Geometric and NB Distributions

Questions

Q1.

Indre works on reception in an office and deals with all the telephone calls that arrive. Calls arrive randomly and, in a 4-hour morning shift, there are on average 80 calls.

(a) Using a suitable model, find the probability of more than 4 calls arriving in a particular 20-minute period one morning.

(3)

Indre is allowed 20 minutes of break time during each 4-hour morning shift, which she can take in 5-minute periods. When she takes a break, a machine records details of any call in the office that Indre has missed.

One morning Indre took her break time in 4 periods of 5 minutes each.

(b) Find the probability that in exactly 3 of these periods there were no calls.

(2)

On another occasion Indre took 1 break of 5 minutes and 1 break of 15 minutes.

(c) Find the probability that Indre missed exactly 1 call in each of these 2 breaks.

(3)

(Total for question = 8 marks)

Q2.

In a game a spinner is spun repeatedly. When the spinner is spun, the probability of it landing on blue is 0.11

(a) Find the probability that the spinner lands on blue

(i) for the first time on the 6th spin,

(2)

(ii) for the first time before the 6th spin,

(2)

(iii) exactly 4 times during the first 6 spins,

(2)

(iv) for the 4th time on or before the 6th spin.

(4)

Zac and Izana play the game. They take turns to spin the spinner. The winner is the first one to have the spinner land on blue. Izana spins the spinner first.

(b) Show that the probability of Zac winning is 0.471 to 3 significant figures.

(3)

(Total for question = 13 marks)

Q3.

A spinner can land on red or blue. When the spinner is spun, there is a probability of $\frac{1}{3}$ that it

lands on blue. The spinner is spun repeatedly.

The random variable *B* represents the number of the spin when the spinner first lands on blue.

(a) Find (i) P(B = 4)

(ii) P($B \leq 5$)

(4)

(b) Find E(B^2)

(3)

Steve invites Tamara to play a game with this spinner.

Tamara must choose a colour, either red or blue.

Steve will spin the spinner repeatedly until the spinner first lands on the colour Tamara has chosen. The random variable *X* represents the number of the spin when this occurs.

If Tamara chooses red, her score is e^X

If Tamara chooses blue, her score is X^2

(c) State, giving your reasons and showing any calculations you have made, which colour you would recommend that Tamara chooses.

(5)

(Total for question = 12 marks)

Q4.

The probability of Richard winning a prize in a game at the fair is 0.15

Richard plays a number of games.

(a) Find the probability of Richard winning his second prize on his 8th game,

(2)

(b) State two assumptions that have to be made, for the model used in part (a) to be valid.

(2)

Mary plays the same game, but has a different probability of winning a prize. She plays until she has won r prizes. The random variable G represents the total number of games Mary plays.

(c) Given that the mean and standard deviation of G are 18 and 6 respectively, determine whether Richard or Mary has the greater probability of winning a prize in a game.

(4)

(Total for question = 8 marks)

Mark Scheme

Q1.

Qu	Scheme	Marks	AO
(a)	{Let C = no of calls in a 20 min period} $C \sim \text{Po}(\ldots)$	M1	3.3
	80 calls per 4-hour period gives $\frac{20}{3}$ per 20 mins i.e. $C \sim \text{Po}(\frac{20}{3})$	M1	3.4
	$[\text{P}(C > 4)] = 1 - \text{P}(C \leqslant 4)$		
	$= 0.79437\ldots$ awrt **<u>0.794</u>**	A1	1.1b
		(3)	
(b)	{X = no. of 5 min periods with no calls} $X \sim \text{B}(4, \text{e}^{-\frac{5}{3}})$	M1	3.3
	$\text{P}(X = 3) = 0.02186125\ldots$ awrt **<u>0.0219</u>**	A1	1.1b
		(2)	
(c)	P(exactly one call) $\text{e}^{-\frac{5}{3}} \times \frac{5}{3}$ or $\text{e}^{-5} \times 5$	M1	2.1
	P(exactly one call in each break) = $\left(\text{e}^{-\frac{5}{3}} \times \frac{5}{3}\right) \times \left(\text{e}^{-5} \times 5\right)$	M1	1.1b
	$= 0.0106052\ldots$ awrt **<u>0.0106</u>**	A1	1.1b
		(3)	
		(8 marks)	
	Notes		
(a)	1st M1 for selecting a Poisson model – written or used. May be implied by 2nd M1 or a correct Answer.		
	2nd M1 for the correct Poisson $\text{Po}(\frac{20}{3})$ or Po(6.67) or better seen <u>and</u> writing or using $1 - \text{P}(C \leqslant 4)$		
	A1 for awrt 0.794 (correct ans with no incorrect working scores 3/3)		
(b)	M1 for selecting a correct model B(4, 0.189) or better (calc: 0.188875…)		
	A1 for using the model to get awrt 0.0219 (correct ans with no incorrect working scores 2/2)		
(c)	1st M1 for <u>a</u> correct prob of 1 call (expressions in e or values) (allow 0.31479… or awrt 0.315 <u>or</u> 0.033689… or awrt 0.0337)		
	2nd M1 for a correct probability statement or expression. E.g. $\text{P}(S = 1 \mid S \sim \text{Po}(\frac{5}{3})) \times \text{P}(T = 1 \mid T \sim \text{Po}(5))$		
SC	e.g. $F \sim \text{Po}(\lambda)$ used in (b) to find $\text{P}(F = 0)$. Then if we see $Y \sim \text{Po}(3\lambda)$ and statement $\text{P}(F = 1) \times \text{P}(Y = 1)$ award M0M1		
	A1 for awrt 0.0106 (correct ans with no incorrect working scores 3/3)		

Q2.

Question	Scheme	Marks	AOs
(a) (i)	$[W \sim \text{Geo}(0.11)]$ $\quad \text{P}(W=6)=(0.89)^5(0.11)$	M1	3.3
	$= 0.06142\ldots$ | awrt <u>0.0614</u>	A1	1.1b
		(2)	
(ii)	$\text{P}(W \leqslant 5) = 1-(0.89)^5$	M1	3.1b
	$= 0.44159\ldots$ | awrt <u>0.442</u>	A1	1.1b
		(2)	
(iii)	$X \sim \text{B}(6, 0.11)$	M1	3.3
	$\text{P}(X=4)=0.001739\ldots$ | awrt <u>0.00174</u>	A1	1.1b
		(2)	
(iv)	$[Y \sim \text{NB}(4, 0.11)]$ using a neg bin | **or** $V\sim\text{B}(6, 0.11)$	M1	3.3
	$\text{P}(Y \leqslant 6)=\text{P}(Y=4)+\text{P}(Y=5)+\text{P}(Y=6)$ | **and** $\text{P}(V \geqslant 4)$ **for M2**	M1	3.1b
	$= (0.11)^4+\binom{4}{3}(0.11)^3(0.89)^1\times 0.11+\binom{5}{3}(0.11)^3(0.89)^2\times 0.11$	M1	3.4
	$= 0.001827$ | awrt <u>0.00183</u>	A1	1.1b
		(4)	
(b)	$\text{P(Zac wins)} = 0.89\times 0.11+(0.89)^3\times 0.11+(0.89)^5\times 0.11+\ldots$	M1	3.1b
	$= \dfrac{0.89\times 0.11}{1-(0.89)^2}$ oe	M1	1.1b
	$= 0.47089\ldots = 0.471^*$	A1cso*	2.1
		(3)	
			Total 13

(a)(i)	M1	Correct method to find $\text{P}(W=6)$ eg $(p)^5(1-p)$ for $p = 0.11$ or 0.89	
	A1	awrt 0.0614 (Correct ans with no incorrect working 2/2)	
(ii)	M1	Correct method to find $\text{P}(W \leqslant 5)$	
	A1	awrt 0.442 (Correct ans with no incorrect working 2/2)	
(iii)	M1	For using the model B(6, 0.11) allow B(6, 0.89) [Implied by 0.0017 or awrt 0.114]	
	A1	awrt 0.00174 (Correct ans with no incorrect working 2/2)	
(iv)		In part (iv) we can accept correct expressions or values for probabilities	
	1st M1	For using a negative binomial model implied by correct $\text{P}(Y=5)$ or $\text{P}(Y=6)$	
	2nd M1	Correct method to find $\text{P}(Y \leqslant 6)$	see table below
	3rd M1	At least two correct terms or $1-0.99817..$ from $1-\text{P}(V \leqslant 3)$	
	A1	awrt 0.00183	
(b)	1st M1	Forming the correct probability of Zac winning or identify a and r of GP	Allow for $p=(0.11)\times 0+(1-0.11)(1-p)$
	2nd M1	Using sum to infinity of a GP	Allow for $p=\dfrac{0.89}{1+0.89}$
	A1*	Previous method marks must be seen leading to an answer 0.471 (NOT awrt 0.471)	

a	4	5	6
$\text{P}(Y=a)$	1.46×10^{-4}	5.21×10^{-4}	1.16×10^{-3}
$\text{P}(V=a)$	1.74×10^{-3}	8.60×10^{-5}	1.77×10^{-6}

Q3.

Qu	Scheme	Marks	AO
(a)(i)	$[B \sim \text{Geo}(\frac{1}{3})]$ $\text{P}(B=4) = \left(\frac{2}{3}\right)^3 \times \frac{1}{3}$	M1	3.3
	$= \underline{\frac{8}{81}}$	A1	1.1b
(ii)	$\text{P}(B \leqslant 5) = 1 - \text{P}(B > 5)$ <u>or</u> $1 - \left(\frac{2}{3}\right)^5$	M1	2.1
	$= \underline{\frac{211}{243}}$	A1	1.1b
		(4)	
(b)	$\text{E}(B^2) = \text{Var}(B) + [\text{E}(B)]^2$	M1	2.1
	From formula booklet: $\text{E}(B) = \frac{1}{\frac{1}{3}} = 3$ and $\text{Var}(B) = \frac{1-\frac{1}{3}}{\left(\frac{1}{3}\right)^2} = 6$	B1	1.1b
	So $\text{E}(B^2) = 6 + 9 = \underline{\mathbf{15}}$	A1	1.1b
		(3)	
(c)	[Let R = no. of the spin when it first lands on red] $X = R \sim \text{Geo}(\frac{2}{3})$	M1	3.3
	Require $\text{E}(\text{e}^X) = \sum_{x=1}^{\infty} \text{e}^x \left(\frac{1}{3}\right)^{x-1} \frac{2}{3}$	M1	3.1a
	$= \frac{2\text{e}}{3} \sum_{x=1}^{\infty} \left(\frac{\text{e}}{3}\right)^{x-1}$	M1	2.1
	$= \frac{2\text{e}}{3} \times \frac{1}{1-\frac{\text{e}}{3}}$ or $\frac{2\text{e}}{3-\text{e}}$	A1	1.1b
	$\text{E}(\text{e}^X) = 19.297\ldots$ {> 15 = $\text{E}(B^2)$} so Tamara should **choose red** since it has the greater expected score	A1	2.2a
		(5)	
		(12 marks)	

	Notes
(a)(i)	M1 for selecting the correct model i.e. Geo(p) (May be implied by a correct expression) A1 for $\frac{8}{81}$ (= 0.098765… accept awrt 0.0988)
(ii)	M1 for a suitable strategy to use the geometric model to find a correct expression A1 for $\frac{211}{243}$ (= 0.868312…accept awrt 0.868)
(b)	M1 for a suitable strategy to find $\text{E}(B^2)$ [allow $\text{G}''(1) + \text{G}'(1)$] B1 for use of the correct formulae to find $\text{E}(B) = 3$ <u>and</u> $\text{Var}(B) = 6$ <u>or</u> $\text{G}''(1) = 12$ A1 for 15
SC	**Formula for E(B^2)** Allow M1B1A0 for $\text{E}(B^2) = \frac{2-p}{p^2}$ (o.e.)
(c)	1st M1 for choosing a suitable geometric model (sight of Geo($\frac{2}{3}$) or at least 3 correct probabilities) 2nd M1 for realising the need for appropriate expected value and using E(g(X)) [Need sum and f(x)] NB simply finding $\text{e}^{\text{E}(X)} = \text{e}^{1.5}$ = awrt 4.48 is M0 and probably no more marks. 3rd M1 for a suitable strategy to turn the expression into a sum that can be found 1st A1 for correct use of sum to infinity of geometric series 2nd A1 for interpreting the outcome of the calculations in terms of a solution to the problem must choose red and see the awrt 19.3 (and allow ft of their $\text{E}(B^2) < 19$)

Q4.

Question	Scheme	Marks	AOs
(a)	$\binom{7}{1}\times 0.15^2\times(0.85)^6$	M1	3.3
	$= 0.05940\ldots \quad = \text{awrt } \underline{\mathbf{0.0594}}$	A1	1.1b
		(2)	
(b)	The model is only valid if:		
	the games (trials) are **independent**	B1	3.5b
	the probability of winning a prize, 0.15, is **constant** for each game	B1	3.5b
		(2)	
(c)	$18=\frac{r}{p}$ and $6^2=\frac{r(1-p)}{p^2}$	M1 A1	3.1b 1.1b
	Solving: $2p=1-p$	M1	1.1b
	$p=\frac{1}{3}$ (>0.15) so **Mary** has the greater chance of winning a prize	A1	3.2a
		(4)	
	(8 marks)		

Notes	
(a)	**M1:** For selecting an appropriate model negative binomial or B(7, 0.15) with an extra success in 8th trial e.g. $\binom{7}{1}0.15\times(0.85)^6\times 0.15$ Allow $\binom{7}{1}0.85\times(0.15)^6\times 0.85$ may be implied by awrt 0.0594 **A1:** awrt 0.0594
(b)	**B1:** Stating the first assumption that games are independent **B1:** Stating the second assumption that the probability remains constant
(c)	**M1:** Forming an equation for the mean or for the standard deviation. **A1:** Both equations correct **M1:** Solving the 2 equations leading to $2p=1-p$ **A1:** For $p=\frac{1}{3}$ followed by a correct deduction