Centres of Mass and Plane Figures

Questions

Q1.

Three particles of masses $3m$, $4m$ and $2m$ are placed at the points $(-2, 2)$, $(3, 1)$ and (p, p) respectively.

The value of p is such that the distance of the centre of mass of the three particles from the point $(0, 0)$ is as small as possible.

Find the value of p.

(Total for question = 7 marks)

Q2.

Three particles of masses $2m$, $3m$ and km are placed at the points with coordinates $(3a, 2a)$, $(a, -4a)$ and $(-3a, 4a)$ respectively.

The centre of mass of the three particles lies at the point with coordinates $(\bar{x}, \bar{y})$.

(a) (i) Find $\bar{x}$ in terms of a and k

(ii) Find $\bar{y}$ in terms of a and k

(4)

Given that the distance of the centre of mass of the three particles from the point $(0, 0)$ is $\frac{1}{3}a$

(b) find the possible values of k

(2)

(Total for question = 6 marks)

Q3.

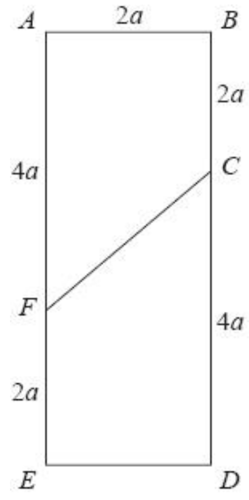

Figure 1

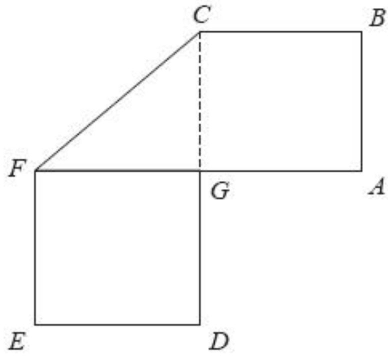

Figure 2

The uniform rectangular lamina *ABDE*, shown in Figure 1, has side *AB* of length 2*a* and side *BD* of length 6*a*. The point *C* divides *BD* in the ratio 1 : 2 and the point *F* divides *EA* in the ratio 1 : 2. The rectangular lamina is folded along *FC* to produce the folded lamina *L*, shown in Figure 2.

(a) Show that the centre of mass of *L* is $\frac{16}{9}a$ from *EF*.

(5)

The folded lamina, *L*, is freely suspended from *C* and hangs in equilibrium.

(b) Find the size of the angle between *CF* and the downward vertical.

(4)

(Total for question = 9 marks)

Q4.

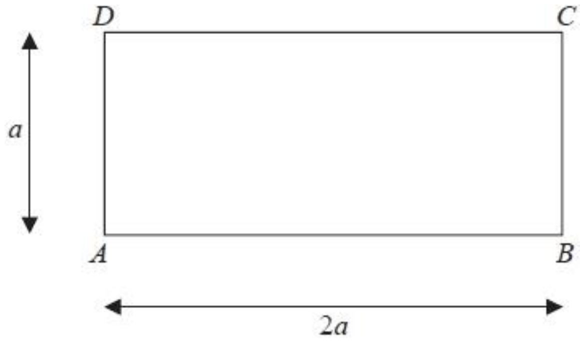

Figure 1

Figure 1 shows a uniform rectangular lamina *ABCD* with $AB = 2a$ and $AD = a$
The mass of the lamina is $6m$.

A particle of mass $2m$ is attached to the lamina at *A*, a particle of mass m is attached to the lamina at *B* and a particle of mass $3m$ is attached to the lamina at *D*, to form a loaded lamina *L* of total mass $12m$.

(a) Write down the distance of the centre of mass of *L* from *AB*. You must give a reason for your answer.

(2)

(b) Show that the distance of the centre of mass of *L* from *AD* is $\frac{2a}{3}$

(3)

A particle of mass km is now also attached to *L* at *D* to form a new loaded lamina *N*.

(c) Show that the distance of the centre of mass of *N* from *AB* is $\frac{(k+6)a}{(k+12)}$

(4)

When *N* is freely suspended from *A* and is hanging in equilibrium, the side *AB* makes an angle α with the vertical, where $\tan \alpha = \frac{3}{2}$

(d) Find the value of k.

(6)

(Total for question = 15 marks)

Q5.

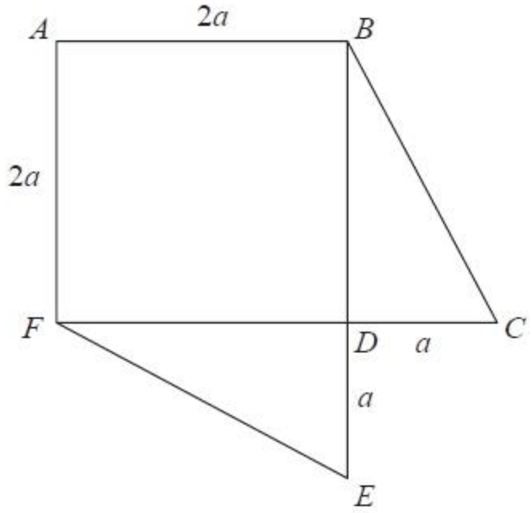

Figure 2

The lamina *L*, shown in Figure 2, consists of a uniform square lamina *ABDF* and two uniform triangular laminas *BDC* and *FDE*. The square has sides of length 2*a*. The two triangles are identical.

The straight lines *BDE* and *FDC* are perpendicular with *BD* = *DF* = 2*a* and *DC* = *DE* = *a*.
The mass per unit of area of the square is *M*.
The mass per unit area of each triangle is 3*M*.
The centre of mass of *L* is at the point *G*.

(a) Without doing any calculations, explain why *G* lies on *AD*.

(1)

(b) Show that the distance of *G* from *D* is $\frac{\sqrt{2}}{2}a$

(7)

The lamina *L* is freely suspended from *B* and hangs in equilibrium.

(c) Find the size of the angle between *BE* and the downward vertical.

(3)

(Total for question = 11 marks)

Q6.

Unless otherwise indicated, whenever a numerical value of g is required, take g = 9.8 m s^{-2} and give your answer to either 2 significant figures or 3 significant figures.

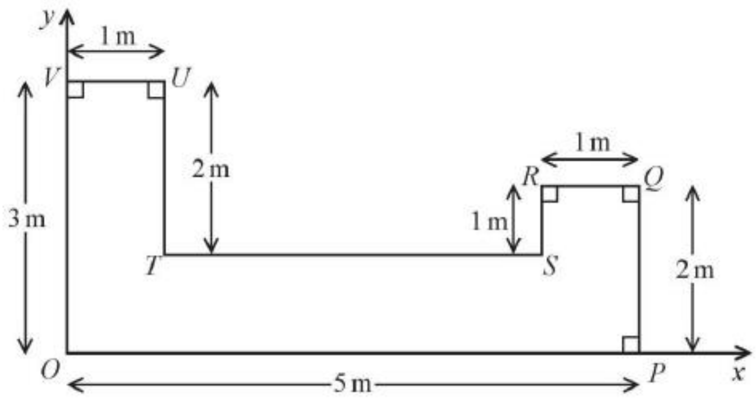

Figure 1

Figure 1 shows the shape and dimensions of a template *OPQRSTUV* made from thin uniform metal.
OP = 5 m, *PQ* = 2 m, *QR* = 1 m, *RS* = 1 m, *TU* = 2 m, *UV* = 1 m, *VO* = 3 m.
Figure 1 also shows a coordinate system with *O* as origin and the *x*-axis and *y*-axis along *OP* and *OV* respectively. The unit of length on both axes is the metre.
The centre of mass of the template has coordinates ($\bar{x}$, $\bar{y}$).

(a) (i) Show that $\bar{y}$ = 1

(ii) Find the value of $\bar{x}$.

(7)

A new design requires the template to have its centre of mass at the point (2.5, 1). In order to achieve this, two circular discs, each of radius *r* metres, are removed from the template which is shown in Figure 1, to form a new template *L*. The centre of the first disc is (0.5, 0.5) and the centre of the second disc is (0.5, *a*) where *a* is a constant.

(b) Find the value of *r*.

(4)

(c) (i) Explain how symmetry can be used to find the value of *a*.

(ii) Find the value of *a*.

(2)

The template *L* is now freely suspended from the point *U* and hangs in equilibrium.

(d) Find the size of the angle between the line *TU* and the horizontal.

(3)

(Total for question = 16 marks)

Q7.

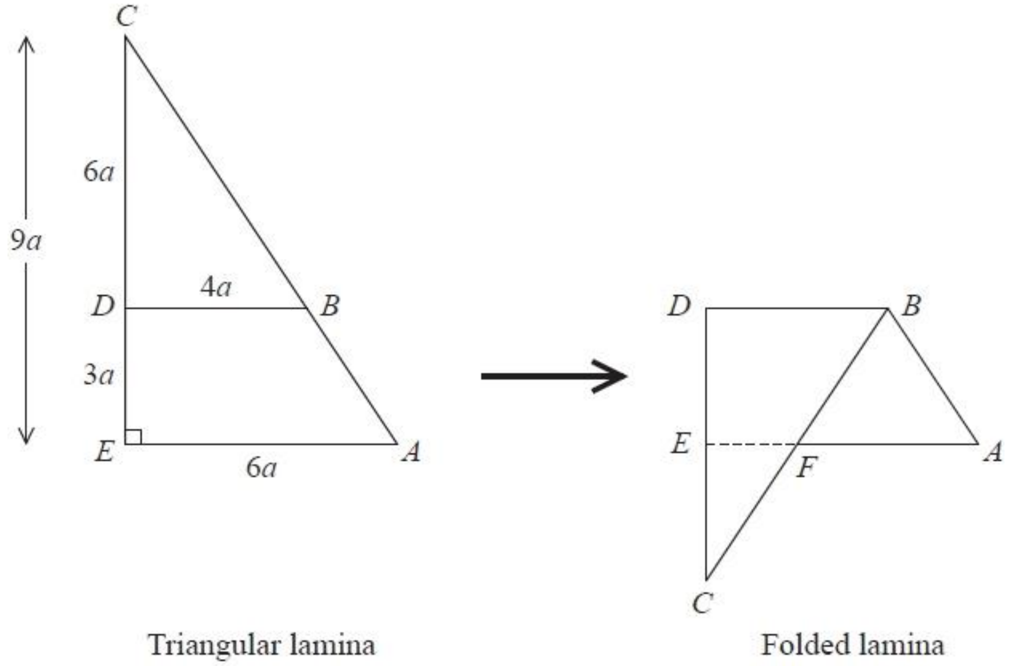

Figure 2

The uniform triangular lamina *ABCDE* is such that angle *CEA* = 90°, *CE* = 9*a* and *EA* = 6*a*. The point *D* lies on *CE*, with *DE* = 3a. The point *B* on *CA* is such that *DB* is parallel to *EA* and *DB* = 4*a*. The triangular lamina is folded along the line *DB* to form the folded lamina *ABDECF*, as shown in Figure 2.

The distance of the centre of mass of the triangular lamina from *DC* is d_1

The distance of the centre of mass of the folded lamina from *DC* is d_2

(a) Explain why $d_1 = d_2$

(1)

The folded lamina is freely suspended from *B* and hangs in equilibrium with *BA* inclined at an angle α to the downward vertical through *B*.

(b) Find, to the nearest degree, the size of angle α.

(9)

(Total for question = 10 marks)

Q8.

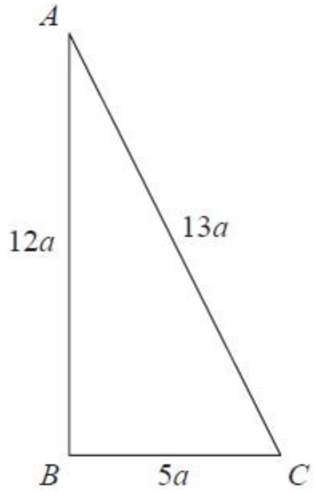

Figure 1

A thin uniform rod, of total length $30a$ and mass M, is bent to form a frame. The frame is in the shape of a triangle ABC, where $AB = 12a$, $BC = 5a$ and $CA = 13a$, as shown in Figure 1.

(a) Show that the centre of mass of the frame is $\frac{3}{2}a$ from AB.

(4)

The frame is freely suspended from A. A horizontal force of magnitude kMg, where k is a constant, is applied to the frame at B. The line of action of the force lies in the vertical plane containing the frame. The frame hangs in equilibrium with AB vertical.

(b) Find the value of k.

(3)

(Total for question = 7 marks)

Q9.

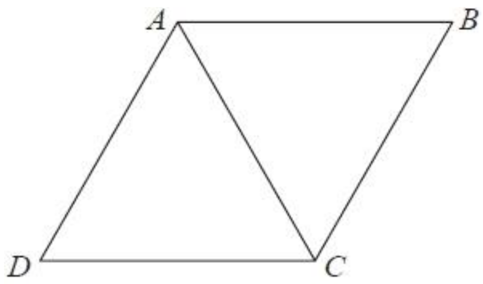

Figure 1

Five identical uniform rods are joined together to form the rigid framework *ABCD* shown in Figure 1. Each rod has weight W and length $4a$. The points A, B, C and D all lie in the same plane.

The centre of mass of the framework is at the point G.

(a) Explain why G is the midpoint of AC.

(1)

The framework is suspended from the ceiling by two vertical light inextensible strings. One string is attached to the framework at A and the other string is attached to the framework at B. The framework hangs freely in equilibrium with AB horizontal.

(b) Find

(i) the tension in the string attached at A,
(ii) the tension in the string attached at B.

(4)

A particle of weight kW is now attached to the framework at D and a particle of weight $2kW$ is now attached to the framework at C. The framework remains in equilibrium with AB horizontal and the strings vertical.

Either string will break if the tension in it exceeds $6W$.

(c) Find the greatest possible value of k.

(4)

(Total for question = 9 marks)

Q10.

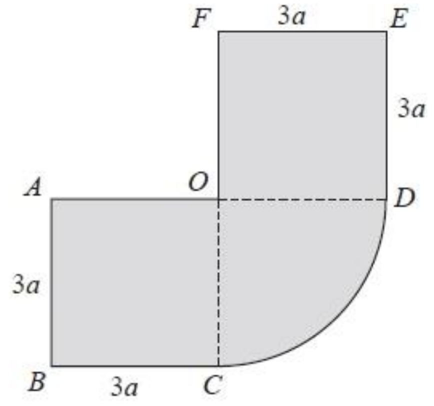

Figure 3

The uniform plane lamina shown in Figure 3 is formed from two squares, *ABCO* and *ODEF*, and a sector *ODC* of a circle with centre *O*. Both squares have sides of length 3*a* and *AO* is perpendicular to *OF*. The radius of the sector is 3*a*

[*In part* (a) *you may use, without proof, any of the centre of mass formulae given in the formulae booklet*.]

(a) Show that the distance of the centre of mass of the sector *ODC* from *OC* is $\frac{4a}{\pi}$

(3)

(b) Find the distance of the centre of mass of the lamina from *FC*

(4)

The lamina is freely suspended from *F* and hangs in equilibrium with *FC* at an angle $\theta°$ to the downward vertical.

(c) Find the value of θ

(4)

(Total for question = 11 marks)

Q11.

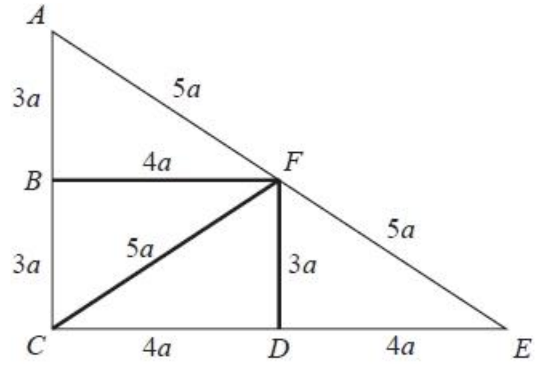

Figure 1

Nine uniform rods are joined together to form the rigid framework *ABCDEFA*, with *AB* = *BC* = *DF* = 3*a*, *BF* = *CD* = *DE* = 4*a* and *AF* = *FE* = *CF* = 5*a*, as shown in Figure 1. All nine rods lie in the same plane.

The mass per unit length of each of the rods *BF*, *CF* and *DF* is twice the mass per unit length of each of the other six rods.

(a) Find the distance of the centre of mass of the framework from *AC*

(4)

The mass of the framework is *M*. A particle of mass *kM* is attached to the framework at *E* to form a loaded framework.

When the loaded framework is freely suspended from *F*, it hangs in equilibrium with *CE* horizontal.

(b) Find the exact value of *k*

(3)

(Total for question = 7 marks)

Q12.

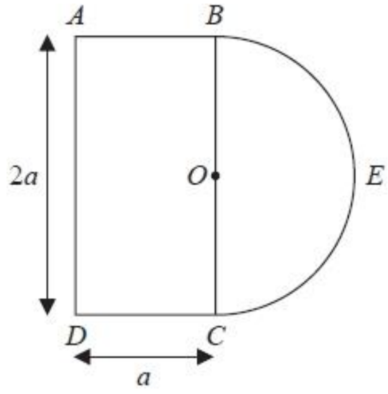

Figure 2

Uniform wire is used to form the framework shown in Figure 2.

In the framework

- *ABCD* is a rectangle with $AD = 2a$ and $DC = a$
- *BEC* is a semicircular arc of radius a and centre O, where O lies on BC

The diameter of the semicircle is *BC* and the point *E* is such that *OE* is perpendicular to *BC*.

The points *A*, *B*, *C*, *D* and *E* all lie in the same plane.

(a) Show that the distance of the centre of mass of the framework from *BC* is

$$\frac{a}{6+\pi}$$

(5)

The framework is freely suspended from *A* and hangs in equilibrium with *AE* at an angle $\theta°$ to the downward vertical.

(b) Find the value of θ.

(4)

The mass of the framework is M.

A particle of mass kM is attached to the framework at *B*.

The centre of mass of the loaded framework lies on *OA*.

(c) Find the value of k.

(3)

(Total for question = 12 marks)

Q13.

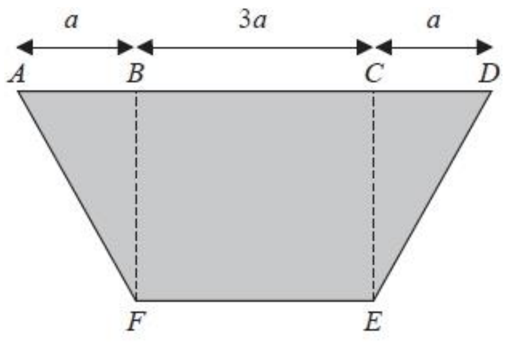

Figure 1

A uniform plane lamina is in the shape of an isosceles trapezium *ABCDEF*, as shown shaded in Figure 1.

- *BCEF* is a square
- *AB* = *CD* = *a*
- *BC* = 3a

(a) Show that the distance of the centre of mass of the lamina from *AD* is $\frac{11a}{8}$

(5)

The mass of the lamina is *M*

The lamina is suspended by two light vertical strings, one attached to the lamina at *A* and the other attached to the lamina at *F*

The lamina hangs freely in equilibrium, with *BF* horizontal.

(b) Find, in terms of *M* and *g*, the tension in the string attached at *A*

(2)

(Total for question = 7 marks)

Q14.

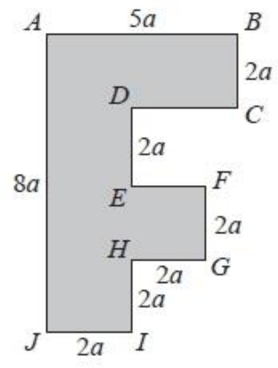

Figure 3

The uniform lamina *ABCDEFGHIJ* is shown in Figure 3.

The lamina has $AJ = 8a$, $AB = 5a$ and $BC = DE = EF = FG = GH = HI = IJ = 2a$.

All the corners are right angles.

(a) Show that the distance of the centre of mass of the lamina from *AJ* is $\frac{49}{26}a$

(5)

A light inextensible rope is attached to the lamina at *A* and another light inextensible rope is attached to the lamina at *B*. The lamina hangs in equilibrium with both ropes vertical and *AB* horizontal. The weight of the lamina is *W*.

(b) Find, in terms of *W*, the tension in the rope attached to the lamina at *B*.

(3)

The rope attached to *B* breaks and subsequently the lamina hangs freely in equilibrium, suspended from *A*.

(c) Find the size of the angle between *AJ* and the downward vertical.

(5)

(Total for question = 13 marks)

Q15.

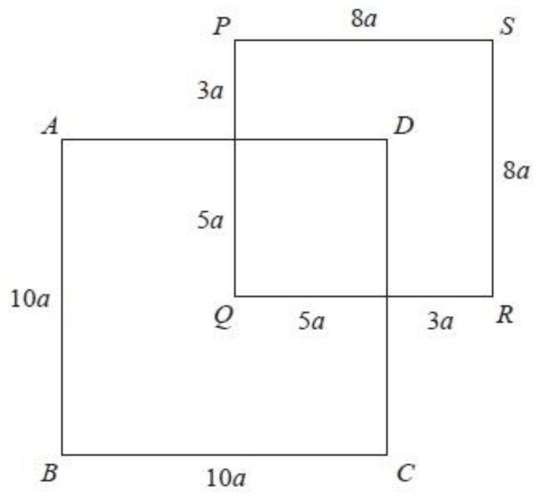

Figure 1

A uniform rod of length $72a$ is cut into pieces. The pieces are used to make two rigid squares, *ABCD* and *PQRS*, with sides of length $10a$ and $8a$ respectively. The two squares are joined to form the rigid framework shown in Figure 1.

The squares both lie in the same plane with the rod *AB* parallel to the rod *PQ*.

Given that

- *AD* cuts *PQ* in the ratio 3 : 5
- *DC* cuts *QR* in the ratio 5 : 3

(a) explain why the centre of mass of square *ABCD* is at *Q*.

(1)

(b) Find the distance of the centre of mass of the framework from *B*.

(5)

(Total for question = 6 marks)

Q16.

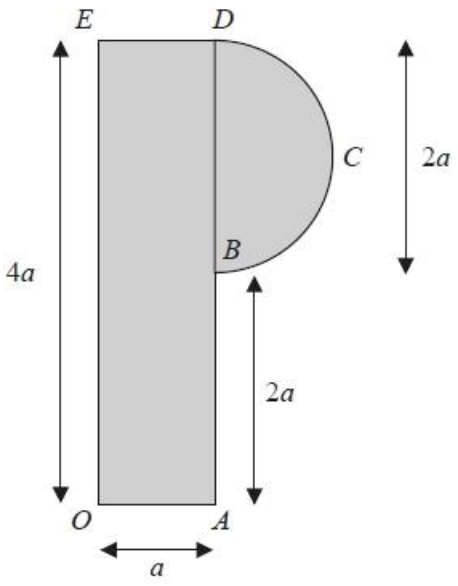

Figure 1

A letter P from a shop sign is modelled as a uniform plane lamina which consists of a rectangular lamina, *OABDE*, joined to a semicircular lamina, *BCD*, along its diameter *BD*.

$OA = ED = a$, $AB = 2a$, $OE = 4a$, and the diameter $BD = 2a$, as shown in Figure 1.

Using the model,

(a) find, in terms of π and a, the distance of the centre of mass of the letter P,

from (i) *OE*
(ii) *OA*

(6)

The letter P is freely suspended from *O* and hangs in equilibrium. The angle between *OE* and the downward vertical is α.

Using the model,

(b) find the exact value of $\tan \alpha$

(2)

(Total for question = 8 marks)

Q17.

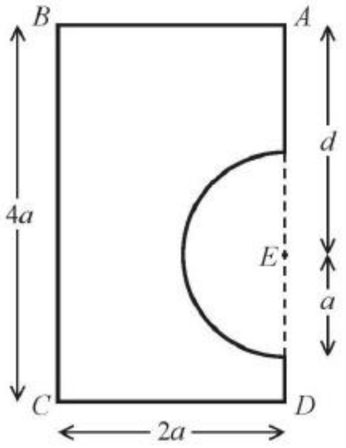

Figure 3

A shop sign is modelled as a uniform rectangular lamina *ABCD* with a semicircular lamina removed.

The semicircle has radius *a*, *BC* = 4*a* and *CD* = 2*a*.

The centre of the semicircle is at the point *E* on *AD* such that *AE* = *d*, as shown in Figure 3.

(a) Show that the centre of mass of the sign is $\frac{44a}{3(16-\pi)}$ from *AD*.

(4)

The sign is suspended using vertical ropes attached to the sign at *A* and at *B* and hangs in equilibrium with *AB* horizontal.
The weight of the sign is *W* and the ropes are modelled as light inextensible strings.

(b) Find, in terms of *W* and *π*, the tension in the rope attached at *B*.

(2)

The rope attached at *B* breaks and the sign hangs freely in equilibrium suspended from *A*, with *AD* at an angle *α* to the downward vertical.

Given that tan $\alpha = \frac{11}{18}$

(c) find *d* in terms of *a* and *π* .

(6)

(Total for question = 12 marks)

Mark Scheme – Centres of Mass of Plane Figures

Q1.

Question	Scheme	Marks	AOs
	Correct method to find an equation in $\bar{x}$	M1	1.1b
	$-3\times2+4\times3+2\times p=9\bar{x}\quad(6+2p=9\bar{x})$	A1	1.1b
	Correct method to find an equation in $\bar{y}$	M1	1.1b
	$3\times2+4\times1+2\times p=9\bar{y}\quad 10+2p=9\bar{y}$	A1	1.1b
	$(9\bar{x})^2+(9\bar{y})^2=(6+2p)^2+(10+2p)^2$ $(=136+64p+8p^2)$	M1	1.1b
	$=8\left[(p+4)^2+17-16\right]$	M1	3.1a
	$\Rightarrow p=-4$	A1	2.2a
	(7 marks)		

Notes:

M1	Take moments about axis parallel to $x=0$. Need all terms and dimensionally correct.
A1	Correct unsimplified equation in $\bar{x}$. Seen or implied
M1	Take moments about axis parallel to $y=0$. Need all terms and dimensionally correct.
A1	Correct unsimplified equation in $\bar{y}$. Seen or implied
M1	Use of Pythagoras to find distance (or square of distance) from origin
M1	Correct strategy to find value of p to minimise the distance e.g. use of calculus or complete the square
A1	Correct answer only

Q2.

Question	Scheme	Marks	AOs
(a)	Moments about y-axis	M1	3.4
	$((5+k)m\bar{x} = -3kma + 6ma + 3ma)$ $\quad \bar{x} = \dfrac{(9-3k)a}{5+k}$	A1	1.1b
	Moments about x-axis	M1	3.4
	$((5+k)m\bar{y} = 4kma + 4ma - 12ma)$ $\quad \bar{y} = \dfrac{(4k-8)a}{5+k}$	A1	1.1b
		(4)	
	$\Rightarrow 9\left[(9-3k)^2 + (4k-8)^2\right] = (5+k)^2$ $(224k^2 - 1072k + 1280 = 0)$	M1	3.1a
	$\Rightarrow k = \dfrac{5}{2}$, or $k = \dfrac{16}{7}$	A1	2.2a
		(2)	
(b)			
			(6 marks)

Notes:	
(a)	
M1	Moments equation to find $\bar{x}$ – need all terms and dimensionally correct Allow with m cancelled throughout Allow if they have a common factor of g
A1	Correct expression for $\bar{x}$ Any equivalent form. Allow recovery
M1	Moments equation to find $\bar{y}$ – need all terms and dimensionally correct Allow with m cancelled throughout Allow if they have a common factor of g
A1	Correct expression for $\bar{y}$ Any equivalent form. Allow recovery
(b)	
M1	Use their moments equations to form a quadratic equation in k only with no square root (need not simplify)
A1	Obtain both correct values. Accept 2.5 and 2.3 or better (2.2857…)

Q3.

<table>
<tr><th>Q.</th><th>Scheme</th><th>Marks</th><th>Notes</th></tr>
<tr><td rowspan="2">a</td><td rowspan="2">
<table>
<tr><td></td><td>FGDE</td><td>2 of CFG</td><td>CBAG</td><td>L</td></tr>
<tr><td>Mass ratio</td><td>4</td><td>4</td><td>4</td><td>12</td></tr>
<tr><td>C of M from EF</td><td>a</td><td>$\frac{4}{3}a$</td><td>$3a$</td><td>d</td></tr>
</table>
</td><td>B1</td><td>Mass ratios</td></tr>
<tr><td>B1</td><td>Distances from EF or an alternative vertical axis</td></tr>
<tr><td></td><td>$12d = 4a + 4\times\frac{4}{3}a + 4\times 3a$</td><td>M1</td><td>Moments about EF or equivalent
Need all terms and dimensionally correct</td></tr>
<tr><td></td><td></td><td>A1</td><td>Correct unsimplified equation</td></tr>
<tr><td></td><td>$12d = 16a\times\frac{4}{3},\ d = \frac{16}{9}a$</td><td>A1</td><td>Sufficient working to justify ***given answer***</td></tr>
<tr><td></td><td></td><td>(5)</td><td></td></tr>
<tr><td>a alt</td><td>Splitting the rectangle into a pair of trapeziums gives mass ratios 1: 1: 2</td><td>B1</td><td></td></tr>
<tr><td></td><td>Distance of c of m of ABCF is
$\frac{8}{9}a$ from AF and $\frac{22}{9}a$ from EF</td><td>B1</td><td></td></tr>
<tr><td></td><td>$2a - a\times\frac{8}{9} + a\times\frac{22}{9}\left(=\frac{32}{9}a\right) = 2d$</td><td>M1A1</td><td></td></tr>
<tr><td></td><td>$d = \frac{16}{9}a$</td><td>A1</td><td></td></tr>
<tr><td></td><td></td><td>(5)</td><td></td></tr>
<tr><td rowspan="2">a alt</td><td rowspan="2">Square -square -triangle+triangle
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<tr><td></td><td>L sq</td><td>S sq</td><td>-tri</td><td>+tri</td><td>L</td></tr>
<tr><td>mass</td><td>4</td><td>1</td><td>$\frac{1}{2}$</td><td>$\frac{1}{2}$</td><td>3</td></tr>
<tr><td>From EF</td><td>2a</td><td>3a</td><td>$\frac{2a}{3}$</td><td>$\frac{4a}{3}$</td><td>d</td></tr>
</table>
</td><td>B1</td><td></td></tr>
<tr><td>B1</td><td></td></tr>
<tr><td></td><td>$4\times 2a - 3a - \frac{1}{2}\times\frac{2a}{3} + \frac{1}{2}\times\frac{4a}{3}\left(=\frac{16a}{3}\right) = 3d$</td><td>M1A1</td><td></td></tr>
<tr><td></td><td>$d = \frac{16}{9}a$</td><td>A1</td><td></td></tr>
<tr><td></td><td></td><td>(5)</td><td></td></tr>
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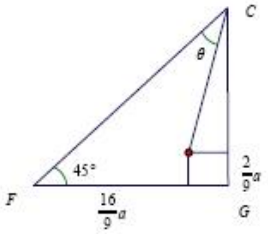
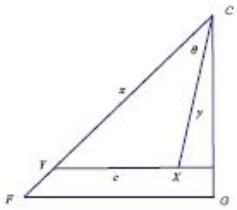
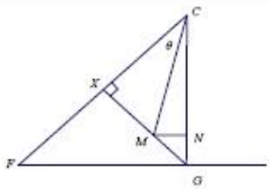
Q.	Scheme	Marks	Notes
b			
	Symmetry $\Rightarrow$ c of m $\frac{16}{9}a$ from C	B1	For vertical distance – allow for $\frac{20}{9}a$ or equivalent
	$\tan^{-1}\frac{1}{8}$ $\left(\tan^{-1}8\right)$	M1	Correct trig to find relevant angle (using $\frac{2}{9}a$ horizontally and their vertical $\neq 2a$)
	$7.125°$	A1	$(7.1°, 82.9°, 0.124\text{rads}, 1.45\text{rads})$
	$\theta = 37.9°$	A1 (4)	$38°$ or better $(37.874....°,\ 0.66 \text{ rads})$
b alt			Using cosine rule With $x = \frac{16}{9}\sqrt{2}a$, $c = \frac{14}{9}a$ $y = \sqrt{65} \times \frac{2a}{9}$
	Symmetry $\Rightarrow$ c of m $\frac{2}{9}a$ from FG	B1	
	$\cos\theta = \frac{x^2+y^2-c^2}{2xy} = \frac{9}{\sqrt{130}} = 0.789...$	M1A1	
	$\theta = 37.9°$	A1 (4)	
balt			
	Symmetry $\Rightarrow$ c of m $\frac{2}{9}a$ from FG	B1	i.e. on bisector of angle G
	$\tan\theta = \frac{MX}{CX} = \frac{\sqrt{2}a - \frac{2\sqrt{2}}{9}a}{\sqrt{2}a} = \frac{7}{9}$	M1A1	Using Isosceles triangles
	$\theta = 37.9°$	A1 (4)	
		[9]	

Q4.

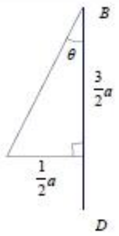
Question	Scheme	Marks	AOs
(a)	$\frac{1}{2}a$	B1	1.1b
	Loaded lamina has a mass distribution which is symmetrical about the perpendicular bisector of AD	B1	2.4
		(2)	
(b)	Moments about AD	M1	3.1a
	$6ma + m.2a = 12m\overline{x}$	A1	1.1b
	$\overline{x} = \frac{2a}{3}$ *	A1*	2.2a
		(3)	
(c)	Moments about AB	M1	3.1a
	$kma + 12m.\frac{1}{2}a = (k+12)m\overline{y}$	A1	1.1.b
		A1	1.1b
	$\overline{y} = \frac{(k+6)a}{(k+12)}$ *	A1*	2.2a
		(4)	

Question	Scheme	Marks	AOs
(d)	Moments about AD	M1	3.1a
	$\overline{x}_1 = \frac{8a}{(k+12)}$	A1	1.1b
	Use of $\tan\alpha = \frac{\overline{y}}{\overline{x}_1}$	M1	1.1b
	$\frac{3}{2} = \frac{\frac{(k+6)a}{(k+12)}}{\frac{8a}{(k+12)}}$	A1	1.1b
	Solve for k	M1	1.1b
	$k = 6$	A1	1.1b
	SC: For use of $(\tan\alpha =)\frac{\text{their } \overline{y}}{\text{their } \overline{x}} = \frac{3}{2}$, M1A1M0A0M0A0		
		(6)	
			(15 marks)

Notes
(a) **B1**: cao **B1**: clear explanation
(b) **M1**: Correct no. of terms and dimensionally correct (allow cancelled m's) **A1**: A correct equation **A1***: Correctly obtained printed answer
(c) **M1**: Correct no. of terms and dimensionally correct (allow cancelled m's) **A1**: Correct equation with one error **A1**: Correct equation **A1***:Correctly obtained printed answer
(d) **M1**: Correct no. of terms and dimensionally correct (allow cancelled m's) **A1**: Correct distance **M1**: Correct use of tan but allow reciprocal **A1**: Correct equation **M1**: Solve for k **A1**: cao

Q5.

Question	Scheme	Marks	AOs
(a)	L is symmetrical about AD	B1	2.4
		(1)	
(b)	<table><tr><td></td><td>$ABDF$</td><td>BCD</td><td>DEF</td><td>L</td></tr><tr><td>Mass ratio</td><td>$4a^2 \times M$</td><td>$a^2 \times 3M$</td><td>$a^2 \times 3M$</td><td>$10a^2 \times M$</td></tr><tr><td>C of M from BE</td><td>$-a$</td><td>$+\frac{a}{3}$</td><td>$-\frac{2a}{3}$</td><td>x</td></tr></table>		
	Mass ratios	B1	1.2
	Distances from BE	B1	1.2
	Moments equation	M1	2.1
	$-a \times 4a^2M + \frac{a}{3} \times 3a^2M - \frac{2a}{3} \times 3a^2M = 10a^2M \times x$ $(-4a + a - 2a = 10x)$	A1	1.1b
	$x = -\frac{5a}{10} = -\frac{a}{2}$	A1	1.1b
	Use symmetry and Pythagoras	M1	1.1a
	Distance from $D = \sqrt{\frac{a^2}{4} + \frac{a^2}{4}} = \frac{\sqrt{2}}{2}a$ *	A1*	2.2a
		(7)	

(c)			
	Trig ratio of a relevant angle	M1	1.2
	$\tan\theta = \frac{1}{3}$ or $\cos\theta = \dfrac{\frac{10}{4}a^2 + 4a^2 - \frac{2}{4}a^2}{2\times\frac{\sqrt{10}}{2}a\times 2a} = \frac{6}{2\sqrt{10}}$	A1ft	1.1b
	$\theta = 18.4°$	A1	1.1b
		(3)	
			(11 marks)

Notes
(a) B1: Any equivalent statement about the symmetry
(b) B1: Correct mass ratios **B1:** Distance ratios from any horizontal or vertical axis **M1:** Moments equation for complete lamina about any horizontal or vertical axis. Must be dimensionally correct **A1:** Correct unsimplified equation for their axes **A1:** Correct horizontal or vertical distance from *D* **M1:** Use of Pythagoras with their distance **A1*:** Obtain given answer from correct working.
(c) M1: Trig ratio of θ or $90° - \theta$ or equivalent **A1ft:** Correct unsimplified expression using their $\frac{a}{2}$ **A1:** Correct angle. Accept 0.322 radians

Q6.

Question	Scheme	Marks	AOs
(a)	Rel. Mass: 2 5 1 8	B1	1.2
	y : 2 0.5 1.5 $\bar{y}$	B1	1.2
	x : 0.5 2.5 4.5 $\bar{x}$	B1	1.2
	$(2 \times 2) + (5 \times 0.5) + (1 \times 1.5) = 8\bar{y}$	M1	2.1
	$\bar{y} = 1$ *	A1*	1.1b
	$(2 \times 0.5) + (5 \times 2.5) + (1 \times 4.5) = 8\bar{x}$	M1	2.1
	$\bar{x} = 2.25$	A1	1.1b
		(7)	
(b)	Use of correct strategy to solve the problem by use of 'moments equation'	M1	3.1b
	$(8 \times 2.25) - (2\pi r^2 \times 0.5) = (8 - 2\pi r^2)2.5$	A1ft	1.1b
	Solving for r	M1	1.1b
	$r = \frac{1}{\sqrt{2\pi}} = 0.399$	A1	1.1b
(c)	Since $\bar{y}$ for original plate is 1, holes must be symmetrically placed about the line $y = 1$	B1	2.4
	$a = 1.5$	B1	2.2a
		(2)	
(d)	Use of tan from an appropriate triangle	M1	1.1a
	$\tan\alpha = \frac{2}{1.5} = \frac{4}{3}$	A1 ft	1.1b
	$\alpha = 53.1°$	A1	1.1b
		(3)	
			(16 marks)

Notes:

(a)
B1: for correct relative masses
B1: for correct y values
B1: for correct x values
M1: for a moments equation, correct no. of terms, condone sign errors
A1*: for a correct given answer (1)
M1: for a moments equation, correct no. of terms
A1: for 2.25

(b)
M1: for a moments equation, correct no. of terms, condone sign errors

A1ft:	for a correct equation, follow through on their $\bar{x}$
M1:	for solving for r
A1:	for 0.399 or 0.40
(c)	
B1:	for consideration of symmetry about $y = 1$
B1:	for $a = 1.5$
(d)	
M1:	for use of tan from an appropriate triangle
A1ft:	for a correct equation, follow through on their a
A1:	for a correct angle

Q7.

Question	Scheme	Marks	AOs
(a)	In the folding process, each point of the lamina remains the same distance from CD	B1	2.4
		(1)	
(b)	For the folded lamina: $\bar{x} = 2a \quad (= d_2)$ oe	B1	1.1b
	Distances from EA		

Large triangle (ACE)	Removed triangle (BCD)	Added triangle (BCD)	Folded lamina
$27a^2$	$12a^2$	$12a^2$	$27a^2$
$3a$	$5a$	a	$\bar{y}$

Alternative 1

Distances from BD

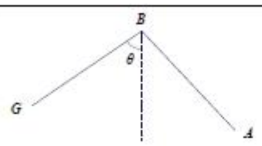
Rectangle $EDBH$	Triangle BHA	Triangle DBC	Folded lamina
$12a^2$	$3a^2$	$12a^2$	$27a^2$
$1.5a$	$2a$	$2a$	$\bar{y}$

Alternative 2

Distances from BD

Triangle FAB	Triangle EFC	2 x Rectangle $DGEF$	2 x Triangle BGF	Folded lamina
$6a^2$	$3a^2$	$12a^2$	$6a^2$	$27a^2$
$2a$	$4a$	$1.5a$	a	$\bar{y}$

	Area ratios	B1	1.2
	Distances from EA	B1	1.2
	Moments about EA:	M1	2.1
	$27\times 3a - 12\times 5a + 12\times a = 27\bar{y}$	A1ft	1.1b
	$\bar{y} = \dfrac{11a}{9}$	A1	1.1b

(b) cont			
	$\theta = \tan^{-1}\dfrac{4a-\bar{x}}{3a-\bar{y}}\left(=\tan^{-1}\dfrac{9}{8}\right)$ or $(90° - \theta) = \tan^{-1}$ (reciprocal)	M1	1.1b
	$\alpha = \tan^{-1}\dfrac{4a-\bar{x}}{3a-\bar{y}} + \tan^{-1}\dfrac{2}{3}$ or oe	M1	3.1b
	$= 82°$ (nearest degree)	A1	1.1b
	Alternative for the final 3 marks:		
	$\overrightarrow{BA}.\overrightarrow{BG} = \dfrac{2}{9}\begin{pmatrix}-9\\-8\end{pmatrix}.\begin{pmatrix}2\\-3\end{pmatrix}\left(=\dfrac{4}{3}\right)$	M1	1.1b
	$\cos\alpha = \dfrac{4/3}{\frac{2}{9}\sqrt{145}\sqrt{13}}(=0.138...)$	M1	3.1b
	$\theta = 82°$	A1	1.1b
		(9)	
			(10 marks)

Notes		
(a)	B1	Any equivalent explanation e.g. folding doesn't change the mass distribution relative to *CD*. A calculation to verify is not the same as an explanation. Allow use of 'vertical' for *CD*.
(b)	B1	Seen anywhere

Distances from *EA*

Large triangle (*ACE*)	Removed triangle (*BCD*)	Added triangle (*BCD*)	Folded lamina
$27a^2$	$12a^2$	$12a^2$	$27a^2$
$3a$	$5a$	a	$\bar{y}$

N.B. B marks only available for viable dissections

Other dissections are possible:

<u>Alternative 1</u>

Distances from *BD*

Rectangle *EDBH*	Triangle *BHA*	Triangle *DBC*	Folded lamina
$12a^2$	$3a^2$	$12a^2$	$27a^2$
$1.5a$	$2a$	$2a$	$\bar{y}$

EDBH + *BHA* + *DBC*, where *H* is midpoint of *AF*

<u>Alternative 2</u>

Distances from *BD*

Triangle *FAB*	Triangle *EFC*	2 x Rectangle *DGEF*	2 x Triangle *BGF*	Folded lamina
$6a^2$	$3a^2$	$12a^2$	$6a^2$	$27a^2$
$2a$	$4a$	$1.5a$	a	$\bar{y}$

FAB + *EFC* + (2 x *DGEF*) + (2 x *GBF*) , where G is midpoint of *DB*

	B1	Any equivalent form for the mass (area) ratios
	B1	Or correct distances from an alternative axis parallel to *AE* e.g. *BD*
	M1	Moments about *AE* or a parallel axis. Need all terms. Must be dimensionally correct. Condone sign errors.
	A1ft	Correct unsimplified moments equation **ft** on their 'table'
	A1	Correct (for their axis) only
	M1	Correct use of trigonometry to find a relevant angle
	M1	Correct strategy for the required angle.
	A1	Correct answer only

Q8.

Question	Scheme	Marks	AOs
(a)	Complete strategy to find d	M1	3.1b
	$\frac{5}{30}M\times\frac{5}{2}a+\frac{13}{30}M\times\frac{5}{2}a=M\times d$	A1	1.1b
	$\left(\frac{25}{2}a+\frac{65}{2}a=30d\right)$	A1	1.1b
	$90a=60d \Rightarrow d=\frac{3}{2}a$ *	A1*	2.1
		(4)	
(b)	Complete strategy to find k, e.g. by use of a moments equation	M1	3.1b
	$Mg\times\frac{3}{2}a=kMg\times 12a$	A1	1.1b
	$k=\frac{1}{8}$	A1	1.1b
		(3)	
(b) alt	Moments equation	M1	
	$12a\times kM=\frac{13}{30}M\times 2.5a+\frac{5}{30}M\times 2.5a$	A1	
	$12k=\frac{45}{30}$, $k=\frac{1}{8}$	A1	
		(3)	
			(7 marks)

Notes

(a) M1: Complete strategy to find d e.g. moments about AB or a parallel axis. Needs all relevant terms. Must be dimensionally correct.
Condone sign errors. M's might cancel from the start.
A1: Unsimplified equation with at most one error
A1: Correct unsimplified equation
A1*: Obtain the **given answer** from a convincing argument

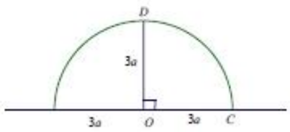
(b) M1: Complete strategy to find k e.g. moments about A.
Needs all relevant terms. Must be dimensionally correct.
Condone sign errors. Condone if a, M, g missing throughout
A1: Correct unsimplified equation in k
A1: Correct answer – any equivalent form

Q9.

Question	Scheme	Marks	AOs
(a)	The rods are uniform and the axes of symmetry intersect at midpoint of AC.	B1	2.4
		(1)	
(b)	Use moments: e.g. M(A): $(2aW + aW + 3aW = 4aT_B + aW)$	M1	2.1
	e.g. M(A): $5W.2a\cos 60^\circ = 4aT_B$ or M(B): $3a \times 5W = 4aT_A$	A1	1.1b
	Resolving vertically: $T_A + T_B = 5W$	M1	2.1
	$\Rightarrow T_A = \frac{15W}{4},\ T_B = \frac{5W}{4}$	A1	1.1b
		(4)	
(c)	T_A will be the larger, so the first to exceed $6W$ so need to use $T_A = 6W$ (e.g. by M(B) but they may use two equations) to form an equation in k only.	M1	3.1a
	$6W \times 4a = 5W \times 3a + kW \times 6a + 2kW \times 2a$	A1	1.1b
	$24aW = 15aW + 10kaW$	A1	1.1b
	$k = 0.9$	A1	1.1b
		(4)	
			(9 marks)

Notes		
(a)	B1	Any equivalent clear justification. Needs to mention uniformity and symmetry and the midpoint of AC
(b)	M1	Form ANY moments equation. Require all terms. Dimensionally correct. Condone sign errors.
	A1	Correct unsimplified (including trig) equation e.g. M(G) : $T_A.2a\cos 60^\circ = T_B.(4a - 2a\cos 60^\circ)$ or $T_B.(4a\cos^2 30^\circ)$
	M1	Form a second equation in T_A and/or T_B e.g. by resolving vertically or a second moments equation, and solve for T_A and T_B
	A1	Both tensions correct. If answers reversed, allow M marks.
(c)	M1	Realise that the first to break will be the rope at A and complete method to form an equation in k only (allow uncancelled W's) using $T_A = 6W$. Require all terms (in all equations used). Dimensionally correct. Condone sign errors. M0 if they use $T_B = 6W$ to find k (this gives $k = 9.5$)
	A1	Unsimplified equation or inequality in k only with at most one error
	A1	Correct unsimplified equation or inequality in k only
	A1	Correct only. Decimal or fraction.

Q10.

Question	Scheme	Marks	AOs
(a)	Using sector: distance $OG = \dfrac{2 \times 3a \sin\frac{\pi}{4}}{3 \times \frac{\pi}{4}}$	B1	1.1b
	Using Pythagoras: $2d^2 = \dfrac{32a^2}{\pi^2} \quad (d^2 + d^2 = OG^2)$ Or using trigonometry: Distance from $OC = OG\cos 45° = OG\sin 45°$	M1	2.1
	$d = \sqrt{\dfrac{16a^2}{\pi^2}} = \dfrac{4a}{\pi}$ *	A1*	2.2a
		(3)	
(a) alt	Using semicircle of radius $3a$: $\bar{y} = \dfrac{4 \times 3a}{3\pi}\left(= \dfrac{4a}{\pi}\right)$ 	B1	1.1b
	Moments about diameter: $\dfrac{9\pi a^2}{2} \times \dfrac{4a}{\pi} = 2 \times \dfrac{9\pi a^2}{4} \times d$	M1	2.1
	$\Rightarrow d = \dfrac{4a}{\pi}$ *	A1*	2.2a
		(3)	

<table>
<tr><td>(b)</td><td><table><tr><td></td><td>$ABCO$</td><td>$ODEF$</td><td>ODC</td></tr><tr><td>Mass ratio</td><td>9</td><td>9</td><td>$\frac{9\pi}{4}$</td></tr><tr><td>From FC</td><td>$-\frac{3a}{2}$</td><td>$\frac{3a}{2}$</td><td>$\frac{4a}{\pi}$</td></tr></table></td><td>B1</td><td>1.2</td></tr>
<tr><td></td><td>Moments about FC:</td><td>M1</td><td>3.1a</td></tr>
<tr><td></td><td>$-9 \times \frac{3a}{2} + 9 \times \frac{3a}{2} + \frac{9\pi}{4} \times \frac{4a}{\pi} = \left(18 + \frac{9\pi}{4}\right)\bar{x}\,(= 9a)$</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td>$\bar{x} = \frac{4a}{8+\pi}$</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(4)</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
</table>

(b) alt	<table><tr><td></td><td>*ABCO*</td><td>*ODEF*</td><td>*ODC*</td></tr><tr><td>Mass ratio</td><td>9</td><td>9</td><td>$\frac{9\pi}{4}$</td></tr><tr><td>From *BOE*</td><td>0</td><td>0</td><td>$\frac{4\sqrt{2}a}{\pi}$</td></tr></table>	B1	1.2
	Moments about *BOE*:	M1	3.1a
	$\left(18+\frac{9\pi}{4}\right)d=\frac{9\pi}{4}\times\frac{4\sqrt{2}a}{\pi}$	A1	1.1b
	$\bar{x}=d\cos 45°=\frac{4a}{8+\pi}$	A1	1.1b
		(4)	
(c)	$\bar{y}=\frac{4a}{8+\pi}$ from *OD* or $\bar{y}=3a+\frac{4a}{8+\pi}$ from *FE*	B1ft	1.1b
	Complete method to find a relevant angle	M1	3.1a
	$\theta°=\tan^{-1}\left(\frac{\bar{x}}{3a+\bar{y}}\right)=\tan^{-1}\left(\frac{4a}{28a+3\pi a}\right)$	A1ft	1.1b
	$\theta=6.1$	A1	1.1b
		(4)	
			(11 marks)

Notes:	
(a)B1	Correct application of standard result from formula booklet. Must substitute for a but need not simplify Implied if you see $\left(=\frac{4\sqrt{2}a}{\pi}\right)$
M1	Correct strategy to find the distance for the quadrant Need to see use of $\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$ somewhere in the solution
A1*	Obtain the given result from correct working.
(b)B1	Correct masses and distance from *FC* or a parallel axis or *BOE* Seen or implied (a bright candidate might realise that if taking moments about *FC* then the two squares cancel each other).
M1	Moments about *FC* or a parallel axis or *BOE*. All terms required, and dimensionally correct. Condone sign errors. Accept as part of a vector equation.

A1	Correct unsimplified equation for their axis
A1	Or equivalent with no errors seen Accept $0.36a$ or better $(0.3590\ldots a)$
(c)B1ft	Allow use of symmetry seen or implied. Accept $\overline{y} = \overline{x}$ (From FE, $\overline{y} = \dfrac{28a + 3\pi a}{8+\pi}$) Accept + / -
M1	Correct strategy to find a relevant angle (θ or $90 - \theta$) Need to substitute their values of $\overline{x}$ and distance from $F \neq \dfrac{4a}{\pi}$.
A1ft	Correct unsimplified expression for a relevant angle. Follow their $\overline{x}$ and $\overline{y}$
A1	6.1 or better (6.10067…) The question defines θ as measured in degrees. 0.106 can score B1M1A1ftA0 Do not ISW

Q11.

Question	Scheme	Marks	AOs
(a)	Moments about AC:	M1	3.1a
	rod: CD, DE, EF, FA, AB, BC, BF, DF, CF; Mass ratio: 4, 4, 5, 5, 3, 3, 8, 6, 10; From AC: $2a$, $6a$, $6a$, $2a$, 0, 0, $2a$, $4a$, $2a$		
	$8a\times 4a+2\times 3a\times 4a+2\times 5a\times 2a+10a\times 4a+2\times 4a\times 2a=48a\overline{x}$	A1 A1	1.1b 1.1b
	$(132a=48\overline{x})\Rightarrow \overline{x}=\frac{11}{4}a$	A1	1.1b
		(4)	
(b)	Moments about F:	M1	3.1a
	$Mg(4a-\overline{x})=kMg\times 4a$	A1ft	1.1b
	$\Rightarrow k=\frac{5}{16}$	A1	1.1b
		(3)	
	Moments about C:	M1	
	$4a(M+kM)=M\overline{x}+8akM$	A1ft	
	$\Rightarrow k=\frac{5}{16}$	A1	1.1b
		(3)	
		(7 marks)	

rod	CD	DE	EF	FA	AB	BC	BF	DF	CF
Mass ratio	4	4	5	5	3	3	8	6	10
From AC	$2a$	$6a$	$6a$	$2a$	0	0	$2a$	$4a$	$2a$

Notes:	
(a)M1	Dimensionally correct equation for moments about AC or a parallel axis. **All terms** needed and horizontal distances Must be using the mass ratio. Allow slips, but not consistently density and not consistently lengths. Condone without a
A1 A1	One side of the equation correct Both sides of the equation correct
A1	Or equivalent single term Condone if a is missing in the working and appears at the end
(b)M1	Dimensionally correct moments equation. Accept any complete alternative method using M and kM to obtain an equation in k only. Condone if g and / or M cancelled throughout Condone incorrect distances Condone if use $M = 48$ throughout
A1	Correct unsimplified equation (accept without g and/or M) Correct mass and distance combination for their $\overline{x}$
A1	Or 0.3125 Condone 0.31 or 0.313

Q12.

Question	Scheme	Marks	AOs
(a)	*ABCD* *BEC* framework		
	$6a$ πa $6a + \pi a$	B1	1.2
	$\frac{1}{2}a$ $(-)\frac{2a}{\pi}$ $\overline{x}$	B1	1.2
	Moments about *BC*	M1	2.1
	$6a\times\frac{1}{2}a-\pi a\times\frac{2a}{\pi}=(6a+\pi a)\overline{x}$	A1	1.1b
	$\overline{x}=\frac{a}{6+\pi}$ *	A1*	2.2a
		(5)	
(b)	Angle $DAE=\tan^{-1}\left(\frac{2a}{a}\right)$	M1	1.1b
	Angle $DAG=\tan^{-1}\left(\frac{a-\frac{a}{6+\pi}}{a}\right)=\tan^{-1}\left(\frac{5+\pi}{6+\pi}\right)$	M1	1.1b
	Angle $= DAE - DAG$	M1	3.1a
	21.74637…	A1	1.1b
		(4)	
(c)	Moments about *OA*	M1	2.1
	$kMa\sin 45^{o}=M\overline{x}\sin 45^{o}$	A1	1.1b
	$k=\frac{1}{6+\pi}$ $(=0.10939\ldots)$	A1	1.1b
		(3)	

(c) alt	Moments about *O*	M1	2.1
	$kM\begin{pmatrix}0\\a\end{pmatrix}-M\begin{pmatrix}\frac{a}{6+\pi}\\0\end{pmatrix}=(k+1)M\begin{pmatrix}-\lambda\\\lambda\end{pmatrix}$	A1	1.1b
	$k=\frac{1}{6+\pi}$ $(=0.10939\ldots)$	A1	1.1b
		(3)	
		(12 marks)	

Notes:

a	B1	Any equivalent ratios
	B1	Or correct distances from a parallel axis
	M1	Or moments about a parallel axis Must be using framework. If *BC* included twice mark as a misread.

	A1	Correct unsimplified equation for their axis. Allow within a vector equation
	A1*	Correct given answer correctly obtained
b	M1	Correct relevant angle (or side if they use the cosine rule) Do not need to evaluate: accept $\tan\alpha = ...$ or $\alpha = \tan^{-1}...$ (e.g. $63.4...°$ or $90° - 63.4...°$)
	M1	Another correct relevant angle (or side if they use the cosine rule) Do not need to evaluate: accept $\tan\beta = ...$ or $\beta = \tan^{-1}...$ (e.g. $41.68...°$ or $90° - 41.68...°$)
	M1	Correct method for finding the required angle
	A1	$22°$ or better
c	M1	Complete method to give an equation in k only
	A1	Correct equation in k only
	A1	0.11 or better

Q13.

Question	Scheme				Marks	AOs
(a)	ABF	$BCEF$	CDE	lamina		
	$\frac{3}{2}a^2$	$9a^2$	$\frac{3}{2}a^2$	$12a^2$	B1	1.2
	a	$\frac{3}{2}a$	a	$\bar{y}$	B1	1.2
	Moments about AD				M1	2.1
	$(\frac{3}{2}a^2 \times a) + (9a^2 \times \frac{3}{2}a) + (\frac{3}{2}a^2 \times a) = 12a^2\bar{y}$				A1	1.1b
	$\bar{y} = \frac{11a}{8}$ *				A1*	2.2a
					(5)	
(b)	Moments about F, $Mg \times (3a - \frac{11a}{8}) = 3aT$				M1	3.1a
	$T = \frac{13Mg}{24}$ $\quad (0.54166666\ldots\ Mg)$				A1	1.1b
					(2)	
					(7 marks)	

Notes:		
a	B1	Any equivalent ratios e.g. 3 : 18 : 3 : 24
	B1	Or correct distances from a parallel axis
	M1	Or moments about a parallel axis
	A1	Correct unsimplified equation for their axis
	A1*	Correct given answer correctly obtained If they have centre of mass at (xa, ya) then the a might not be seen in the working. Otherwise, with no a in the working the maximum score is B1B0M1A0A0
b	M1	A complete method to obtain an equation in T only
	A1	$0.54Mg$ or better

Q14.

Question	Scheme	Marks	AOs
(a)	Mass ratio: 16, 6, 4, 26; → from AJ: a, $3.5a$, $3a$, $\bar{x}$; ↓ from AB: $4a$, a, $5a$, $\bar{y}$	B1 B1	1.2 1.2
	M(AJ)	M1	2.1
	$16a+21a+12a(=49a)=26\bar{x}$	A1	1.1b
	$26\bar{x}=49a \Rightarrow \bar{x}=\frac{49}{26}a$ *	A1*	1.1b
		(5)	
(b)	M(A)	M1	3.1b
	$5a\times T=\frac{49}{26}a\times W$	A1	1.1b
	$T=\frac{49}{130}W$	A1	1.1b
		(3)	
(c)	Distances from AB (as in table above)	B1	1.2
	M(AB): $64a+6a+20a=26\bar{y}$	M1	2.1
	$\bar{y}=\frac{90}{26}a$	A1	1.1b
	$\tan\theta=\frac{49}{90}$	M1	1.1a
	$\theta=28.6°\ (29°)$	A1	2.2a
		(5)	
			(13 marks)

Notes:		
a	B1	Correct mass ratios seen or implied
	B1	Correct distances from their vertical axis
	M1	Correct strategy to find distance including appropriate division of the lamina and moments about an axis parallel to AJ. Terms dimensionally correct. Condone sign errors.
	A1	Correct unsimplified moments equation for a correct division of the template.
	A1*	Obtain **given answer** from complete and correct working
b	M1	Complete method to find the tension. Dimensionally correct equations.
	A1	Correct unsimplified equation for the tension
	A1	Correct simplified $(0.38W$ or better$)$
c	B1	Distances from a horizontal axis for a complete correct division of the lamina (could be found in (a) but need to be used here to score the B1)
	M1	Moments about a horizontal axis. Terms dimensionally correct. Condone sign errors.
	A1	Correct vertical distance (any equivalent form) $\left(\frac{59}{13}a \text{ from } JI\right)$ seen or implied.
	M1	Use trig. with $\frac{49}{26}a$ and their $\bar{y}$, or equivalent, to find a relevant angle.
	A1	$29°$ or better. $28.5658...°$, 0.499 rads

Q15.

Question	Scheme	Marks	AOs
a	By symmetry, centre of mass at centre of square.	B1	2.4
		(1)	
b	Mass ratios 40 : 32 : 72	B1	1.2
	Distances $5a$: $9a$: ($\bar{x}$)	B1	1.2
	Moments about AB: $(40\times 5a + 32\times 9a = 72\bar{x} \quad (488a = 72\bar{x}))$ (or $2\times 10\times 5a + 10\times 10a + 2\times 8\times 9a + 8\times 5a + 8\times 13a = 72\bar{x}$) $\left(\bar{x} = \dfrac{61a}{9}\right)$	M1	2.1
	Complete method to find distance $= \sqrt{\bar{x}^2 + \bar{x}^2}$	M1	3.1b
	Distance $= \dfrac{61\sqrt{2}a}{9}$	A1	1.1b
		(5)	
b alt	Mass ratios 40 : 32 : 72	B1	1.2
	Distances $5\sqrt{2}a : 9\sqrt{2}a : (d)$	B1	1.2
	Moments about B: $(40\times 5\sqrt{2}a + 32\times 9\sqrt{2}a = 72d)$	M1	2.1
	Complete method to find distance	M1	3.1b
	Distance $= \dfrac{61\sqrt{2}a}{9}$	A1	1.1b
		(5)	
			(6 marks)

Notes:	
(a)	
B1	Any clear explanation
(b)	
B1	Correct mass ratios seen or implied
B1	Correct distances seen or implied
M1	Moments equation for the whole framework about an axis parallel to AB or to BC.
M1	Use of symmetry of the framework and Pythagoras with their $\bar{x}$ to find the required distance
A1	Or equivalent. $9.6a$ $(9.585...a)$ or better
	Alternative approach:
	2nd M1 moments equation using their distances and masses
	1st M1 complete method to find distances from B

Q16.

Question	Scheme	Marks	AOs
(a)	Mass ratios: $4a^2$, $\frac{1}{2}\pi a^2$, $(4a^2+\frac{1}{2}\pi a^2)$	B1	1.2
	x: $\frac{1}{2}a$, $a+\frac{4a}{3\pi}$, $\bar{x}$ $\quad$ y: $2a$, $3a$, $\bar{y}$	B1	1.2
	Moments about OE	M1	3.1b
	$\bar{x}=\frac{(16+3\pi)a}{3(8+\pi)}$	A1	1.1b
	Moments about OA	M1	3.1b
	$\bar{y}=\frac{(16+3\pi)a}{(8+\pi)}$	A1	1.1b
		(6)	
(b)	$\tan\alpha=\frac{\bar{x}}{\bar{y}}$ and substitute for their $\bar{x}$ and $\bar{y}$	M1	3.1b
	$\tan\alpha=\frac{1}{3}$	A1	1.1b
		(2)	
			(8 marks)

Notes:

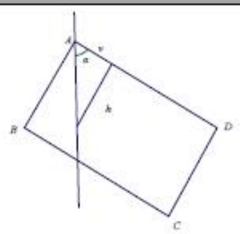
a	B1	All correct
	B1	Distances could be measured from a parallel axis
	M1	All terms needed and must be dimensionally correct
	A1	cao (must be in terms of π and a)
	M1	All terms needed and must be dimensionally correct
	A1	cao (must be in terms of π and a)
b	M1	Do not allow the reciprocal
	A1	cao

Q17.

Question	Scheme	Marks	AOs
(a)			

	Mass	From AD
Rectangle	$8a^2$	a
Semicircle	$\frac{1}{2}\pi a^2$	$\frac{4a}{3\pi}$
Sign	$a^2\left(8-\frac{\pi}{2}\right)$	h

Question	Scheme	Marks	AOs
(a)	Mass ratios	B1	1.2
	Moments about AD	M1	2.1
	$a^2\left(8-\frac{\pi}{2}\right)h = 8a^2 \times a - \frac{1}{2}\pi a^2 \times \frac{4a}{3\pi}\left(= 8a^3 - \frac{2}{3}a^3 = \frac{22}{3}a^3\right)$	A1	1.1b
	$\Rightarrow h = \frac{22}{3}a \div \left(8-\frac{\pi}{2}\right) = \frac{44a}{3(16-\pi)}$ *	A1*	2.2a
		(4)	
(b)	Moments about A $2aT = \frac{44a}{3(16-\pi)}W$	M1	3.1b
	$T = \frac{hW}{2a} = \frac{22W}{3(16-\pi)}$	A1	1.1b
		(2)	

Question	Scheme	Marks	AOs
(c)			
	Take moments about AB to find distance of com from AB	M1	3.1b
	$8a^2 \times 2a - \frac{1}{2}\pi a^2 \times d = \left(8-\frac{1}{2}\pi\right)a^2 \times v$	A1	1.1b
	$v = \frac{32a - \pi d}{16-\pi}$	A1	1.1b
	Correct trig for the given angle	M1	3.1b
	$\tan\alpha = \frac{11}{18} = \frac{h}{v} = \frac{44a}{3(32a-\pi d)}$	A1ft	1.1b
	$(\ 24a = 32a - \pi d\ ,\ \ 8a = \pi d\ )\quad d = \frac{8a}{\pi}$	A1	1.1b
		(6)	
			(12 marks)

Notes:	
(a)	
B1:	correct mass ratios
M1:	need all three terms, must be dimensionally correct
A1:	Correct unsimplified equation
A1*: has	Show sufficient working to justify the **given answer** and a 'statement' that the required result been achieved.
(b)	
M1:	Could also take moments about B or about the c.o.m. and use $T_A + T_B = W$
A1:	cso
(c)	
M1:	all terms and dimensionally correct
A1:	Correct unsimplified equation
A1:	or equivalent
M1:	Condone tan the wrong way up.
A1:	Equation in a and d; follow through on their v
A1:	cao.