Mark Scheme (Results)

November 2021

Pearson Edexcel GCE
In Mathematics (9MAO)
Paper 32 Mechanics

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October 2021
Question Paper Log Number P68824A
Publications Code 9MAO_03_2111_MS
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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.


## EDEXCEL GCE MATHEMATICS <br> General Instructions for Marking

1. The total number of marks for the paper is 100 .
2. The Edexcel Mathematics mark schemes use the following types of marks:

- M marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of $M$ marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod - benefit of doubt
- ft - follow through
- the symbol $\sqrt{ }$ will be used for correct ft
- cao - correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw - ignore subsequent working
- awrt - answers which round to
- SC: special case
- oe - or equivalent (and appropriate)
- dep - dependent
- indep - independent
- dp decimal places
- sf significant figures
-     * The answer is printed on the paper
- $\square$ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.
6. Ignore wrong working or incorrect statements following a correct answer.
7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

## General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or $\sin$ ) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- $d M$ indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of $g=9.8$ should be given to 2 or 3 SF.
- Use of $g=9.81$ should be penalised once per (complete) question.
N.B. Over-accuracy or under-accuracy of correct answers should only be penalised once per complete question. However, premature approximation should be penalised every time it occurs.
- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),......then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads - if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft
- Mechanics Abbreviations
$\mathrm{M}(\mathrm{A})$ Taking moments about A
N2L Newton's Second Law (Equation of Motion)
NEL Newton's Experimental Law (Newton's Law of Impact)
HL Hooke's Law
SHM Simple harmonic motion
PCLM Principle of conservation of linear momentum
RHS, LHS Right hand side, left hand side

| Question |  | Scheme | Marks | AOs |
| :---: | :---: | :---: | :---: | :---: |
| 1(a) |  | Use of $\mathbf{v}=\mathbf{u}+\mathbf{a} t$ with $t=2: \quad \mathbf{v}=4 \mathbf{i}+2(2 \mathbf{i}-3 \mathbf{j})$ <br> OR integration: <br> $\mathbf{v}=(2 \mathbf{i}-3 \mathbf{j}) t+4 \mathbf{i}$, with $t=2$ | M1 | 3.1a |
|  |  | $\mathbf{v}=8 \mathbf{i}-6 \mathbf{j}$ | A1 | 1.1b |
|  |  |  | (2) |  |
| 1(b) |  | Use of $\mathbf{r}=\mathbf{u} t+\frac{1}{2} \mathbf{a} t^{2}$ at $t=3$ : $(\mathbf{i}+\mathbf{j})+\left[3 \times 4 \mathbf{i}+\frac{1}{2} \times(2 \mathbf{i}-3 \mathbf{j}) \times 3^{2}\right]$ <br> OR: find $\mathbf{v}$ at $t=3: 4 \mathbf{i}+3(2 \mathbf{i}-3 \mathbf{j})=(10 \mathbf{i}-9 \mathbf{j})$ then use $\mathbf{r}=\frac{1}{2}(\mathbf{u}+\mathbf{v}) t$ $(\mathbf{i}+\mathbf{j})+\left[\frac{1}{2}[4 \mathbf{i}+(10 \mathbf{i}-9 \mathbf{j})] \times 3\right]$ <br> or $\quad \mathbf{r}=\mathbf{v} t-\frac{1}{2} \mathbf{a} t^{2}$ $(\mathbf{i}+\mathbf{j})+\left[3 \times(10 \mathbf{i}-9 \mathbf{j})-\frac{1}{2} \times(2 \mathbf{i}-3 \mathbf{j}) \times 3^{2}\right]$ <br> OR integration: $\mathbf{r}=(\mathbf{i}+\mathbf{j})+\left[(2 \mathbf{i}-3 \mathbf{j}) \frac{1}{2} t^{2}+4 t \mathbf{i}\right]$, with $t=3$ | M1 | 3.1a |
|  |  | $\mathbf{r}=22 \mathbf{i}-12.5 \mathbf{j}$ | A1 | 2.2a |
|  |  |  | (2) |  |
| (4 marks) |  |  |  |  |
| Notes: Accept column vectors throughout |  |  |  |  |
| 1a | M1 | Complete method to find $\mathbf{v}$, using ruvat or integration (M0 if $\mathbf{i}$ and/or $\mathbf{j}$ is missing) |  |  |
|  | A1 | Apply isw if they also find the speed |  |  |
| 1b | M1 | Complete method to find the p.v. but this mark can be scored if they omit $(\mathbf{i}+\mathbf{j})$ i.e. the M1 is for the expression in the square bracket <br> If they integrate, the M1 is earned once the expression in the square bracket is seen with $t=3$ <br> (M0 if $\mathbf{i}$ and/or $\mathbf{j}$ is missing) |  |  |
|  | A1 | cao |  |  |


| Question |  | Scheme | Marks | AOs |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Mark parts (a) and (b) together |  |  |
| 2(a) |  | Equation of motion for $A$ | M1 | 3.3 |
|  |  | $3 m g \sin \alpha-F-T=3 m a$ | A1 | 1.1b |
|  |  |  | (2) |  |
| 2(b) |  | Resolve perpendicular to the plane | M1 | 3.4 |
|  |  | $R=3 m g \cos \alpha$ | A1 | 1.1b |
|  |  | $F=\frac{1}{6} R$ | B1 | 1.2 |
|  |  | Equation of motion for $B$ OR for whole system | M1 | 3.3 |
|  |  | $T-m g=m a \quad$ OR $3 m g \sin \alpha-F-m g=3 m a+m a$ | A1 | 1.1b |
|  |  | Complete method to solve for $a$ | DM1 | 3.1b |
|  |  | $a=\frac{1}{10} g$ * | A1* | 2.2a |
|  |  |  | (7) |  |
| 2(c) |  |  | B1 | 1.1b |
|  |  | e.g. acceleration (of $B$ ) is constant; dependent on first B1 | DB1 | 2.4 |
|  |  |  | (2) |  |
| 2(d) |  | e.g. the tensions in the two equations of motion would be different. Tension on $A$ would be different to tension on $B$ | B1 | 3.5a |
|  |  |  | (1) |  |
| (12 marks) |  |  |  |  |
| Notes: N.B. If m's are consistently missing treat as a MR, so max <br> (a) M1A0 <br> (b) M1A0B0M1A1M1A1 <br> (c) B1B1 <br> (d) B1 <br> For (a) and (b), allow verification, but must see full equations of motion. |  |  |  |  |
| 2a | M1 | Equation in $T$ and $a$ with correct no. of terms, condone sign errors and $\sin / \cos$ confusion (If one of the 3's is missing, allow M1) <br> N.B. Treat $\sin (3 / 5)$ etc as an A error but allow recovery |  |  |
|  | A1 | Correct equation (allow ( $-a$ ) instead of $a$ in both equations) |  |  |


| 2b | M1 | Correct no. of terms, condone sign errors and sin/cos confusion <br> Allow if appears in (a) |
| :--- | :--- | :--- |
|  | A1 | Correct equation |
|  | B1 | Seen anywhere in (a) or (b), including on a diagram |
|  | M1 | Equation (for $B$ ) in $T$ and $a$ with correct no. of terms, condone sign errors and sin/cos <br> confusion <br> OR Whole system equation with correct no. of terms, condone sign errors and <br> sin/cos confusion |
|  | A1 | Correct equation |
|  | DM1 | Complete method (trig may not be substituted), dependent on M1 in (a) and second <br> M1 in (b) if they use two equations, or second M1 in (b) if they use one equation. |
| A1* | Correct answer correctly obtained. |  |
| 2c | B1 | Straight line starting at the origin (could be reflected in the $t$-axis). B0 if continuous <br> vertical line at the end. |
|  | DB1 | Dependent on first B1, for any equivalent statement |
| 2d | B1 | B0 if incorrect extras |


| Question | Scheme | Marks | AOs |
| :---: | :---: | :---: | :---: |
|  | Part (a) is a 'Show that..' so equations need to be given in full to earn A marks |  |  |
| 3(a) |  |  |  |
|  | Moments equation: (M1A0 for a moments inequality) | M1 | 3.3 |
|  | $\mathrm{M}(A), m g a \cos \theta=2 S a \sin \theta$ <br> $\mathrm{M}(B), m g a \cos \theta+2 F a \sin \theta=2 R a \cos \theta$ <br> $\mathrm{M}(C), F \times 2 a \sin \theta=m g a \cos \theta$ <br> $\mathrm{M}(D), 2 R a \cos \theta=m g a \cos \theta+2 S a \sin \theta$ <br> $\mathrm{M}(G), R a \cos \theta=F a \sin \theta+S a \sin \theta$. | A1 | 1.1b |
|  | ( $\downarrow$ ) $R=m g$ OR ( $\leftrightarrow$ ) $F=S$ | B1 | 3.4 |
|  | Use their equations (they must have enough) and $F \leq \mu R$ to give an inequality in $\mu$ and $\theta$ only (allow DM1 for use of $F=\mu R$ to give an equation in $\mu$ and $\theta$ only) | DM1 | 2.1 |
|  | $\mu \geq \frac{1}{2} \cot \theta^{*}$ | A1* | 2.2a |
|  |  | (5) |  |
| 3(b) |  |  |  |
|  | Moments equation: | M1 | 3.4 |
|  | $\begin{aligned} & \mathrm{M}(A), m g a \cos \theta=2 N a \sin \theta \\ & \mathrm{M}(B), m g a \cos \theta+2 k m g a \sin \theta=2 R a \cos \theta+\frac{1}{2} m g 2 a \sin \theta \\ & \mathrm{M}(D), 2 R a \cos \theta=m g a \cos \theta+N 2 a \sin \theta \\ & \mathrm{M}(G), k m g a \sin \theta+N a \sin \theta=\frac{1}{2} m g a \sin \theta+R a \cos \theta \end{aligned}$ | A1 | 1.1b |



| Question |  | Scheme | Marks | AOs |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Note that $\boldsymbol{g}=10$; penalise once for whole question if $\boldsymbol{g}=9.8$ |  |  |
| 4(a) |  | Use $s=u t+\frac{1}{2} a t^{2}$ vertically or any complete method to give an equation in $t$ only | M1 | 3.4 |
|  |  | $-70=65 \sin \alpha \times t-\frac{1}{2} \times g \times t^{2}$ | A1 | 1.1b |
|  |  | M (A) 1 | 1.1b |
|  |  | $t=7$ (s) | A1 | 1.1b |
|  |  |  | (4) |  |
| 4(b) |  |  | Horizontal velocity component at $A=65 \cos \alpha$ (60) | B1 | 3.4 |
|  |  | Complete method to find vertical velocity component at $A$ | M1 | 3.4 |
|  |  | $65 \sin \alpha-g \times 7 \quad$ OR $\quad \sqrt{(-25)^{2}+2 g \times 70}$ | A1ft | 1.1b |
|  |  | Sub for trig and square, add and square root : $\sqrt{60^{2}+(-45)^{2}}$ | M1 | 3.1b |
|  |  | 75 Accept $80\left(\mathrm{~m} \mathrm{~s}^{-1}\right)$ | A1 | 1.1b |
|  |  |  | (5) |  |
| 4(c) |  | e.g. an approximate value of $g$ has been used, the dimensions of the stone could affect its motion, spin of the stone, $g=10$ instead of 9.8 has been used, $g$ has been assumed to be constant, wind effect, shape of the stone | B1 | 3.5b |
|  |  |  | (1) |  |
| (10 marks) |  |  |  |  |
| Notes: |  |  |  |  |
| 4a | M1 | Complete method, correct no. of terms, condone sign errors and sin/cos confusion |  |  |
|  | A1 | Correct equation in $t$ only with at most one error |  |  |
|  | M(A) 1 | Correct equation in $t$ only |  |  |
|  |  | N.B. For 'up and down' methods etc, the two A marks are for all the equations that they use, lose a mark for each error. |  |  |
|  | A1 | Cao $(g=9.8,7.1$ or 7.11$) \quad(g=9.81,7.1$ or 7.12$)$ |  |  |
| 4b | B1 | Seen, including on a diagram. |  |  |
|  | M1 | Condone sign errors and sin/cos confusion |  |  |
|  | A1ft | Correct expression; accept negative of this, follow their $t$ |  |  |
|  | M1 | Sub for trig and use Pythagoras |  |  |
|  | A1 | Cao ( $g=9.8$ or $9.81,75$ or 74.8 ) |  |  |


| Question | Scheme | Marks | AOs |
| :---: | :---: | :---: | :---: |
|  | Allow column vectors throughout this question |  |  |
| 5(a) | Differentiate $\mathbf{v}$ wrt $t$ | M1 | 3.1a |
|  | $\frac{3}{2} t^{-\frac{1}{2}} \mathbf{i}-2 \mathbf{j}$ isw | A1 | 1.1b |
|  |  | (2) |  |
| 5(b) | $3 t^{\frac{1}{2}}=2 t$ | M1 | 2.1 |
|  | Solve for $t$ | DM1 | 1.1b |
|  | $t=\frac{9}{4}$ | A1 | 1.1b |
|  |  | (3) |  |
| 5(c) | Integrate $\mathbf{v}$ wrt $t$ | M1 | 3.1a |
|  | $\mathbf{r}=2 t^{\frac{3}{2}} \mathbf{i}-t^{2} \mathbf{j}(+\mathbf{C})$ | A1 | 1.1b |
|  | $t=1, \mathbf{r}=-\mathbf{j}=>\mathbf{C}=-2 \mathbf{i}$ so $\mathbf{r}=2 t^{\frac{3}{2}} \mathbf{i}-t^{2} \mathbf{j}-2 \mathbf{i}$ | A1 | 2.2a |
|  |  | (3) |  |
| 5(d) | $\sqrt{\left(3 t^{\frac{1}{2}}\right)^{2}+(2 t)^{2}}=10 \quad$ or $\left(3 t^{\frac{1}{2}}\right)^{2}+(2 t)^{2}=10^{2}$ | M1 | 2.1 |
|  | $9 t+4 t^{2}=100$ | M(A)1 | 1.1b |
|  | $t=4$ | A1 | 1.1b |
|  | $\mathbf{r}=14 \mathbf{i}-16 \mathbf{j}$ | M1 | 1.1b |
|  | $\sqrt{14^{2}+(-16)^{2}}$ | M1 | 3.1a |
|  | $\sqrt{452}(2 \sqrt{113})(\mathrm{m})$ | A1 | 1.1b |
|  |  | (6) |  |
| (14 marks) |  |  |  |

## Notes:

| $\mathbf{5 a}$ | M1 | Both powers decreasing by $1 \quad$ (M0 if vector(s) disappear but allow recovery) |
| :--- | :--- | :--- |
|  | A1 | cao |
| $\mathbf{5 b}$ | M1 | Complete method, using v, to obtain an equation in $t$ only, allow a sign error |
|  | DM1 | Dependent on M1,solve for $t$ |


|  | A1 | cao |
| :--- | :--- | :--- |
| $\mathbf{5 c}$ | M1 | Both powers increasing by 1 (M0 if vectors disappear but allow recovery) |
|  | A1 | Correct expression without $\mathbf{C}$ |
|  | A1 | cao |
| $\mathbf{5 d}$ | M1 | Use of Pythagoras on $\mathbf{v}$ and 10 to set up equation in $t$ |
|  | M(A)1 | Correct 3 term quadratic in $t$ |
|  | A1 | cao |
|  | M1 | Substitute their numerical $t$ value into their $\mathbf{r}$ |
|  | M1 | Use of Pythagoras to find the magnitude of their $\mathbf{r}$ |
|  | A1 | cso |

