Circular Motion

Questions

Q1.

A light inextensible string has length $8a$. One end of the string is attached to a fixed point A and the other end of the string is attached to a fixed point B, with A vertically above B and $AB = 4a$. A small ball of mass m is attached to a point P on the string, where $AP = 5a$.

The ball moves in a horizontal circle with constant speed v, with both AP and BP taut.

The string will break if the tension in it exceeds $\frac{3mg}{2}$

By modelling the ball as a particle and assuming the string does not break,

(a) show that $\frac{9ag}{4} < v^2 \leqslant \frac{27ag}{4}$

(7)

(b) find the least possible time needed for the ball to make one complete revolution.

(2)

(Total for question = 9 marks)

Q2.

A car moves round a bend which is banked at a constant angle of $\theta°$ to the horizontal.

When the car is travelling at a constant speed of 80 km h^{-1} there is no sideways frictional force on the car. The car is modelled as a particle moving in a horizontal circle of radius 500 m.

(a) Find the value of θ.

(7)

(b) Identify one limitation of this model.

(1)

The speed of the car is increased so that it is now travelling at a constant speed of 90 km h^{-1} The car is still modelled as a particle moving in a horizontal circle of radius 500 m.

(c) Describe the extra force that will now be acting on the car, stating the direction of this force.

(1)

(Total for question = 9 marks)

Q3.

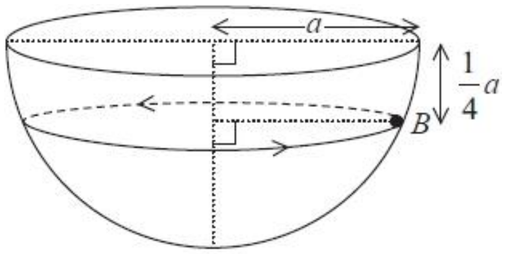


Figure 1

A hemispherical shell of radius *a* is fixed with its rim uppermost and horizontal. A small bead, *B*, is moving with constant angular speed, *ω*, in a horizontal circle on the smooth inner surface of the shell. The centre of the path of *B* is at a distance $\frac{1}{4}a$ vertically below the level of the rim of the hemisphere, as shown in Figure 1.

Find the magnitude of *ω*, giving your answer in terms of *a* and *g*.

(Total for question = 6 marks)

Q4.

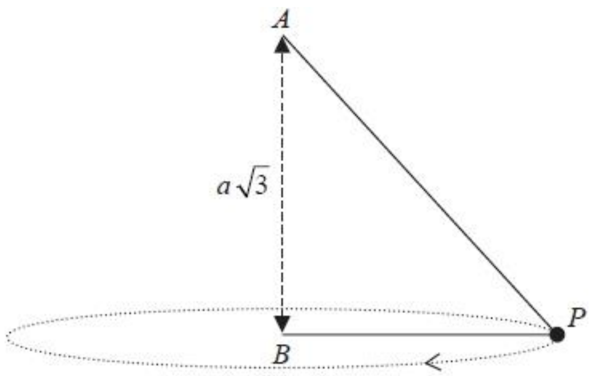


Figure 2

One end of a string of length 3*a* is attached to a point *A* and the other end is attached to a point *B* on a smooth horizontal table. The point *B* is vertically below *A* with *AB* = $a\sqrt{3}$
A small smooth bead, *P*, of mass *m* is threaded on to the string. The bead *P* moves on the table in a horizontal circle, with centre *B*, with constant speed *U*. Both portions, *AP* and *BP*, of the string are taut, as shown in Figure 2.

The string is modelled as being light and inextensible and the bead is modelled as a particle.

(a) Show that *AP* = 2*a*

(2)

(b) Find, in terms of m, U and a, the tension in the string.

(4)

(c) Show that $U^2 < ag\sqrt{3}$

(5)

(d) Describe what would happen if $U^2 > ag\sqrt{3}$

(1)

(e) State briefly how the tension in the string would be affected if the string were not modelled as being light.

(1)

(Total for question = 13 marks)

Q5.

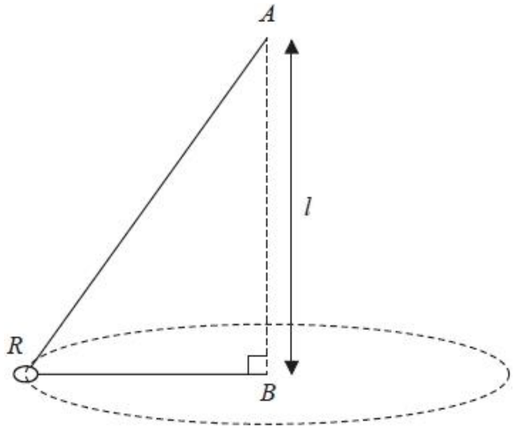


Figure 4

One end of a light inextensible string of length $2l$ is attached to a fixed point A. A small smooth ring R of mass m is threaded on the string and the other end of the string is attached to a fixed point B. The point B is vertically below A, with $AB = l$. The ring is then made to move with constant speed V in a horizontal circle with centre B. The string is taut and BR is horizontal, as shown in Figure 4.

(a) Show that $BR = \frac{3l}{4}$

(2)

Given that air resistance is negligible,

(b) find, in terms of m and g, the tension in the string,

(4)

(c) find V in terms of g and l.

(4)

(Total for question = 10 marks)

Q6.

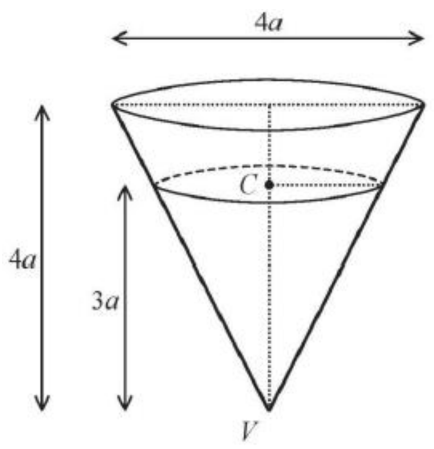


Figure 1

A hollow right circular cone, of base diameter 4*a* and height 4*a* is fixed with its axis vertical and vertex *V* downwards, as shown in Figure 1.

A particle of mass *m* moves in a horizontal circle with centre *C* on the rough inner surface of the cone with constant angular speed ω.

The height of *C* above *V* is 3*a*.

The coefficient of friction between the particle and the inner surface of the cone is $\frac{1}{4}$

Find, in terms of *a* and g, the greatest possible value of ω.

(8)

(Total for question = 8 marks)

Q7.

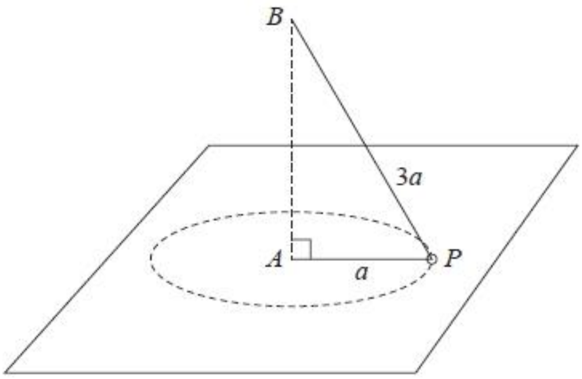


Figure 2

A small smooth ring P, of mass m, is threaded onto a light inextensible string of length $4a$.
One end of the string is attached to a fixed point A on a smooth horizontal table.
The other end of the string is attached to a fixed point B which is vertically above A.
The ring moves in a horizontal circle with centre A and radius a, as shown in Figure 2.

The ring moves with constant angular speed $\sqrt{\frac{2g}{3a}}$ about AB.

The string remains taut throughout the motion.

(a) Find, in terms of m and g, the magnitude of the normal reaction between P and the table.

(6)

The angular speed of P is now gradually increased.

(b) Find, in terms of a and g, the angular speed of P at the instant when it loses contact with the table.

(3)

(c) Explain how you have used the fact that P is smooth.

(1)

(Total for question = 10 marks)

Q8.

Unless otherwise indicated, whenever a numerical value of g is required, take g = 9.8 m s^{-2} and give your answer to either 2 significant figures or 3 significant figures.

A light inextensible string has length $7a$. One end of the string is attached to a fixed point A and the other end of the string is attached to a fixed point B, with A vertically above B and $AB = 5a$. A particle of mass m is attached to a point P on the string where $AP = 4a$. The particle moves in a horizontal circle with constant angular speed ω, with both AP and BP taut.

(a) Show that

(i) the tension in AP is $\frac{4m}{25}$ $(9a\omega^2 + 5g)$

(ii) the tension in BP is $\frac{3m}{25}$ $(16a\omega^2 - 5g)$.

(10)

The string will break if the tension in it reaches a magnitude of $4mg$.

The time for the particle to make one revolution is S.

(b) Show that

$$3\pi\sqrt{\frac{a}{5g}} < S < 8\pi\sqrt{\frac{a}{5g}}$$

(5)

(c) State how in your calculations you have used the assumption that the string is light.

(1)

(Total for question = 16 marks)

Q9.

A cyclist is travelling around a circular track which is banked at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$

The cyclist moves with constant speed in a horizontal circle of radius r.

In an initial model,

- the cyclist and her cycle are modelled as a particle
- the track is modelled as being rough so that there is sideways friction between the tyres

of the cycle and the track, with coefficient of friction μ, where $\mu < \frac{4}{3}$

Using this model, the maximum speed that the cyclist can travel around the track in a horizontal circle of radius r, without slipping sideways, is V.

(a) Show that $V = \sqrt{\frac{(3+4\mu)rg}{4-3\mu}}$

(7)

In a new simplified model,

- the cyclist and her cycle are modelled as a particle
- the motion is now modelled so that there is **no** sideways friction between the tyres of

the cycle and the track

Using this new model, the speed that the cyclist can travel around the track in a horizontal circle of radius r, without slipping sideways, is U.

(b) Find U in terms of r and g.

(2)

(c) Show that $U < V$.

(2)

(Total for question = 11 marks)

Q10.

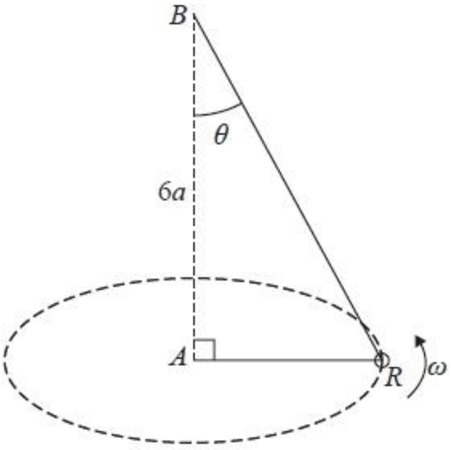


Figure 2

A small smooth ring R of mass m is threaded onto a light inextensible string. One end of the string is attached to a fixed point A and the other end of the string is attached to the fixed point B such that B is vertically above A and $AB = 6a$

The ring moves with constant angular speed ω in a horizontal circle with centre A. The string is taut and BR makes a constant angle θ with the downward vertical, as shown in Figure 2.

The ring is modelled as a particle.

Given that $\tan\theta = \frac{8}{15}$

(a) find, in terms of m and g, the magnitude of the tension in the string,

(3)

(b) find ω in terms of a and g

(5)

(Total for question = 8 marks)

Q11.

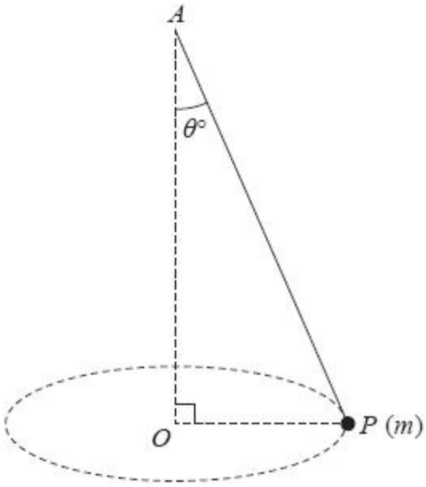


Figure 2

A particle P of mass m is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point A. The particle moves in a horizontal circle with constant angular speed $\sqrt{58.8}$ rad s^{-1}. The centre O of the circle is vertically below A and the string makes a constant angle $\theta°$ with the downward vertical, as shown in Figure 2.

Given that the tension in the string is $1.2mg$, find

(i) the value of θ

(ii) the length of the string.

(8)

(Total for question = 8 marks)

Q12.

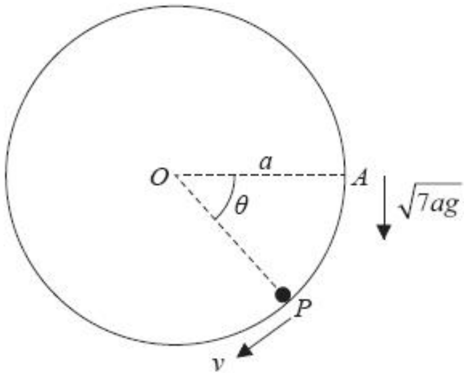


Figure 4

A hollow cylinder is fixed with its axis horizontal. A particle P moves in a vertical circle, with centre O and radius a, on the smooth inner surface of the cylinder. The particle moves in a vertical plane which is perpendicular to the axis of the cylinder. The particle is projected vertically downwards with speed $\sqrt{7ag}$ from the point A, where OA is horizontal and $OA = a$. When angle $AOP = \theta$, the speed of P is v, as shown in Figure 4.

(a) Show that $v^2 = ag(7 + 2\sin\theta)$

(4)

(b) Verify that P will move in a complete circle.

(6)

(c) Find the maximum value of v.

(2)

(Total for question = 12 marks)

Q13.

A small bead B of mass m is threaded on a circular hoop.

The hoop has centre O and radius a and is fixed in a vertical plane.

The bead is projected with speed $\sqrt{\frac{7}{2}ga}$ from the lowest point of the hoop.

The hoop is modelled as being smooth.

When the angle between OB and the downward vertical is θ, the speed of B is v.

(a) Show that $v^2 = ga\left(\frac{3}{2} + 2\cos\theta\right)$

(3)

(b) Find the size of θ at the instant when the contact force between B and the hoop is first zero.

(5)

(c) Give a reason why your answer to part (b) is not likely to be the actual value of θ.

(1)

(d) Find the magnitude and direction of the acceleration of B at the instant when B is first at instantaneous rest.

(5)

(Total for question = 14 marks)

Q14.

A particle, P, of mass m is attached to one end of a light rod of length L. The other end of the rod is attached to a fixed point O so that the rod is free to rotate in a vertical plane about O. The particle is held with the rod horizontal and is then projected vertically downwards with speed u. The particle first comes to instantaneous rest at the point A.

(a) Explain why the acceleration of P at A is perpendicular to OA.

(1)

At the instant when P is at the point A the acceleration of P is in a direction making an angle θ with the horizontal. Given that $u^2 = \frac{2gL}{3}$,

(b) find

(i) the magnitude of the acceleration of P at the point A,
(ii) the size of θ.

(6)

(c) Find, in terms of m and g, the magnitude of the tension in the rod at the instant when P is at its lowest point.

(5)

(Total for question = 12 marks)

Q15.

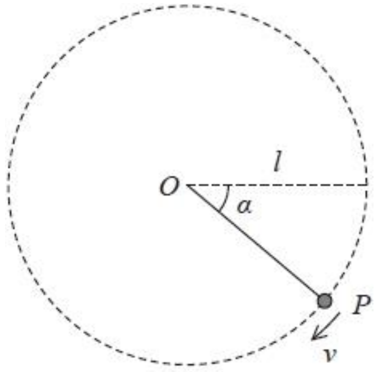


Figure 5

A particle P of mass m is attached to one end of a light inextensible string of length l. The other end of the string is attached to a fixed point O. The particle is held with the string taut and OP horizontal. The particle is then projected vertically downwards with speed u, where $u^2 = \frac{9}{5}gl$. When OP has turned through an angle α and the string is still taut, the speed of P is v, as shown in Figure 5. At this instant the tension in the string is T.

(a) Show that $T = 3mg \sin \alpha + \frac{9}{5} mg$

(6)

(b) Find, in terms of g and l, the speed of P at the instant when the string goes slack.

(3)

(c) Find, in terms of l, the greatest vertical height reached by P above the level of O.

(4)

(Total for question = 13 marks)

Q16.

A light inextensible string of length *a* has one end attached to a fixed point *O*. The other end of the string is attached to a small stone of mass *m*. The stone is held with the string taut and horizontal. The stone is then projected vertically upwards with speed *U*.

The stone is modelled as a particle and air resistance is modelled as being negligible.

Assuming that the string does not break, use the model to

(a) find the least value of *U* so that the stone will move in complete vertical circles.

(6)

The string will break if the tension in it is equal to $\frac{11mg}{2}$

Given that $U = 2\sqrt{ag}$, use the model to

(b) find the total angle that the string has turned through, from when the stone is projected vertically upwards, to when the string breaks,

(6)

(c) find the magnitude of the acceleration of the stone at the instant just before the string breaks.

(4)

(Total for question = 16 marks)

Q17.

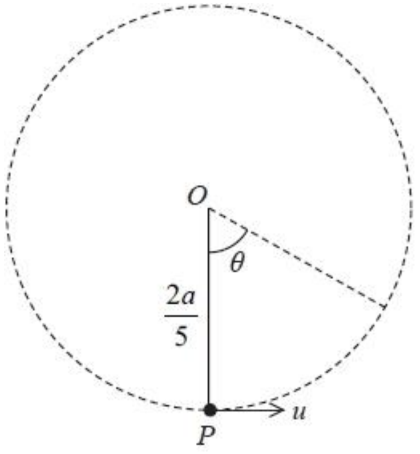


Figure 5

A package P of mass m is attached to one end of a string of length $\frac{2a}{5}$. The other end of the string is attached to a fixed point O. The package hangs at rest vertically below O with the string taut and is then projected horizontally with speed u, as shown in Figure 5.

When OP has turned through an angle θ and the string is still taut, the tension in the string is T

The package is modelled as a particle and the string as being light and inextensible.

(a) Show that $T = 3mg\cos\theta - 2mg + \frac{5mu^2}{2a}$

(6)

Given that P moves in a complete vertical circle with centre O

(b) find, in terms of a and g, the minimum possible value of u

(2)

Given that $u = 2\sqrt{ag}$

(c) find, in terms of g, the magnitude of the acceleration of P at the instant when OP is horizontal.

(3)

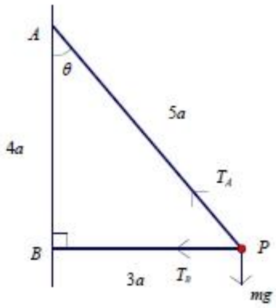
(d) Apart from including air resistance, suggest one way in which the model could be refined to make it more realistic.

(1)

(Total for question = 12 marks)

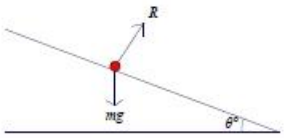
<u>Mark Scheme</u> – Circular Motion

Q1.

Question	Scheme	Marks	AOs
(a)			
	No vertical motion: $T_A \cos\theta = mg$	M1	1.1b
	$T_A = \frac{5mg}{4}$	A1	1.1b
	Circular motion: $T_B + T_A \sin\theta = m \times \frac{v^2}{r}$	M1	3.1b
	$T_B + \frac{3}{5}T_A = m\frac{v^2}{3a}$	A1	1.1b
	$T_B > 0 \left(\Rightarrow v^2 > \frac{9ag}{4}\right)$	DM1	2.1
	$T_B \le \frac{3mg}{2} \Rightarrow m\frac{v^2}{3a} - \frac{3}{4}mg \le \frac{3}{2}mg, \left(m\frac{v^2}{3a} \le \frac{9mg}{4}\right)$	DM1	2.1
	$\Rightarrow \frac{9ag}{4} < v^2 \le \frac{27ag}{4}$ *	A1*	2.2a
		(7)	
(b)	Use $v^2 = \frac{27ag}{4}$ and $T = \frac{2\pi r}{v}$ oe	M1	3.1b
	$T = 4\pi\sqrt{\frac{a}{3g}}$	A1	1.1b
		(2)	
			(9 marks)

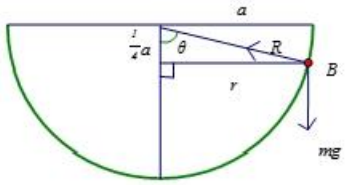
Notes		
(a)		**N.B.** If they have the same tension in both parts of the string, can score ONLY first M1A1 for a correct equation. **N.B**. If no right angle at B, could score max: M1A0M1A0DM1DM1A0
	M1	One equation in T_A and / or T_B . Dimensionally correct, with all relevant terms. Condone sign errors and sin/cos oe confusion
	A1	Correct equation (no trig)
	M1	Form a second equation in T_A and / or T_B. Dimensionally correct, with all relevant terms. Condone sign errors and sin/cos oe confusion. Allow $mr\omega^2$
	A1	Correct equation (no trig)
	DM1	Use the model to form one inequality or equation in v^2, a and g **only, dependent on both M's**
	DM1	Use the model to form a second inequality or equation in v^2, a and g **only dependent on both M's** **Allow use of** $T_B = \frac{3mg}{2}$ or $T_B < \frac{3mg}{2}$
	A1*	Deduce the given answer from correct working. Only available if working with inequalities throughout and fully correct
(b)	M1	Correct method to find T in terms of a and (g) **only.** They may sub 9.8 for g of course
	A1	Any equivalent form but no fractions within fractions. If they use 9.8 for g, the numerical part needs to be to 2 sf or 3sf. i.e $2.3\sqrt{a}$ or $2.32\sqrt{a}$

Q2.

Question	Scheme	Marks	AOs
(a)	Complete strategy to find value of θ	M1	3.1b
			
	Resolve vertically	M1	3.1b
	$R\cos\theta° = mg$	A1	1.1b
	Resolve horizontally	M1	3.1b
	$R\sin\theta° = \frac{mv^2}{r}$	A1	1.1b
	$v = 80 \text{ km h}^{-1} = \frac{80\times1000}{60^2} \text{ m s}^{-1}$	B1	1.2
	Solve simultaneous equations and substitute v in correct units to obtain θ : $\tan\theta° = \frac{v^2}{rg} = \frac{640000}{36^2\times500\times9.8}$, $\theta = 5.8$	A1	2.2a
		(7)	
(b)	All weight acting at a single point	B1	3.5b
		(1)	
(c)	Friction acting down the slope	B1	2.2a
		(1)	
			(9 marks)

Notes
(a) **M1:** Complete strategy involving resolving in perpendicular directions, change of units and solution of simultaneous equations **M1:** Complete strategy to form one equation involving θ e.g. resolve vertically. Condone sin/cos confusion **A1:** Or equivalent **M1:** Complete strategy to form a second equation involving θ e.g. resolve horizontally. Condone sin/cos confusion **A1:** Correct unsimplified – need not substitute for v or r **B1:** Correct conversion km h^{-1} to m s^{-1} (22.2) **A1:** Accept 5.8 or 5.75 (follows use of 9.8)
(b) **B1:** Any appropriate comment e.g. Only one point of contact with the road The centre of mass of the car is on the road.
(c) **B1:** Need to include the direction

Q3.

Question	Scheme	Marks	AOs
			
	$\updownarrow R\cos\theta = mg$	M1	3.1b
	$\leftrightarrow R\sin\theta = mr\omega^2$	M1	3.3
		A1	1.1b
	$\tan\theta = \dfrac{r}{a/4}$ $\left(\tan\theta = \sqrt{15}\right)$	B1	1.1b
	Complete strategy to find ω	M1	3.1b
	$\tan\theta = \dfrac{mr\omega^2}{mg} = \dfrac{4r}{a}$, $\Rightarrow \omega^2 = \dfrac{4g}{a}$, $\omega = 2\sqrt{\dfrac{g}{a}}$	A1	1.1b
		(6)	
	Alternative:		
	$\updownarrow R\cos\theta = mg$	M1	3.1b
	$\leftrightarrow R\sin\theta = ma\sin\theta\omega^2$	M1	3.3
		A1	1.1b
	$\cos\theta = \frac{1}{4}$	B1	1.1b
	Complete strategy to find ω	M1	3.1b
	$\Rightarrow R = 4mg,\ 4mg = ma\omega^2 \Rightarrow \omega = 2\sqrt{\dfrac{g}{a}}$	A1	1.1b
			(6 marks)

Question	Marks	Marking Guidance
		Check their diagram to see where they have put θ Check the working carefully, particularly the value of r: some errors in the working can lead to a fortuitously correct answer
	M1	Resolve vertically. Must be dimensionally correct.
	M1	Resolve horizontally and form equation for circular motion. Must be dimensionally correct.
	A1	Correct pair of equations for their unknowns (any r)
	B1	Correct trig ratio(s) seen or implied. Allow for $r = \frac{\sqrt{15}}{4}a$
	M1	Complete strategy to form and solve a set of equations with $r \neq a$ to find ω. For solving their 2 equations – not dependent
	A1	Eliminate additional variables to obtain ω. Accept equivalent exact forms.
	(6)	
		If R does not act through the centre of the hemisphere then the maximum available is M1M1A0B0M0A0: 2/6

Q4.

Question	Scheme	Marks	AOs
(a)	$(a\sqrt{3})^2 + (3a - AP)^2 = AP^2$	M1	1.1b
	$AP = 2a$ *	A1*	1.1b
		(2)	
(b)	Equation of motion horizontally	M1	3.1b
	$T + T \times \frac{1}{2} = \frac{mU^2}{a}$	A1	1.1b
		A1	1.1b
	$T = \frac{2mU^2}{3a}$	A1	2.2a
		(4)	
(c)	Resolving vertically	M1	3.1b
	$R + T \times \frac{\sqrt{3}}{2} = mg$	A1	1.1b
	On the table => $R > 0$	M1	2.1
	$mg - \frac{2mU^2\sqrt{3}}{3a \times 2} > 0$	A1	1.1b
	$U^2 < ag\sqrt{3}$ *	A1*	2.2a
		(5)	
(d)	Bead would lift off the table	B1	2.4
		(1)	
(e)	Tension would vary along the string	B1	3.5b
		(1)	
			(13 marks)

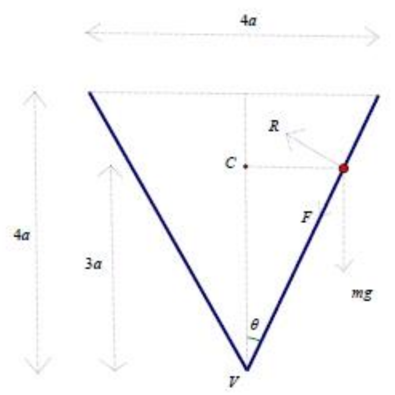
Notes
(a) **M1:** Use of Pythagoras $3a^2 + 9a^2 - 6a \times AP + AP^2 = AP^2 \Rightarrow 6a \times AP = 12a^2$ **A1:** $AP = 2a$. GIVEN ANSWER
(b) **M1:** Use of horizontal equation to solve the problem, with correct no. of terms etc **A1:** Equation with at most one error **A1:** Correct equation **A1:** Correct answer
(c) **M1:** Use of vertical resolution to solve the problem, with correct no. of terms etc **A1:** Correct equation **M1:** Use of $R > 0$ **A1:** Correct inequality **A1*:** Correctly obtained given answer
(d) **B1:** Clear comment
(e) **B1:** Clear explanation

Q5.

Question	Scheme	Marks	AOs
(a)	$l^2 + r^2 = (2l - r)^2$, using Pythagoras	M1	1.1b
	$BR = \frac{3l}{4}$ *	A1*	1.1b
		(2)	
(b)	Resolve vertically	M1	2.1
	$T\cos\alpha = mg$	A1	1.1b
	Overall strategy to solve problem: substitute for $\cos\alpha$ and solve for T	M1	3.1b
	$T = \frac{5mg}{4}$	A1	1.1b
		(4)	
(c)	Equation of motion horizontally	M1	2.1
	$T + T\sin\alpha = \frac{mV^2}{r}$	A1	1.1b
	Overall strategy to solve problem: substitute for T, $\sin\alpha$ and r and solve for V	M1	3.1b
	$V = \sqrt{\frac{3gl}{2}}$	A1	1.1b
		(4)	
			(10 marks)

Notes:		
a	M1	Use of Pythagoras with one unknown
	A1*	Correct length
b	M1	Allow sin/cos confusion
	A1	Correct equation
	M1	Substituting for their trig ratio and solving for T
	A1	cao
c	M1	Correct no. of terms, dimensionally correct
	A1	Correct equation
	M1	Substitute for T, $\sin\alpha$ and r and solve for V
	A1	cao. Accept other equivalent forms

Q6.

Question	Scheme	Marks	AOs
			
	Complete overall strategy	M1	3.1b
	Resolve vertically	M1	3.3
	$mg + F\cos\theta = R\sin\theta$	A1	1.1b
	Horizontal equation of motion	M1	3.3
	$mr\omega^2 = R\cos\theta + F\sin\theta$	A1	1.1b
	Use of limiting friction since maximum ω	M1	3.3
	Substitute for trig ratios: $\dfrac{3a\omega^2}{2g} = \dfrac{9}{2}$	M1	1.1b
	Maximum $\omega = \sqrt{\dfrac{3g}{a}}$	A1	1.1b
			(8 marks)

Notes:

M1: Overall strategy to form equation in ω only e.g.
consider vertical and horizontal motrion and limiting friction

M1: needs all 3 terms. Condone sign errors and sin/cos confusion

A1: correct unsimplified equation

M1: needs all 3 terms. Condone sign errors and sin/cos confusion

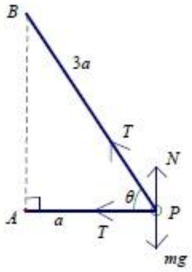
A1: correct unsimplified equation

M1: seen or implied

M1: substitute to achieve equation in a, ω and g only

A1: or equivalent exact form

Q7.

Question	Scheme	Marks	AOs
(a)			
	Resolve vertically	M1	3.1b
	$\updownarrow \quad T\sin\theta + N = mg$	A1	1.1b
	Resolve horizontally	M1	3.1b
	$\leftrightarrow \quad T + T\cos\theta = ma \times \frac{2g}{3a} \quad \left(\frac{4}{3}T = \frac{2}{3}mg, \quad T = \frac{1}{2}mg\right)$	A1	1.1b
	Complete strategy to obtain and solve equations to find N	M1	2.1
	$\Rightarrow \quad \frac{\sqrt{8}}{3} \times \frac{1}{2}mg + N = mg, \qquad N = mg\left(1 - \frac{\sqrt{2}}{3}\right)$	A1	1.1b
		(6)	
(b)	Max speed $\Rightarrow N = 0, \quad \frac{\sqrt{8}}{3}T = mg$	M1	2.1
	$\frac{4}{3}T = ma\omega^2 \Rightarrow \frac{3}{\sqrt{8}}mg \times \frac{4}{3} = ma\omega^2$	M1	1.1b
	$\omega = \sqrt{\frac{4g}{\sqrt{8}a}} = \sqrt{\frac{\sqrt{2}g}{a}}$	A1	1.1b
		(3)	
(c)	The tension in the string is the same on either side of P	B1	2.4
		(1)	
			(10 marks)

Notes:		
a	M1	Forces balance vertically. All terms required. Condone sign errors and sin/cos confusion
	A1	Correct unsimplified equation
	M1	Equation for motion in a horizontal circle. All terms required. Condone sign errors and sin/cos confusion
	A1	Correct unsimplified equation
	M1	Complete strategy to use the model to form simultaneous equations and solve for N
	A1	Any equivalent form. Accept $0.53mg$ or better
b	M1	Resolve vertically and use max speed $\Rightarrow N = 0$ to form equation in T
	M1	Resolve horizontally and substitute for T to form equation in ω
	A1	Any equivalent simplified form. $1.2\sqrt{\frac{g}{a}}$ or better
c	B1	Clear statement explaining how the modelling assumption has been used.

Q8.

Question	Scheme	Marks	AOs
(a)	$\cos\alpha = \frac{4}{5}$ or $\sin\alpha = \frac{3}{5}$	B1	1.1b
	$r = 4a\sin\alpha$	B1	1.1b
	Resolving vertically	M1	3.1b
	$T_1\cos\alpha - T_2\sin\alpha = mg$	A1	1.1b
	Resolving horizontally	M1	3.1b
	$T_1\sin\alpha + T_2\cos\alpha = mr\omega^2$	A1	1.1b
	$T_1\sin\alpha + T_2\cos\alpha = mr\omega^2$	A1	1.1b
	Solving for either tension	M1	2.1
	$T_1 = \frac{4m}{25}(9a\omega^2 + 5g)$ *	A1*	1.1b
	$T_2 = \frac{3m}{25}(16a\omega^2 - 5g)$ *	A1*	1.1b
		(10)	

(b)	$\frac{4m}{25}(9a\omega^2+5g)<4mg$	M1	2.1
	$\frac{3m}{25}(16a\omega^2-5g)>0$	M1	2.1
	$\omega>\sqrt{\frac{5g}{16a}}$ or $\omega<\sqrt{\frac{20g}{9a}}$	A1	2.2a
	$S=\frac{2\pi}{\omega}$	M1	1.1b
	$3\pi\sqrt{\frac{a}{5g}}<S<8\pi\sqrt{\frac{a}{5g}}$ *	A1*	1.1b
		(5)	
(c)	String being light implies that the tension is constant in both portions of the string	B1	3.5b
		(1)	
			(16 marks)

Notes:

(a)

B1: for correct trig. ratio seen

B1: for a correct radius expression seen

M1: for resolving vertically with correct no. of terms and tensions resolved

A1: for a correct equation

M1: for resolving horizontally with correct no. of terms and tensions resolved

A1: for a correct equation

M1: for solving their two equations to find either tension

A1*: for the given answer

A1*: for the given answer

(b)

M1: for use of $T_1<4mg$

M1: for using $T_2>0$

A1: for a correct inequality (either) for ω

M1: for use of $S=\frac{2\pi}{\omega}$ with either critical value

A1*: for given answer

(c)

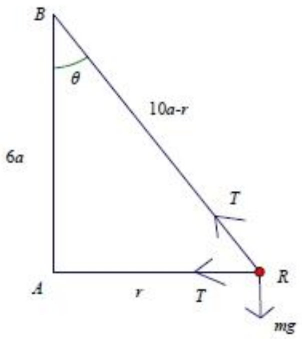
B1: for a clear explanation

Q9.

Question	Scheme	Marks	AOs
(a)	Resolving vertically	M1	3.4
	$R\cos\alpha - F\sin\alpha = mg$	A1	1.1b
	Equation of motion horizontally	M1	3.4
	$R\sin\alpha + F\cos\alpha = \dfrac{mV^2}{r}$	A1	1.1b
	Use of $F = \mu R$	M1	3.4
	Solve for V	M1	3.1b
	$V = \sqrt{\dfrac{(3+4\mu)rg}{4-3\mu}}$ *	A1*	1.1b
		(7)	
(b)	Use of $\mu = 0$ oe	M1	2.1
	$U = \sqrt{\dfrac{3rg}{4}}$	A1	1.1b
		(2)	
(c)	Since $3+4\mu > 3$ and $4-3\mu < 4$ oe	M1	2.1
	$\dfrac{3}{4} < \dfrac{3+4\mu}{4-3\mu}$ and hence $U < V$ *	A1*	2.2a
		(2)	
		(11 marks)	

Notes:		
a	M1	Correct no. of terms, dim correct, condone sin/cos confusion and sign errors
	A1	Correct equation
	M1	Correct no. of terms, dim correct, condone sin/cos confusion and sign errors
	A1	Correct equation
	M1	Independent but must be used in an equation
	M1	Substitute for trig and solve for V. Dependent on preceding M marks.
	A1*	Correct given answer correctly obtained
b	M1	If they don't use $\mu = 0$, we need to see the first 6 marks from (a), without friction
	A1	cao
c	M1	Any convincing argument
	A1*	Given answer correctly obtained
		SC: Allow M1A0 if they work in reverse to show that if $U < V$ then $\mu > 0$ and make an appropriate comment

Q10.

Question	Scheme	Marks	AOs
(a)			
	Resolve vertically	M1	3.4
	$T\cos\theta = mg$	A1	1.1b
	$T = \left(\frac{mg}{\cos\theta} = \frac{6.8mg}{6}\right) = \frac{17mg}{15}$	A1	1.1b
		(3)	
(b)	Equation of motion	M1	3.1b
	$mr\omega^2 = T + T\sin\theta \quad \left(m\times 3.2a\omega^2 = their\ T\left(1+\frac{8}{17}\right)\right)$	A1 A1	1.1b 1.1b
	Solves for ω or ω^2	M1	1.1b
	$\left(\frac{r\omega^2}{g} = \frac{1+\sin\theta}{\cos\theta} = \frac{6.8+3.2}{6},\quad \omega^2 = \frac{10g}{6\times 3.2a}\right) \quad \omega = \sqrt{\frac{25g}{48a}} = \frac{5}{4}\sqrt{\frac{g}{3a}}$	A1	1.1b
		(5)	
			(8 marks)

Notes:	
(a)M1	Need all terms. Condone sin/cos confusion
A1	Correct unsimplified equation.
A1	Correct answer only $1.1mg$ or better ($1.13...mg$) Do not ignore subsequent working if they try to combine this with a tension in AR
(b)M1	Equation for circular motion. Need all terms and dimensionally correct. Condone sin/cos confusion and sign errors. Any correct form for acceleration
A1 A1	Unsimplified equation with at most one error Correct unsimplified equation
	Allow M1A1A0 for $mr\omega^2 = T' + (their\ (a))\sin\theta$
M1	Clear attempt to substitute for trig and tension or divide their two equations to solve for ω or ω^2 in terms of a and g Independent M mark but requires an equation using tension and trig.
A1	Any equivalent form $0.72\sqrt{\frac{g}{a}}$ or better (0.7216...)

Q11.

Question Number	Scheme	Marks
(i)	$1.2mg\cos\theta = mg$ or $T\cos\theta = mg$	M1A1
	$\cos\theta° = \frac{1}{1.2}$ $\theta° = \cos^{-1}\frac{1}{1.2}$, $\theta =$ 33.55... (accept 34, 33.6 or better)	A1
	$1.2mg\sin\theta = mr\omega^2$ or $T\cos\theta = mr\omega^2$	M1A1
	$1.2mg\sin\theta = m \times l\sin\theta\omega^2$	A1
(ii)	$1.2mg = 58.8lm \Rightarrow l = \frac{1.2\times 9.8}{58.8} = 0.2(\text{m})$	dM1A1 (8)

M1	Resolve vertically. Tension to be resolved, weight not resolved.
A1	Fully correct equation with substitution for T made.
(i)A1	Correct value of θ Min 2 sf Use of radians scores A0
M1	Attempt NL2 horizontally. Tension must be resolved, acceleration can be in either form.
A1	LHS correct, RHS can be $mr\omega^2$ or $m\frac{v^2}{r}$ here. T substituted now or later
A1	RHS correct, acceleration as shown. $\sin\theta$ may be numerical $\frac{\sqrt{11}}{6}$ or 0.5527... (min 3 sf) or a numerical value for r ($\frac{\sqrt{11}}{30}$ or 0.110...) may be seen.
dM1	Use the above equation to obtain a numerical value for l. Depends on the second M mark
(ii)A1	Correct value of l. Accept 0.2, 0.20, 0.200. Exceptionally allow $\frac{1}{5}$ here.

Q12.

Question Number	Scheme	Marks
(a)	$\frac{1}{2}mv^2 - \frac{1}{2}m \times 7ag = mga\sin\theta$	M1A1A1
	$v^2 = 7ag + 2ag\sin\theta = ag(7 + 2\sin\theta)$ *	A1 (4)
(b)	At top $v^2 = 5ag$	M1A1
	$R + mg = m\frac{v^2}{a}$ or $m\frac{v^2}{a} > mg$	M1A1
	$R = 4mg$ or substitute for v^2	dM1
	$R > 0$ $\therefore$ complete circles	A1 cso (6)
(c)	Max v at lowest point	
	$\sin\theta = 1 \Rightarrow v^2 = 9ag$	M1
	$v = 3\sqrt{ag}$	A1 (2) [12]

(a)

M1 Energy equation from the point of projection to a general point. Must have 3 terms and the PE term must include a trig function.

A1 Correct difference of KE terms.

A1 Correct PE term and all signs correct.

A1cso Obtain correct **given** expression for v^2 with no errors in the solution.

(b)

M1 Use the result given in (a) with $\theta = 270°$ to obtain v^2 at the top. Substitution for θ may occur later.

A1 Correct expression for v^2. May be implied by correct work later.

M1 Attempt NL2 at the **top.** This mark cannot be awarded if a general position is used but can be awarded later when the motion at the highest point is considered.

A1 Correct NL2 at the top with $R + mg$

dM1 Eliminate v^2 between the 2 equations. Depends on the 2 previous M marks in (b).

A1cso Correct result for R (at the top) seen **and** the conclusion stated. (Do not need to see $R > 0$). If working with the resultant, resultant $> mg$ must be seen.
Full marks can be awarded if it is stated that $v^2 > 0$ and $R > 0$ at the top - mark the work relevant to R.

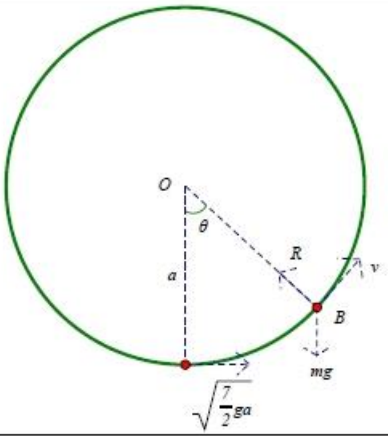
ALT Last 4 marks:
If $m\frac{v^2}{a} > mg$ is seen, give M1A1. M1 substitute for v^2; $5mg > mg$ $\therefore$ complete circles

(c)

M1 Using $\sin\theta = 1$ in the result given in (a) to obtain v^2 at the lowest point. Any other **complete** method may be used, eg an energy equation provided it leads to the speed at the lowest point.

A1 $v = 3\sqrt{ag}$ or $\sqrt{9ag}$ (Watch square root covers all necessary letters.)

Q13.

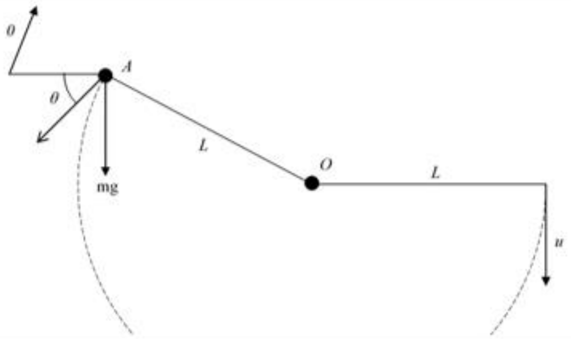
Question	Scheme	Marks	AOs
(a)			
	Conservation of energy	M1	2.1
	$\frac{1}{2}mv^2 + mga(1-\cos\theta) = \frac{1}{2}m\left(\frac{7}{2}ga\right)$	A1	1.1b
	$v^2 = ga\left(\frac{3}{2} + 2\cos\theta\right)$*	A1*	2.2a
		(3)	
(b)	Resolve parallel to OB and use $\frac{mv^2}{a}$	M1	3.1b
	$R - mg\cos\theta = \frac{mv^2}{a}$	A1	1.1b
	Use $R = 0$ $\quad g\cos\theta = -\frac{v^2}{a}$	M1	3.1b
	Solve for θ $\quad \Rightarrow\ g\cos\theta = -g\left(\frac{3}{2} + 2\cos\theta\right)$	M1	1.1b
	$\theta = 120°$	A1	1.1b
		(5)	

(c)	Any appropriate comment e.g. The hoop is unlikely to be smooth	B1	3.5b
		(1)	

Question	Scheme	Marks	AOs
(d)	At rest $\Rightarrow \quad v = 0$	M1	3.1b
	$\Rightarrow \quad \cos\theta = -\frac{3}{4}$	A1	1.1b
	Acceleration is tangential	M1	3.1b
	Magnitude $\lvert g\cos(\theta - 90)\rvert = 6.48 \text{ m s}^{-2}$ or $\frac{\sqrt{7}}{4}g$	A1	1.1b
	At $\left(\cos^{-1}\left(-\frac{3}{4}\right) - 90 = \right) 48.6°$ to the downward vertical	A1	1.1b
		(5)	
			(14 marks)

Notes:
(a) **M1:** All terms required. Must be dimensionally correct **A1:** Correct unsimplified equation **A1*:** Show sufficient working to justify the **given answer** and a 'statement' that the required result has been achieved.
(b) **M1:** Resolve parallel to *OB* **A1:** correct equation **M1:** Use $R = 0$ seen or implied **M1:** Solve for θ **A1:** Accept $\frac{2\pi}{3}$
(c) B1: Any appropriate comment e.g. - hoop may not be smooth; - air resistance could affect the motion
(d) **M1:** $v = 0$ seen or implied **A1:** correct equation in θ **M1:** correct direction for acceleration **A1:** Accept 6.48, 6.5 or exact in g **A1:** Accept 0.848 (radians)

Q14.

Question	Scheme	Marks	AOs
(a)	$v = 0 \Rightarrow \dfrac{v^2}{L} = 0 \Rightarrow$ no acceleration towards O $\Rightarrow$ acceleration is perpendicular to OA	B1	2.4
		(1)	
(b)			
	Conservation of energy	M1	2.1
	$0 = \frac{1}{2}mu^2 - mgL\cos\theta \quad \left(0 = \frac{2gL}{3} - 2gL\cos\theta\right)$	A1	1.1b
	$\Rightarrow \quad \cos\theta = \frac{1}{3}$	A1	1.1b
	Complete strategy to find the angle and |acceleration|	M1	3.1a
	Magnitude: $g\sin\theta = \frac{2\sqrt{2}}{3}g$	A1	1.1b
	$\theta = 71°$ or better	A1	1.1b
		(6)	

(c)	Circular motion	M1	3.1a
	$T - mg = \frac{mv^2}{L}$	A1	1.1b
	Energy equation	M1	2.1
	$v^2 = \frac{2gL}{3} + 2gL\left(= \frac{8gL}{3}\right)$	A1	1.1b
	$T = mg + \frac{8mg}{3} = \frac{11mg}{3}$	A1	2.2a
		(5)	
			(12 marks)

Question	Marks	Marking Guidance
(a)	B1	Clear explanation using $v=0$
	(1)	
(b)		Check their diagram to see where they have put θ .
	M1	All terms required. Must be dimensionally correct. Condone sign errors. $v=0$ seen or implied
	A1	Correct unsimplified equation for their θ
	A1	Or equivalent to give trig ratio for relevant angle (taking account of their θ)
	M1	Complete strategy to find our θ or magnitude of acceleration
	A1	Correct magnitude from correct work only. Accept 9.2, 9.24
	A1	Correct value of θ (1.2 radians or better) from correct work only
	(6)	
(c)	M1	Equation for circular motion. Need all terms and dimensionally correct. Condone sign errors.
	A1	Correct unsimplified equation
	M1	Use of conservation of energy. Require all 3 terms and dimensionally correct.
	A1	Correct unsimplified equation
	A1	Correct only
	(5)	

Q15.

Question	Scheme	Marks	AOs
(a)	Conservation of energy:	M1	3.1a
	$\frac{1}{2}mu^2 + mgl\sin\alpha = \frac{1}{2}mv^2 \quad \left(v^2 = \frac{9gl}{5} + 2gl\sin\alpha\right)$	A1	1.1b
	Equation of motion:	M1	3.1a
	$T - mg\sin\alpha = \frac{mv^2}{l}$	A1	1.1b
	Complete strategy to find T in terms of α	M1	2.1
	$\Rightarrow T = mg\sin\alpha + \frac{mv^2}{l} = mg\sin\alpha + \frac{9mg}{5} + 2mg\sin\alpha$ $= 3mg\sin\alpha + \frac{9mg}{5}$ *	A1*	2.2a
		(6)	
(b)	String slack $\Rightarrow T = 0 \Rightarrow \sin\alpha = -\frac{3}{5}$	B1	3.1a
	Use energy equation to find v:	M1	1.1b
	$v^2 = \frac{9gl}{5} - \frac{3}{5}\times 2gl, \quad v = \sqrt{\frac{3gl}{5}}$	A1	1.1b
		(3)	
(c)	Initial vertical component of speed $= \frac{4}{5}\times\sqrt{\frac{3gl}{5}}$	B1	1.1b
	Use of *suvat*: $0 = u^2 - 2gh = \frac{16}{25}\times\frac{3gl}{5} - 2gh$	M1	3.1a
	$h = \frac{24l}{125}$	A1	1.1b
	Total height above $O = \frac{3l}{5} + \frac{24l}{125} = \frac{99l}{125}$	A1	2.2a
		(4)	

Question	Scheme	Marks	AOs
alt	Initial horizontal component of speed $= \frac{3}{5}\times\sqrt{\frac{3gl}{5}}$	B1	1.1b
	Conservation of energy:	M1	3.1a
	$mgh = \frac{1}{2}m\left(\frac{9}{5}\right)gl - \frac{1}{2}m\left(\frac{9}{25}\times\frac{3gl}{5}\right)$	A1	1.1b
	$h = \frac{99l}{125}$	A1	2.2a
		(4)	
		(13 marks)	

Notes:		
	M1	Must include all terms. Condone sign errors and sin/cos confusion
	A1	Correct unsimplified equation
	M1	Must include all terms. Condone sign errors and sin/cos confusion
	A1	Correct unsimplified equation
	M1	Complete strategy to form an expression for T in terms of α e.g. by using conservation of energy and the circular motion to form sufficient equations to obtain an equation in T only.
	A1*	Obtain given answer from correct working
(b)	B1	Correct deduction
	M1	Substitute value to find v^2
	A1	Correct only
(c)	B1	Correct vertical component of velocity when string goes slack.
	M1	Use of $v^2 = u^2 + 2as$ or alternative complete method to find the additional height.
	A1	Additional height correct
	A1	Total height correct
(c) alt	B1	Correct horizontal component of velocity when string goes slack.
	M1	Use of conservation of energy or alternative complete method to find the height. All terms required. Condone sign errors.
	A1	Correct unsimplified equation in h and l
	A1	Correct answer

Q16.

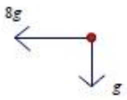
Question	Scheme	Marks	AOs
(a)	Equation of motion along the string at the top of the circle	M1	3.1b
	$T + mg = \dfrac{mv^2}{a}$	A1	1.1b
	Conservation of energy	M1	3.1b
	$\dfrac{1}{2}mU^2 - \dfrac{1}{2}mv^2 = mga$	A1	1.1b
	Overall strategy to solve these equations for T and use $T = 0$	M1	3.1b
	$U = \sqrt{3ga}$	A1	1.1b
		(6)	
(b)	Equation of motion along the string at instant string breaks	M1	3.1b
	$\dfrac{11mg}{2} - mg\cos\alpha = \dfrac{mv^2}{a}$	A1	1.1b
	Conservation of energy	M1	3.1b
	$\dfrac{1}{2}mv^2 - \dfrac{1}{2}m.4ag = mga\cos\alpha$	A1	1.1b
	Solve these equations for $\cos\alpha$ $(= \dfrac{1}{2})$	M1	1.1b
	Angle turned through is 210°	A1	1.1b
		(6)	
(c)	Find radial component of acceleration: $\dfrac{v^2}{a}$ $(= 5g)$	M1	2.1
	Find tangential component of acceleration: $g\sin\alpha$ $(= \dfrac{\sqrt{3}}{2}g)$	M1	2.1
	Square, add and square root	M1	3.1b
	$\dfrac{\sqrt{103}}{2}g$ or 49.7 $(\mathrm{m\,s^{-2}})$ or 50 $(\mathrm{m\,s^{-2}})$	A1	1.1b
		(4)	

Notes:		
a	M1	Correct number of terms
	A1	Correct equation
	M1	All terms needed and dimensionally correct
	A1	Correct equation

	M1	Solve for T and use $T = 0$ (allow $T \geq 0$)
	A1	cao
b	M1	Correct no. of terms with mg resolved and correct acceleration component
	A1	Correct equation
	M1	All terms needed and dimensionally correct
	A1	Correct equation
	M1	Solve for $\cos\alpha$
	A1	cao
c	M1	Uses their value of v from part (b)
	M1	Equation of motion along the tangent oe
	M1	Find the magnitude of the resultant acceleration
	A1	cao

Q17.

Question	Scheme	Marks	AOs
(a)	Conservation of energy:	M1	3.1b
	$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg \times \frac{2a}{5}(1-\cos\theta)$	A1	1.1b
	Equation of motion towards O	M1	3.1b
	$T - mg\cos\theta = \frac{5mv^2}{2a}$	A1	1.1b
	Complete method to find T in terms of u, a and θ	DM1	2.1
	$T = mg\cos\theta + \frac{5m}{2a}\left(u^2 - \frac{4a}{5}g(1-\cos\theta)\right)$ $= 3mg\cos\theta - 2mg + \frac{5mu^2}{2a}$ *	A1*	2.2a
		(6)	
(b)	Require $T \geq 0$ when $\theta = \pi$: $\frac{5mu^2}{2a} \geq mg(2+3)$	M1	2.1
	$u^2 \geq 2ag$, minimum $u = \sqrt{2ag}$	A1	1.1b
		(2)	

(c)	$\theta = \frac{\pi}{2}, u = 2\sqrt{ag} \Rightarrow T = -2mg + \frac{5m}{2a} \times 4ag$	B1	1.1b
			
	Magnitude of acceleration $= g\sqrt{64+1}$	M1	2.1
	$= \sqrt{65}g$	A1	1.1b
		(3)	
(d)	Consider the uniformity / dimensions of the package String might be extensible. include the weight of the string	B1	3.5c
		(1)	

	(12 marks)
Notes:	
(a)M1	Need all terms. Dimensionally correct. Condone sign errors and sin/cos confusion Allow with $\frac{2a}{5}\cos\theta$ in place of $\frac{2a}{5}(1-\cos\theta)$
A1	Correct unsimplified equation
M1	Need all terms. Dimensionally correct. Condone sign errors and sin/cos confusion
A1	Correct unsimplified equation
M1	Complete method, e.g. using conservation of energy and the circular motion, to form sufficient equations to obtain an expression without v A complete method requires the two preceding M marks.
A1*	Obtain given result from correct working
(b)M1	Identify correct condition for complete circle and solve for u. Condone working from $T = 0$
A1	Allow $u \geq \sqrt{2ag}$ Condone $u > \sqrt{2ag}$, and $u = \sqrt{2ag}$
(c)B1	Correct T or v^2 seen or implied
M1	Use of Pythagoras with their horizontal component of acceleration
A1	Correct only, or $8.1g$ $(8.062\ldots g)$ or better
(d) B1	Any valid suggestion **relating to the model**. Allow negatives of statements within the model e.g. not model the package as a particle. B0 if multiple suggestions including one incorrect. B0 for accuracy of g as this is not part of the description of the model.