Conic Sections 1

Questions

Q1.

The rectangular hyperbola H has parametric equations

$$x = 4t, \quad y = \frac{4}{t} \qquad t \neq 0$$

The points P and Q on this hyperbola have parameters $t = \frac{1}{4}$ and $t = 2$ respectively.

The line l passes through the origin O and is perpendicular to the line PQ.

(a) Find an equation for l.

(3)

(b) Find a cartesian equation for H.

(1)

(c) Find the exact coordinates of the two points where l intersects H.

Give your answers in their simplest form.

(3)

(Total for question = 7 marks)

Q2.

The parabola C has equation $y^2 = 4ax$, where a is a constant and $a > 0$
The point $Q(aq^2, 2aq)$, $q > 0$, lies on the parabola C.

(a) Show that an equation of the tangent to C at Q is

$$qy = x + aq^2$$

(4)

The tangent to C at the point Q meets the x-axis at the point $X\left(-\frac{1}{4}a, 0\right)$ and meets the directrix of C at the point D.

(b) Find, in terms of a, the coordinates of D.

(4)

Given that the point F is the focus of the parabola C,

(c) find the area, in terms of a, of the triangle FXD, giving your answer in its simplest form.

(2)

(Total for question = 10 marks)

Q3.

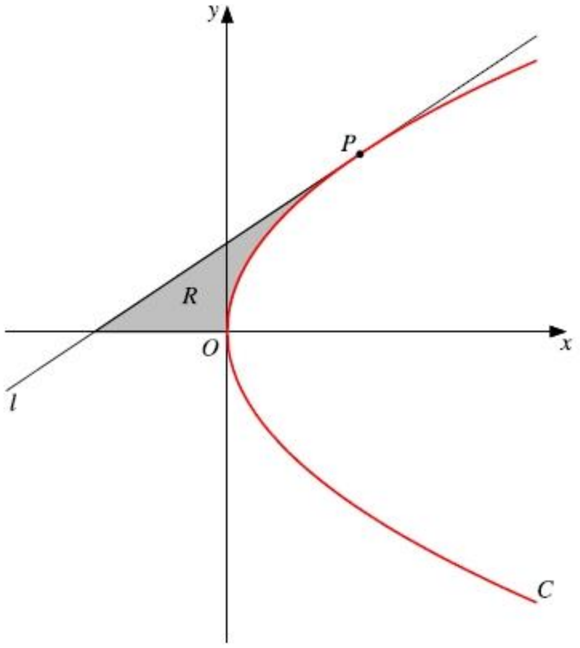

Diagram not drawn to scale

Figure 2

[You may quote without proof that for the general parabola $y^2 = 4ax$, $\frac{dy}{dx} = \frac{2a}{y}$]

The parabola C has equation $y^2 = 16x$.

(a) Deduce that the point $P(4p^2, 8p)$ is a general point on C.

(1)

The line l is the tangent to C at the point P.

(b) Show that an equation for l is

$$py = x + 4p^2$$

(3)

The finite region R, shown shaded in Figure 2, is bounded by the line l, the x-axis and the parabola C.

The line l intersects the directrix of C at the point B, where the y coordinate of B is $\frac{10}{3}$

Given that $p > 0$

(c) show that the area of R is 36

(8)

(Total for question = 12 marks)

Q4.

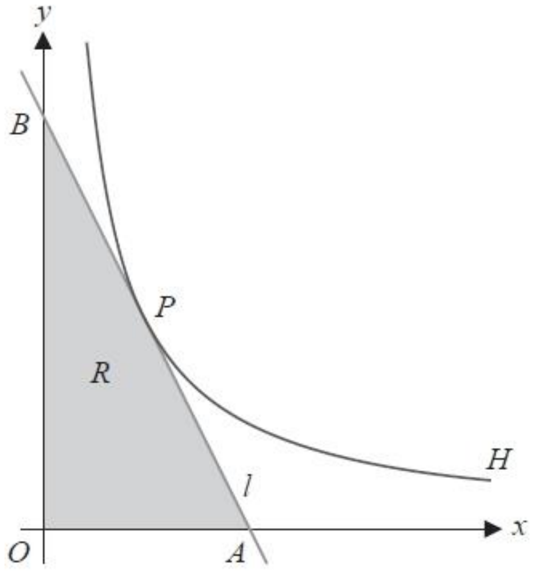

Figure 2

Figure 2 shows a sketch of part of the rectangular hyperbola *H* with equation

$$xy = c^2 \qquad x > 0$$

where *c* is a positive constant.

The point $P\left(ct, \frac{c}{t}\right)$ lies on *H*.

The line *l* is the tangent to *H* at the point *P*.

The line *l* crosses the *x*-axis at the point *A* and crosses the *y*-axis at the point *B*.

The region *R*, shown shaded in Figure 2, is bounded by the *x*-axis, the *y*-axis and the line *l*.

Given that the length *OB* is twice the length of *OA*, where *O* is the origin, and that the area of *R* is 32, find the exact coordinates of the point *P*.

(Total for question = 10 marks)

Q5.

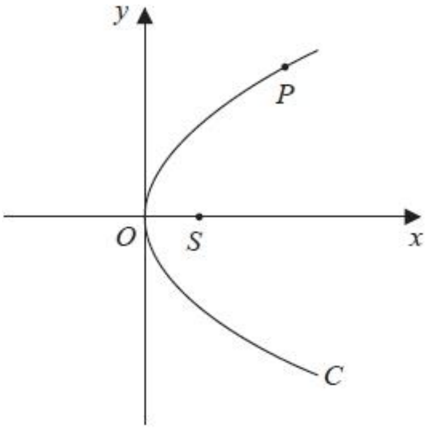

Figure 2

Figure 2 shows a sketch of the parabola *C* with equation $y^2 = 4ax$, where a is a positive constant. The point *S* is the focus of *C* and the point $P(ap^2, 2ap)$ lies on *C* where $p > 0$

(a) Write down the coordinates of *S*.

(1)

(b) Write down the length of *SP* in terms of *a* and *p*.

(1)

The point $Q(aq^2, 2aq)$, where $p \neq q$, also lies on *C*.
The point *M* is the midpoint of *PQ*.

Given that $pq = -1$

(c) prove that, as *P* varies, the locus of *M* has equation

$$y^2 = 2a(x - a)$$

(5)

(Total for question = 7 marks)

Q6.

The point *P* (ap^2, $2ap$), where *a* is a positive constant, lies on the parabola with equation

$$y^2 = 4ax$$

The normal to the parabola at *P* meets the parabola again at the point *Q* (aq^2, $2aq$)

(a) Show that

$$q = \frac{-p^2 - 2}{p}$$

(5)

(b) Hence show that

$$PQ^2 = \frac{ka^2}{p^4}\left(p^2 + 1\right)^n$$

where *k* and *n* are integers to be determined.

(5)

(Total for question = 10 marks)

Q7.

The parabola *C* has equation $y^2 = 10x$

The point *F* is the focus of *C*.

(a) Write down the coordinates of *F*.

(1)

The point *P* on *C* has *y* coordinate *q*, where $q > 0$

(b) Show that an equation for the tangent to *C* at *P* is given by

$$10x - 2qy + q2 = 0$$

(3)

The tangent to *C* at *P* intersects the directrix of *C* at the point *A*.

The point *B* lies on the directrix such that *PB* is parallel to the *x*-axis.

(c) Show that the point of intersection of the diagonals of quadrilateral *PBAF* always lies on the *y*-axis.

(5)

(Total for question = 9 marks)

Q8.

The rectangular hyperbola H has equation $xy = 36$

(a) Use calculus to show that the equation of the tangent to H at the point $P\left(6t, \frac{6}{t}\right)$ is

$$yt^2 + x = 12t$$

(3)

The point $Q\left(12t, \frac{3}{t}\right)$ also lies on H.

(b) Find the equation of the tangent to H at the point Q.

(2)

The tangent at P and the tangent at Q meet at the point R.

(c) Show that as t varies the locus of R is also a rectangular hyperbola.

(4)

(Total for question = 9 marks)

Q9.

The normal to the parabola $y^2 = 4ax$ at the point $P(ap^2, 2ap)$ passes through the parabola again at the point $Q(aq^2, 2aq)$.

The line OP is perpendicular to the line OQ, where O is the origin.

Prove that $p^2 = 2$

(9)

(Total for question = 9 marks)

Mark Scheme – Conic Sections 1

Q1.

Question Number	Scheme		Marks
(a)	$x = 4t,\ y = \frac{4}{t},\ t \neq 0$		
	$t = \frac{1}{4} \Rightarrow P(1, 16),\quad t = 2 \Rightarrow Q(8, 2)$	Coordinates for either P or Q are correctly stated. (Can be implied).	B1
	$m(PQ) = \frac{2-16}{8-1}\ \{= -2\}$	Finds the gradient of the chord PQ with $\frac{y_2 - y_1}{x_2 - x_1}$ then uses in $y = -\frac{1}{m}x$. Condone incorrect sign of gradient.	M1
	$m(l) = \frac{1}{2}$		
	So, $l : y = \frac{1}{2}x$ or $2y = x$	$y = \frac{1}{2}x$ or $2y = x$	A1 oe
			[3]
(b)	$xy = 16$ or $y = \frac{16}{x}$ or $x = \frac{16}{y}$	Correct Cartesian equation. Accept $\frac{4}{y} = \frac{x}{4}$ or $xy = 4^2$	B1 oe
			[1]
(c)	**Way 1** $\frac{1}{2}x = \frac{16}{x}$ $\{x^2 = 32\}$ | **Way 2** $\frac{4}{t} = \frac{1}{2}(4t)$ $\{t^2 = 2\}$ | **Way 3** $2y = \frac{16}{y}$ $\{y^2 = 8\}$	Attempts to substitute their l into either their Cartesian equation or parametric equations of H	M1
	$(4\sqrt{2},\ 2\sqrt{2}),\ (-4\sqrt{2},\ -2\sqrt{2})$	At least one set of coordinates (simplified or un-simplified) or $x = \pm 4\sqrt{2},\ y = \pm 2\sqrt{2}$	A1
		Both sets of simplified coordinates. Accept written in pairs as $x = 4\sqrt{2},\ y = 2\sqrt{2}$; $x = -4\sqrt{2},\ y = -2\sqrt{2}$	A1
			[3] 7

Q2.

Question Number	Scheme		Marks
	$y^2 = 4ax$, at $Q(aq^2, 2aq)$		
(a)	$y = 2\sqrt{a}\,x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = \sqrt{a}\,x^{-\frac{1}{2}}$ or $2y\frac{dy}{dx} = 4a$ or $\frac{dy}{dx} = 2a \times \frac{1}{2aq}$	$\frac{dy}{dx} = \pm k x^{-\frac{1}{2}}$ or $ky\frac{dy}{dx} = c$ or $\frac{\text{their } \frac{dy}{dq}}{\text{their } \frac{dx}{dq}}$	M1
	When $x = aq^2$, $m_T = \frac{dy}{dx} = \frac{\sqrt{a}}{\sqrt{aq^2}} = \frac{\sqrt{a}}{\sqrt{a}\,q} = \frac{1}{q}$ or when $y = 2aq$, $m_T = \frac{dy}{dx} = \frac{4a}{2(2aq)} = \frac{1}{q}$	$\frac{dy}{dx} = \frac{1}{q}$	A1
	T: $y - 2aq = \frac{1}{q}(x - aq^2)$	Applies $y - 2aq = (\text{their } m_T)(x - aq^2)$ or $y = (\text{their } m_T)x + c$ and an attempt to find c with gradient from calculus.	dM1
	T: $qy - 2aq^2 = x - aq^2$ **T**: $qy = x + aq^2$ *	cso	A1 *
			[4]
(b)	$X(-\frac{1}{4}a, 0) \Rightarrow 0 = -\frac{1}{4}a + aq^2$	Substitutes $x = -\frac{1}{4}a$ and $y = 0$ into **T**	M1
	$\Rightarrow \left\{q^2 = \frac{1}{4} \Rightarrow q = -\frac{1}{2}\text{ (reject)}\right\}$ $q = \frac{1}{2}$	$q = \frac{1}{2}$ oe	A1
	So, $\frac{1}{2}y = -a + a\left(\frac{1}{2}\right)^2$	Substitutes their "$q = \frac{1}{2}$" and $x = -a$ in **T** or finds $y_D = \frac{1}{q}(-a + aq^2)$	M1
	giving, $y = -\frac{3a}{2}$. So $D(-a, -\frac{3}{2}a)$ o.e.	$D(-a, -\frac{3}{2}a)$ o.e.	A1
			[4]
(c)			
	$\{\text{focus } F(a, 0)\}$		
Way 1	$\text{Area}(FXD) = \frac{1}{2}\left(\frac{5a}{4}\right)\left(\frac{3a}{2}\right) = \frac{15a^2}{16}$	Applies $\frac{1}{2}(\text{their } \lvert FX \rvert)(\text{their } \lvert y_D \rvert)$. If their $\left\lvert y_D = \frac{1}{q}(-a + aq^2)\right\rvert$ then require an attempt to sub for q to award M.	M1
		$\frac{15a^2}{16}$ or $0.9375a^2$	A1 cso
			[2]

(c) **Way 2**	$\text{Area}(FXD) = \frac{1}{2}\begin{vmatrix} a & -\frac{1}{4}a & -a & a \\ 0 & 0 & -\frac{3}{2}a & 0 \end{vmatrix}$ $= \frac{1}{2}\left\lvert\left(0 + \frac{3}{8}a^2 + 0\right) - \left(0 + 0 - \frac{3}{2}a^2\right)\right\rvert = \frac{15}{16}a^2$	A correct attempt to apply the shoelace method. $\frac{15a^2}{16}$ or $0.9375a^2$	M1 A1cao **[2]**
(c) **Way 3**	Rectangle – triangle 1 – triangle 2 $= 2a.\frac{3a}{2} - \frac{1}{2}.\frac{3a}{4}.\frac{3a}{2} - \frac{1}{2}.2a.\frac{3a}{2} = 3a^2 - \frac{9a^2}{16} - \frac{3a^2}{2}$	$\frac{15a^2}{16}$ or $0.9375a^2$	M1 A1cao
(c) **Way 4**	Attempts sine rule using appropriate choice from $FX = \frac{5a}{4},\ FD = \frac{5a}{2}, DX = \frac{3\sqrt{5}a}{4},\ \sin F = \frac{3}{5},\ \sin X = \frac{2}{\sqrt{5}}$	Uses Area $= \frac{1}{2}ab\sin C$ $\frac{15a^2}{16}$ or $0.9375a^2$	M1 A1cao **10**

	Question Notes
(c) **Way 1**	Do not award M1 if area of wrong triangle found e.g. $\frac{1}{2}.2a.\frac{3a}{2} = \frac{3a^2}{2}$

Q3.

Question	Scheme	Marks	AOs
(a)	$y^2 = (8p)^2 = 64p^2$ and $16x = 16(4p^2) = 64p^2$ $\Rightarrow$ $P(4p^2, 8p)$ is a general point on C	B1	2.2a
		(1)	
(b)	$y^2 = 16x$ gives $a = 4$, or $2y\frac{dy}{dx} = 16$ so $\frac{dy}{dx} = \frac{8}{y}$	M1	2.2a
	$l: y - 8p = \left(\frac{8}{8p}\right)(x - 4p^2)$	M1	1.1b
	leading to $py = x + 4p^2$ *	A1*	2.1
		(3)	
(c)	$B\left(-4, \frac{10}{3}\right)$ into l $\Rightarrow$ $\frac{10p}{3} = -4 + 4p^2$	M1	3.1a
	$6p^2 - 5p - 6 = 0 \Rightarrow (2p-3)(3p+2) = 0 \Rightarrow p = ...$	M1	1.1b
	$p = \frac{3}{2}$ and l cuts x-axis when $\frac{3}{2}(0) = x + 4\left(\frac{3}{2}\right)^2$ $\Rightarrow x = ...$	M1	2.1
	$x = -9$	A1	1.1b
	$p = \frac{3}{2} \Rightarrow P(9, 12) \Rightarrow$ Area$(R) = \frac{1}{2}(9 - -9)(12) - \int_0^9 4x^{\frac{1}{2}}dx$	M1	2.1
	$\int 4x^{\frac{1}{2}}dx = \frac{4x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} (+c)$ or $\frac{8}{3}x^{\frac{3}{2}} (+c)$	M1	1.1b
		A1	1.1b
	Area$(R) = \frac{1}{2}(18)(12) - \frac{8}{3}\left(9^{\frac{3}{2}} - 0\right) = 108 - 72 = 36$ *	A1*	1.1b
		(8)	

(c) ALT 1	$B\left(-4, \frac{10}{3}\right)$ into l $\Rightarrow$ $\frac{10p}{3} = -4 + 4p^2$	M1	3.1a
	$6p^2 - 5p - 6 = 0 \Rightarrow (2p-3)(3p+2) = 0 \Rightarrow p = ...$	M1	1.1b
	$p = \frac{3}{2}$ into l gives $\frac{3}{2}y = x + 4\left(\frac{3}{2}\right)^2$ $\Rightarrow x = ...$	M1	2.1
	$x = \frac{3}{2}y - 9$	A1	1.1b
	$p = \frac{3}{2} \Rightarrow P(9, 12) \Rightarrow$ Area$(R) = \int_0^{12}\left(\frac{1}{16}y^2 - \left(\frac{3}{2}y - 9\right)\right)dy$	M1	2.1
	$\int\left(\frac{1}{16}y^2 - \frac{3}{2}y + 9\right)dy = \frac{1}{48}y^3 - \frac{3}{4}y^2 + 9y \ (+c)$	M1	1.1b
		A1	1.1b
	Area$(R) = \left(\frac{1}{48}(12)^3 - \frac{3}{4}(12)^2 + 9(12)\right) - (0)$ $= 36 - 108 + 108 = 36$ *	A1*	1.1b
		(8)	

Question	Scheme	Marks	AOs
(c) ALT 2	$B\left(-4, \frac{10}{3}\right)$ into $l \Rightarrow \frac{10p}{3} = -4 + 4p^2$	M1	3.1a
	$6p^2 - 5p - 6 = 0 \Rightarrow (2p-3)(3p+2) = 0 \Rightarrow p = ...$	M1	1.1b
	$p = \frac{3}{2}$ and l cuts x-axis when $\frac{3}{2}(0) = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = ...$	M1	2.1
	$x = -9$	A1	1.1b
	$p = \frac{3}{2} \Rightarrow P(9, 12)$ and $x = 0$ in $l: y = \frac{2}{3}x + 6$ gives $y = 6$ $\Rightarrow \text{Area}(R) = \frac{1}{2}(9)(6) + \int_0^9 \left(\left(\frac{2}{3}x + 6\right) - \left(4x^{\frac{1}{2}}\right)\right) dx$	M1	2.1
	$\int \left(\frac{2}{3}x + 6 - 4x^{\frac{1}{2}}\right) dx = \frac{1}{3}x^2 + 6x - \frac{8}{3}x^{\frac{3}{2}} \ (+c)$	M1 A1	1.1b 1.1b
	$\text{Area}(R) = 27 + \left(\left(\frac{1}{3}(9)^2 + 6(9) - \frac{8}{3}\left(9^{\frac{3}{2}}\right)\right) - (0)\right)$ $= 27 + (27 + 54 - 72) = 27 + 9 = 36$ *	A1*	1.1b
		(8)	
		(12 marks)	

Notes	
(a)	
B1	Substitutes $y_p = 8p$ into y^2 to obtain $64p^2$ and substitutes $x_p = 4p^2$ into $16x$ to obtain $64p^2$ and concludes that P lies on C.
(b)	
M1	Uses the given formula to deduce the derivative. Alternatively, may differentiate using chain rule to deduce it.
M1	Applies $y - 8p = m(x - 4p^2)$, with their tangent gradient m, which is in terms of p. Accept use of $8p = m(4p^2) + c$ with a clear attempt to find c.
A1*	Obtains $py = x + 4p^2$ by cso.

Notes Continued	
(c)	
M1	Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into l.
M1	Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$
M1	Substitutes their p (which must be positive) and $y = 0$ into l and solves to give $x = ...$
A1	Finds that l cuts the x-axis at $x = -9$
M1	Fully correct method for finding the area of R. i.e. $\frac{1}{2}(\text{their } x_P - "-9")(\text{their } y_P) - \int_0^{\text{their } x_P} 4x^{\frac{1}{2}}\,dx$
M1	Integrates $\pm\lambda x^{\frac{1}{2}}$ to give $\pm\mu x^{\frac{3}{2}}$, where $\lambda, \mu \neq 0$
A1	Integrates $4x^{\frac{1}{2}}$ to give $\frac{8}{3}x^{\frac{3}{2}}$, simplified or un-simplified.
A1*	Fully correct proof leading to a correct answer of 36
(c) ALT 1	
M1	Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into l.
M1	Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$
M1	Substitutes their p (which must be positive) into l and rearranges to give $x = ...$
A1	Finds l as $x = \frac{3}{2}y - 9$
M1	Fully correct method for finding the area of R. i.e. $\int_0^{\text{their } y_P} \left(\frac{1}{16}y^2 - \text{their}\left(\frac{3}{2}y - 9\right)\right)dy$
M1	Integrates $\pm\lambda y^2 \pm \mu y \pm \nu$ to give $\pm\alpha y^3 \pm \beta y^2 \pm \nu y$, where $\lambda, \mu, \nu, \alpha, \beta \neq 0$
A1	Integrates $\frac{1}{16}y^2 - \left(\frac{3}{2}y - 9\right)$ to give $\frac{1}{48}y^3 - \frac{3}{4}y^2 + 9y$, simplified or un-simplified.
A1*	Fully correct proof leading to a correct answer of 36

Notes Continued	
(c) ALT 2	
M1	Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into l.
M1	Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$
M1	Substitutes their p (which must be positive) and $y = 0$ into l and solves to give $x = ...$
A1	Finds that l cuts the x-axis at $x = -9$
M1	Fully correct method for finding the area of R. i.e. $\frac{1}{2}(\text{their } 9)(\text{their } 6) + \int_0^{\text{their } x_P} \left(\text{their}\left(\frac{2}{3}x + 6\right) - \left(4x^{\frac{1}{2}}\right)\right)dy$
M1	Integrates $\pm\lambda x \pm \mu \pm \nu x^{\frac{1}{2}}$ to give $\pm\alpha x^2 \pm \mu x \pm \beta x^{\frac{3}{2}}$, where $\lambda, \mu, \nu, \alpha, \beta \neq 0$
A1	Integrates $\left(\frac{2}{3}x + 6\right) - \left(4x^{\frac{1}{2}}\right)$ to give $\frac{1}{3}x^2 + 6x - \frac{8}{3}x^{\frac{3}{2}}$, simplified or un-simplified.
A1*	Fully correct proof leading to a correct answer of 36

Q4.

Question	Scheme	Marks	AOs
	$H: xy = c^2,\ c > 0;\ P\left(ct, \frac{c}{t}\right)$ lies on H; $OB = 2OA$; Area$(OAB) = 32$		
Way 1	**Either** $y = \frac{c^2}{x} = c^2x^{-1} \Rightarrow \frac{dy}{dx} = -c^2x^{-2}$ or $-\frac{c^2}{x^2}$ **or** $xy = c^2 \Rightarrow x\frac{dy}{dx} + y = 0$ **or** $x = cp,\ y = \frac{c}{p} \Rightarrow \frac{dy}{dx} = \frac{dy}{dp}.\frac{dp}{dx} = -\left(\frac{c}{p^2}\right)\left(\frac{1}{c}\right)$; condone $t \equiv p$ **and** so, at $P\left(ct, \frac{c}{t}\right)$, $m_T = -\frac{1}{t^2}$	M1	3.1a
	$y - \frac{c}{t} = "-\frac{1}{t^2}"(x - ct)$	M1	1.1b
	or $\frac{c}{t} = "-\frac{1}{t^2}"(ct) + b \Rightarrow y = "-\frac{1}{t^2}"x +$ their $b \Rightarrow y = -\frac{1}{t^2}x + \frac{2c}{t}$	A1	1.1b
	$y = 0 \Rightarrow x = 2ct\ \{\Rightarrow x_A = 2ct\},\ x = 0 \Rightarrow y = \frac{2c}{t}\ \left\{\Rightarrow y_B = \frac{2c}{t}\right\}$	M1 A1	1.1b 1.1b
	$\{OB = 2OA \Rightarrow\}\ \frac{2c}{t} = 2(2ct) \Rightarrow t = ...$	M1	2.1
	$\left\{t^2 = \frac{1}{2} \Rightarrow\right\}\ t = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$ or awrt 0.707	A1	1.1b
	$\{\text{Area } (OAB) = 32 \Rightarrow\}\ \frac{1}{2}(2ct)\left(\frac{2c}{t}\right) = 32 \Rightarrow c = ...\ \{\Rightarrow c = 4\}$	M1	2.1
	Deduces the *numerical* value x_P and y_P using their values of t and c	M1	2.2a
	$P(2\sqrt{2}, 4\sqrt{2})$ **or** P(awrt 2.83, awrt 5.66) **or** $x = 2\sqrt{2}$ **and** $y = 4\sqrt{2}$	A1	1.1b
		(10)	

Question	Scheme	Marks	AOs
Way 2	Same requirement as the 1st M mark in Way 1	M1	3.1a
	e.g. $\left\{t = \frac{1}{\sqrt{2}} \Rightarrow P\left(\frac{c}{\sqrt{2}}, \sqrt{2}c\right) \Rightarrow\right\}\ y - \sqrt{2}c = -2\left(x - \frac{c}{\sqrt{2}}\right)$	M1	1.1b
	using $m_T = -2$ and their P which has been found by a correct method	A1	1.1b
	$y = 0 \Rightarrow x = \sqrt{2}c\ \{\Rightarrow x_A = \sqrt{2}c\},\ x = 0 \Rightarrow y = 2\sqrt{2}c\ \{\Rightarrow y_B = 2\sqrt{2}c\}$	M1 A1	1.1b 1.1b
	$\{OB = 2OA \Rightarrow\}\ m_T = -2$ and their $m_T = -\frac{1}{t^2} = -2 \Rightarrow t = ...$	M1	2.1
	$\left\{t^2 = \frac{1}{2} \Rightarrow\right\}\ t = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$ or awrt 0.707 $\left\{\Rightarrow P\left(\frac{c}{\sqrt{2}}, \sqrt{2}c\right)\right\}$	A1	1.1b
	$\{\text{Area } (OAB) = 32 \Rightarrow\}\ \frac{1}{2}\sqrt{2}c\left(2\sqrt{2}c\right) = 32 \Rightarrow c = ...\ \{\Rightarrow c = 4\}$	M1	2.1
	Deduces the *numerical* value x_P and y_P using their values of t and c	M1	2.2a
	$P(2\sqrt{2}, 4\sqrt{2})$ **or** P(awrt 2.83, awrt 5.66) **or** $x = 2\sqrt{2}$ **and** $y = 4\sqrt{2}$	A1	1.1b
		(10)	
			(10 marks)

Question	Scheme	Marks	AOs
	$H: xy = c^2, c > 0$; $P\left(ct, \frac{c}{t}\right)$ lies on H; $OB = 2OA$; Area$(OAB) = 32$		
Way 3	Same requirement as the 1st M mark in Way 1	M1	3.1a
	e.g. $y - 8\sqrt{2} = -2(x - 0)$ or $y - 0 = -2(x - 4\sqrt{2})$	M1	1.1b
	using $m_T = -2$ and either their $A(4\sqrt{2}, 0)$ or their $B(0, 8\sqrt{2})$ which have been found by a correct method	A1	1.1b
	$\{$Area $(OAB) = 32,\ OB = 2OA \Rightarrow\}$ $\frac{1}{2}(x)(2x) = 32 \Rightarrow x = ...$	M1	2.1
	$x = 4\sqrt{2}$ $\{\Rightarrow x_A = 4\sqrt{2}\}$ or $y = 8\sqrt{2}$ $\{\Rightarrow y_B = 8\sqrt{2}\}$	A1	1.1b
	$\{OB = 2OA \Rightarrow\}$ $m_T = -2$ and their $m_T = -\frac{1}{t^2} = -2 \Rightarrow t = ...$	M1	2.1
	$\left\{t^2 = \frac{1}{2} \Rightarrow\right\}$ $t = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$ or awrt 0.707 $\left\{\Rightarrow P\left(\frac{c}{\sqrt{2}}, \sqrt{2}c\right)\right\}$	A1	1.1b
	$\sqrt{2}c - 8\sqrt{2} = -2\left(\frac{c}{\sqrt{2}} - 0\right) \Rightarrow c = ...$ $\{\Rightarrow c = 4\}$	M1	1.1b
	Deduces the *numerical* value x_P and y_P using their values of t and c	M1	2.2a
	$P(2\sqrt{2}, 4\sqrt{2})$ or P(awrt 2.83, awrt 5.66) or $x = 2\sqrt{2}$ **and** $y = 4\sqrt{2}$	A1	1.1b
		(10)	

Way 4	Complete process substituting their $y - 8\sqrt{2} = -2(x - 0)$ or $y - 0 = -2(x - 4\sqrt{2})$ **into** $xy = c^2$ and applying $b^2 - 4ac = 0$ to their resulting $2x^2 - 8\sqrt{2}x + c^2 = 0$	M1	3.1a
	e.g. $y - 8\sqrt{2} = -2(x - 0)$ or $y - 0 = -2(x - 4\sqrt{2})$	M1	1.1b
	using $m_T = -2$ and either their $A(4\sqrt{2}, 0)$ or their $B(0, 8\sqrt{2})$ which have been found by a correct method	A1	1.1b
	$\{$Area $(OAB) = 32,\ OB = 2OA \Rightarrow\}$ $\frac{1}{2}(x)(2x) = 32 \Rightarrow x = ...$	M1	2.1
	$x = 4\sqrt{2}$ $\{\Rightarrow x_A = 4\sqrt{2}\}$ or $y = 8\sqrt{2}$ $\{\Rightarrow y_B = 8\sqrt{2}\}$	A1	1.1b
	dependent on 2nd M mark $\{xy = c^2 \Rightarrow\}$ $x(-2x + 8\sqrt{2}) = c^2$ $\{\Rightarrow 2x^2 - 8\sqrt{2}x + c^2 = 0\}$	dM1	2.1
	or $\{xy = c^2 \Rightarrow\}$ $\frac{1}{2}\left(8\sqrt{2} - y\right)y = c^2$ $\{\Rightarrow y^2 - 8\sqrt{2}y + 2c^2 = 0\}$	A1	1.1b
	$\{b^2 - 4ac = 0 \Rightarrow\}$ $(8\sqrt{2})^2 - 4(2)(c^2) = 0 \Rightarrow c = ...$ $\{\Rightarrow c = 4\}$	M1	1.1b
	Deduces the *numerical* value x_P and y_P using their value of c	M1	2.2a
	$P(2\sqrt{2}, 4\sqrt{2})$ or P(awrt 2.83, awrt 5.66) or $x = 2\sqrt{2}$ **and** $y = 4\sqrt{2}$	A1	1.1b
		(10)	
Note:	**For the final M1 mark in Way 1, Way 2, Way 3 and Way 4** Allow final M1 for a correct method which gives any of $x_P = 2\sqrt{2}$ or $y_P = 4\sqrt{2}$ or $x_P =$ awrt 2.83 or $y_P =$ awrt 5.66 o.e.		

Notes for Question	
Way 1	
M1:	Establishes the gradient of the tangent by differentiating $xy = c^2$ • to give $\frac{dy}{dx} = \pm kx^{-2}; k \neq 0$, or • by the product rule to give $\pm x\frac{dy}{dx} \pm y$, or • by parametric differentiation to give $\left(\text{their } \frac{dy}{dt}\right) \times \frac{1}{\left(\text{their } \frac{dx}{dt}\right)}$, condoning $p \equiv t$ and attempt to use $P\left(ct, \frac{c}{t}\right)$ to write down the gradient of the tangent to the curve in terms of t
M1:	Correct straight line method for an equation of a tangent where $m_T\ (\neq m_N)$ is found by using calculus. **Note:** m_T must be a function of t for this mark
A1:	Correct equation of the tangent which can be simplified or un-simplified
M1:	Attempts to find either the x-coordinate of A or the y-coordinate of B
A1:	**Both** {x-coordinate of A is} $2ct$ **and** the {y-coordinate of B is} $\frac{2c}{t}$
M1:	See scheme
A1:	See scheme
M1:	See scheme
M1:	See scheme
A1:	See scheme

Way 2	
M1:	Same description as the 1st M mark in Way 1
M1:	See scheme
A1:	Correct equation of the tangent which can be simplified or un-simplified
M1:	Attempts to find either the x-coordinate of A or the y-coordinate of B
A1:	**Both** {x-coordinate of A is} $\sqrt{2}c$ **and** the {y-coordinate of B is} $2\sqrt{2}c$
M1:	Recognising that the gradient of the tangent is -2 and puts this equal to their $\frac{dy}{dx}$ and finds $t = ...$
A1:	See scheme
M1:	See scheme
M1:	See scheme
A1:	See scheme
Way 3	
M1:	Same description as the 1st M mark in Way 1
M1:	See scheme
A1:	Correct equation of the tangent which can be simplified or un-simplified
M1:	Uses $y = 2x$ and Area $(OAB) = 32$ to find either x_A or y_B
A1:	Either {x-coordinate of A is} $4\sqrt{2}$ or the {y-coordinate of B is} $8\sqrt{2}$
M1:	Recognising that the gradient of the tangent is -2 and puts this equal to their $\frac{dy}{dx}$ and finds $t = ...$
A1:	See scheme
M1:	Substitutes their P (which is in terms of c, and has come from a correct method) into the equation of the tangent and finds $c = ...$
M1:	See scheme
A1:	See scheme

Notes for Question	
Way 4	
M1:	See scheme
M1:	See scheme
A1:	Correct equation of the tangent which can be simplified or un-simplified
M1:	Uses $y = 2x$ and Area $(OAB) = 32$ to find either x_A or y_B
A1:	Either {x-coordinate of A is} $4\sqrt{2}$ or the {y-coordinate of B is} $8\sqrt{2}$
M1:	See scheme
A1:	See scheme
M1:	See scheme
M1:	See scheme
A1:	See scheme

Q5.

Question	Scheme	Marks	AOs
(a)	$(a,0)$	B1	1.1b
		(1)	
(b)	$SP = ap^2 + a$ Note that if focus-directrix property not used may use Pythagoras: E.g. $SP = \sqrt{4a^2p^2 + (ap^2 - a)^2} = ... = ap^2 + a$	B1	1.1b
		(1)	
(c)	M has coordinates $\left(\dfrac{ap^2 + aq^2}{2}, \dfrac{2ap + 2aq}{2}\right)$	B1	1.1b
	$y^2 = a^2\left(p^2 + 2pq + q^2\right)$	M1	1.1b
	$y^2 = a^2\left(p^2 - 2 + q^2\right)$	A1	2.1
	$2a(x-a) = 2a\left(\dfrac{1}{2}ap^2 + \dfrac{1}{2}aq^2 - a\right) = a^2\left(p^2 + q^2 - 2\right)$	M1	1.1b
	$\Rightarrow y^2 = 2a(x-a)$*	A1*	2.1
		(5)	
	Alternative for (c)		
	M has coordinates $\left(\dfrac{ap^2 + aq^2}{2}, \dfrac{2ap + 2aq}{2}\right)$	B1	1.1b
	$\dfrac{y}{a} = p + q$	M1	1.1b
	$\dfrac{y^2}{a^2} = p^2 + q^2 + 2pq = p^2 + q^2 - 2$	A1	2.1
	$\dfrac{2x}{a} = p^2 + q^2$	M1	1.1b
	$\dfrac{y^2}{a^2} = \dfrac{2x}{a} - 2 \Rightarrow y^2 = 2a(x-a)$*	A1*	2.1
		(5)	
			(7 marks)

Notes
(a) B1: Correct coordinates (b) B1: Correct expression (c) B1: Correct coordinates for the midpoint M1: Squares their y coordinate of the midpoint A1: Uses $pq = -1$ to obtain a correct expression for y^2 M1: Attempts $2a(x - a)$ using the x coordinate of their midpoint and attempts to simplify A1*: Fully correct completion to show $y^2 = 2a(x-a)$ Alternative B1: Correct coordinates for the midpoint M1: Uses their y coordinate of the midpoint to find $p + q$ A1: Square and uses $pq = -1$ to obtain a correct expression for y^2/a^2 M1: Uses the x coordinate of their midpoint to find $p^2 + q^2$ A1*: Fully correct completion to show $y^2 = 2a(x-a)$

Q6.

Question	Scheme	Marks	AOs
(a)	$\frac{dy}{dx}=\frac{2a}{2ap}=\frac{1}{p}$ or $y=2\sqrt{a}\sqrt{x}\Rightarrow\frac{dy}{dx}=\frac{\sqrt{a}}{\sqrt{x}}=\frac{1}{p}$ or $2y\frac{dy}{dx}=4a\Rightarrow\frac{dy}{dx}=\frac{2a}{y}=\frac{1}{p}$	B1	1.1b
	$y-2ap=-p\left(x-ap^2\right)$	M1	2.1
	$2aq-2ap=-p\left(aq^2-ap^2\right)$ $pq^2+2q-2p-p^3=0$	A1	1.1b
	$(q-p)\left(pq+p^2+2\right)=0\Rightarrow q=\ldots$	M1	3.1a
	$q=\frac{-p^2-2}{p}$ *	A1*	1.1b
		(5)	
(b)	$PQ^2=\left(ap^2-aq^2\right)^2+(2ap-2aq)^2$	M1	1.1b
	$=a^2(p-q)^2(p+q)^2+4a^2(p-q)^2$ $=a^2(p-q)^2\left[(p+q)^2+4\right]$ $=a^2\left(2p+\frac{2}{p}\right)^2\left[\left(-\frac{2}{p}\right)^2+4\right]$	M1 A1	2.1 1.1b
	$=\frac{4a^2}{p^2}\left(p^2+1\right)^2\frac{4}{p^2}\left(p^2+1\right)=\frac{16a^2}{p^4}\left(p^2+1\right)^3$	A1 A1	1.1b 1.1b
		(5)	
			(10 marks)

Notes
(a) B1: Deduces the correct tangent gradient M1: Correct strategy for the equation of the normal A1: Correct equation in terms of p and q M1: Applies a correct strategy for finding q in terms of p. E.g. uses the fact that $q = p$ is known and uses inspection or long division to find the other root A1*: Correct proof with no errors **Alternative:** B1: As above M1A1: $\frac{2aq-2ap}{aq^2-ap^2}\times\frac{1}{p}=-1$ M1: Finds gradient of PQ and uses product of gradients $= -1$ A1: Correct equation M1A1: As above (b) M1: Applies Pythagoras correctly to find PQ^2 M1: Uses their q in terms of p to obtain an expression in terms of p only A1: Correct expression in any form in terms of p only A1: $k = 16$ or $n = 3$ A1: $k = 16$ and $n = 3$

Q7.

Question	Scheme	Marks	AOs
(a)	$\left(\frac{5}{2},0\right)$ o.e.	B1	2.2a
		(1)	
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{5}{q}$	B1	1.1b
	At P, $x=\frac{q^2}{10}$ so tangent has equation $y-q=\text{their}\frac{5}{q}\left(x-\frac{q^2}{10}\right)$ or $q=\left(\text{their}\frac{5}{q}\right)\left(\frac{q^2}{10}\right)+c\Rightarrow c=\ldots$ to reach an equation for y	M1	1.1b
	$\Rightarrow qy-q^2=5x-\frac{q^2}{2}\Rightarrow 10x-2qy+q^2=0$ * cso or $\Rightarrow y=\frac{5}{q}x+\frac{q}{2}\Rightarrow 10x-2qy+q^2=0$ * cso	A1*	2.1
		(3)	

(c)	B is $\left(-\frac{5}{2}, q\right)$ o.e.	**B1**	2.2a
	So diagonal BF has equation $\frac{y-0}{q-0} = \frac{x-\frac{5}{2}}{-\frac{5}{2}-\frac{5}{2}}$ or $y = -\frac{q}{5}\left(x - \frac{5}{2}\right)$	**M1**	1.1b
	(AP is a tangent so) diagonals meet when $10x - 2q\left(-\frac{q}{5}\left(x - \frac{5}{2}\right)\right) + q^2 = 0$ or $x = \frac{2qy - q^2}{10}$ therefore $y = -\frac{q}{5}\left(\frac{2qy - q^2}{10} - \frac{5}{2}\right)$ leading to $y = \dots$ $\left\{y = \frac{25q + q^3}{50 + 2q^2}\right\}$	**dM1**	3.1a
	$\Rightarrow 10x + \frac{2q^2}{5}x - q^2 + q^2 = 0 \Rightarrow x\left(10 + \frac{2q^2}{5}\right) = 0$ or $x = \frac{1}{10}\left(2q\left(\frac{25q + q^3}{50 + 2q^2}\right) - q^2\right)$	**M1**	1.1b
	But $10 + \frac{2q^2}{5} > 0$ so not zero, hence $x = 0$, so the intersection lies on the y-axis.	**A1**	2.4

	Or achieves $x = 0$ (with no errors), so the intersection lies on the y axis.		
		(5)	
	Alternative for the last three marks		
	When $x = 0$ for BF $y = -\frac{q}{5}\left(-\frac{5}{2}\right) = ...$ or for AP $2qy = q^2 \Rightarrow y = ...$	M1	1.1b
	For BF y intercept is $\frac{q}{2}$ **and** for AP y intercept is $\frac{q}{2}$	M1	3.1a
	Since both diagonals always cross the y-axis at the same place, their intersection must always be on the y axis.	A1	2.4
			(9 marks)

Notes:

(a)
B1: Deduces correct coordinates.

(b)
B1: Using or deriving $\frac{dy}{dx} = \frac{5}{q}$

M1: Finds the equation of the tangent using the equation of a line formula with $y_1 = q$, $x = \frac{q^2}{10}$ (or clear attempt at it) and $m = \frac{2 \times \text{their } 'a'}{q}$.

If uses $y = mx + c$ must find a value for c and substitute back to find an equation for the tangent
A1*: Completes correctly to the given equation, no errors seen.

(c)
B1: B is $\left(-\frac{5}{2}, q\right)$ seen or used.

M1: A correct method to find the equation of the diagonal BF using their coordinates of F and B
dM1: Uses the printed answer in (b) and their equation of the diagonal BF to form an equation just involving x or solves the two diagonals simultaneously to find an expression for y
M1: Correctly factors out the x to achieve $x(...) = 0$ or uses their expression for y to find an expression for x

A1: Conclusion given including reference to $10 + \frac{2q^2}{5} \neq 0$

Alternative for last three marks
M1: Attempts to find the y intercept for at least one of the two diagonals.
M1: Finds y intercept for both diagonals in order to compare
A1: Both intercepts correct and suitable conclusion giving reference to both diagonals always crossing y-axis at same point.

Q8.

Question	Scheme	Marks	AOs
(a)	$y + x\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x} = \frac{\frac{6}{t}}{6t} = -\frac{1}{t^2}$ or $y = \frac{36}{x} \Rightarrow \frac{dy}{dx} = -\frac{36}{x^2} = -\frac{36}{(6t)^2} = -\frac{1}{t^2}$ or $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{-6t^{-2}}{6} = -\frac{1}{t^2}$	M1	1.1b
	$y - \frac{6}{t} = "-\frac{1}{t^2}"(x - 6t)$	M1	1.1b
	$yt^2 + x = 12t$*	A1 *	2.1
		(3)	
(b)	$\frac{dy}{dx} = -\frac{y}{x} = \frac{\frac{3}{t}}{12t} = -\frac{1}{4t^2}$ and $y - \frac{3}{t} = '-\frac{1}{4t^2}'(x - 12t)$	M1	1.1b
	$y - \frac{3}{t} = -\frac{1}{4t^2}(x - 12t)$ o.e such as $4yt^2 + x = 24t$	A1	1.1b
		(2)	
(c)	E.g. $\left.\begin{array}{r} 4yt^2 + x = 24t \\ yt^2 + x = 12t \end{array}\right\} 3yt^2 = 12t \Rightarrow y = \ldots$ and $x = 12t - yt^2 = \ldots$	M1	2.1
	$x = 8t$ and $y = \frac{4}{t}$	A1	1.1b
	$xy = \ldots$	dM1	1.1b
	$xy = 32$ hence **rectangular hyperbola**	A1	2.4
		(4)	
			(9 marks)

Notes:

(a)

M1: Differentiates implicitly, directly or parametrically to find the gradient at the point P in terms of t. Allow slips in coefficients, as long as method is clear.

M1: Finds the equation of the tangent at the point P using their gradient (not reciprocal etc). If using $y = mx + c$ must proceed to find c and substitute back in to equation.

A1*: The correct equation for the tangent at the point P from correct working.

(b)

M1: Finds the new gradient (any method as above) and proceeds to find the equation of the tangent at the point Q. Alternatively replaces t by $2t$ in the answer to (a).

A1: Correct equation - any form, need not be simplified and isw after a correct equation.

(c)

M1: Solves their simultaneous equations to find both the x and y coordinate for the point R.

A1: Correct point of intersection, it does not need to be simplified.

dM1: Dependent on the first method mark. Multiplies x by y to reach a constant.

A1: Shows that $xy = 32$ and hence **rectangular hyperbola**

Q9.

Question	Scheme	Marks	AOs
	$y^2 = 4ax \Rightarrow 2y\frac{dy}{dx} = 4a$	M1	2.1
	$\frac{dy}{dx} = \frac{2a}{y} \Rightarrow$ Gradient of normal is $\frac{-y}{2a} = -p$	A1	1.1b
	Equation of normal is : $y - 2ap = -p(x - ap^2)$	M1	1.1b
	Normal passes through $Q(aq^2, 2aq)$ so $2aq + apq^2 = 2ap + ap^3$	M1	3.1a
	Grad OP × Grad $OQ = -1 \Rightarrow \frac{2ap}{ap^2}\frac{2aq}{aq^2} = -1$	M1	2.1
	$q = \frac{-4}{p}$	A1	1.1b
	$2a\left(\frac{-4}{p}\right) + ap\left(\frac{16}{p^2}\right) = 2ap + ap^3 \Rightarrow p^4 + 2p^2 - 8 = 0$	M1	2.1
	$(p^2 - 2)(p^2 + 4) = 0 \Rightarrow p^2 = ...$	M1	1.1b
	Hence (as $p^2 + 4 \neq 0$), $p^2 = 2$*	A1*	1.1b
		(9)	
ALT 1	First three marks as above and then as follows.	M1 A1 M1	2.1 1.1b 1.1b
	Solves $y^2 = 4ax$ and their normal simultaneously to find, in terms of a and p, either $x_Q\left(= ap^2 + 4a + \frac{4a}{p^2}\right)$ or $y_Q\left(= -2ap - \frac{4a}{p}\right)$	M1	3.1a
	Finds the second coordinate of Q in terms of a and p	M1	1.1b
	Both $x_Q = ap^2 + 4a + \frac{4a}{p^2}$ and $y_Q = -2ap - \frac{4a}{p}$	A1	1.1b
	Grad OP × Grad $OQ = -1 \Rightarrow \frac{2ap}{ap^2} \times \frac{-2ap - \frac{4a}{p}}{ap^2 + 4a + \frac{4a}{p^2}} = -1$	M1	2.1
	Simplifies expression and solves: $4p^2 + 8 = p^4 + 4p^2 + 4$ $\Rightarrow p^4 - 4 = 0 \Rightarrow (p^2 - 2)(p^2 + 2) = 0 \Rightarrow p^2 = ...$	M1	2.1
	Hence (as $p^2 + 2 \neq 0$), $p^2 = 2$*	A1*	1.1b
		(9)	

Question	Scheme	Marks	AOs
ALT 2	First three marks as above and then as follows.	M1 A1 M1	2.1 1.1b 1.1b
	Solves $y^2=4ax$ and their normal simultaneously to find, in terms of a and p, either $x_Q\left(=ap^2+4a+\dfrac{4a}{p^2}\right)$ or $y_Q\left(=-2ap-\dfrac{4a}{p}\right)$	M1	3.1a
	Forms a relationship between p and q from their first coordinate: either $y_Q=2a\left(-p-\dfrac{2}{p}\right)\Rightarrow q=-p-\dfrac{2}{p}$ or $x_Q=a\left(p+\dfrac{2}{p}\right)^2\Rightarrow q=\pm\left(p+\dfrac{2}{p}\right)$	M1	2.1
	$q=-p-\dfrac{2}{p}$ (if x coordinate used the correct root must be clearly identified before this mark is awarded).	A1	1.1b
	Grad OP × Grad $OQ=-1\Rightarrow\dfrac{2ap}{ap^2}\times\dfrac{2aq}{aq^2}=-1\left(\Rightarrow q=-\dfrac{4}{p}\right)$.	M1	2.1
	Sets $q=-p-\dfrac{2}{p}=-\dfrac{4}{p}$ and solves to give $p^2=\ldots$	M1	1.1b
	Hence $\left(\text{as } q=p+\dfrac{2}{p}=-\dfrac{4}{p}\text{ gives no solution}\right)$, $p^2=2$ (only)*	A1*	1.1b
		(9)	
			(9 marks)

Notes	
M1	Begins proof by differentiating and using the perpendicularity condition at point P in order to find the equation of the normal.
A1	Correct gradient of normal, $-p$ only.
M1	Use of $y-y_1=m(x-x_1)$. Accept use of $y=mx+c$ and then substitute to find c.
M1	Substitute coordinates of Q into their equation to find an equation relating p and q.
M1	Use of $m_1m_2=-1$ with OP and OQ to form a second equation relating p and q.
A1	$q=\dfrac{-4}{p}$ only.
M1	Solves the simultaneous equations and cancels a from their results to obtain a quadratic equation in p^2 only.
M1	Attempts to solve their quadratic in p^2. Usual rules.
A1*	Correct solution leading to given answer stated. No errors seen.

Notes continued	
ALT 1	
M1A1M1	As main scheme.
M1	Solves $y^2 = 4ax$ and their normal simultaneously to find one of the coordinates for Q in terms of a and p as shown.
M1	Finds the second coordinate of Q in terms of a and p.
A1	Both coordinates correct in terms of a and p.
M1	Use of $m_1 m_2 = -1$ with OP and OQ. i.e. $\frac{2ap}{ap^2} \times \frac{\text{their } y_Q}{\text{their } x_Q} = -1$ with coordinates of P and their expressions for x_Q and y_Q.
M1	Cancels the a's, simplifies to a quadratic in p^2 and solves the quadratic. Usual rules.
A1*	Correct solution leading to the given answer stated. No errors seen.
ALT 2	
M1A1M1	As main scheme.
M1	Solves $y^2 = 4ax$ and their normal simultaneously to find one of the coordinates for Q in terms of a and p as shown.
M1	Uses their coordinate to form a relationship between p and q. Allow $q = \left(p + \frac{2}{p}\right)$ for this mark.
A1	For $q = -p - \frac{2}{p}$. If the x coordinate was used to find q then consideration of the negative root is needed for this mark. Allow for $q = \pm\left(p + \frac{2}{p}\right)$.
M1	Use of $m_1 m_2 = -1$ with OP and OQ to form a second equation relating p and q only.
M1	Equates expressions for q and attempts to solve to give $p^2 = ...$
A1*	Correct solution leading to the given answer stated. No errors seen. If x coordinate used, invalid solution must be rejected.