropriate for a substitution), subtracts the correct way round and combines the ln terms from separate integrals to a single term with evidence of correct ln laws at least once. **A1:** Correct answer