Oxford Cambridge and RSA

# A Level Mathematics A (H240) 

## Formulae Booklet

## Formulae

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## Arithmetic series

$S_{n}=\frac{1}{2} n(a+l)=\frac{1}{2} n\{2 a+(n-1) d\}$

## Geometric series

$S_{n}=\frac{a\left(1-r^{n}\right)}{1-r}$
$S_{\infty}=\frac{a}{1-r}$ for $|r|<1$

## Binomial series

$(a+b)^{n}=a^{n}+{ }^{n} \mathrm{C}_{1} a^{n-1} b+{ }^{n} \mathrm{C}_{2} a^{n-2} b^{2}+\ldots+{ }^{n} \mathrm{C}_{r} a^{n-r} b^{r}+\ldots+b^{n} \quad(n \in \mathbb{N})$
where ${ }^{n} \mathrm{C}_{r}={ }_{n} \mathrm{C}_{r}=\binom{n}{r}=\frac{n!}{r!(n-r)!}$
$(1+x)^{n}=1+n x+\frac{n(n-1)}{2!} x^{2}+\ldots+\frac{n(n-1) \ldots(n-r+1)}{r!} x^{r}+\ldots \quad(|x|<1, n \in \mathbb{R})$

## Differentiation

| $\mathrm{f}(x)$ | $\mathrm{f}^{\prime}(x)$ |
| :--- | :--- |
| $\tan k x$ | $k \sec ^{2} k x$ |
| $\sec x$ | $\sec x \tan x$ |
| $\cot x$ | $-\operatorname{cosec}^{2} x$ |
| $\operatorname{cosec} x$ | $-\operatorname{cosec} x \cot x$ |

Quotient rule $y=\frac{u}{v}, \frac{\mathrm{~d} y}{\mathrm{~d} x}=\frac{v \frac{\mathrm{~d} u}{\mathrm{~d} x}-u \frac{\mathrm{~d} v}{\mathrm{~d} x}}{v^{2}}$

## Differentiation from first principles

$\mathrm{f}^{\prime}(x)=\lim _{h \rightarrow 0} \frac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$

## Integration

$\int \frac{\mathrm{f}^{\prime}(x)}{\mathrm{f}(x)} \mathrm{d} x=\ln |\mathrm{f}(x)|+c$
$\int \mathrm{f}^{\prime}(x)(\mathrm{f}(x))^{n} \mathrm{~d} x=\frac{1}{n+1}(\mathrm{f}(x))^{n+1}+c$
Integration by parts $\int u \frac{\mathrm{~d} v}{\mathrm{~d} x} \mathrm{~d} x=u v-\int v \frac{\mathrm{~d} u}{\mathrm{~d} x} \mathrm{~d} x$

## Small angle approximations

$\sin \theta \approx \theta, \cos \theta \approx 1-\frac{1}{2} \theta^{2}, \tan \theta \approx \theta$ where $\theta$ is measured in radians

## Trigonometric identities

$\sin (A \pm B)=\sin A \cos B \pm \cos A \sin B$
$\cos (A \pm B)=\cos A \cos B \mp \sin A \sin B$
$\tan (A \pm B)=\frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad\left(A \pm B \neq\left(k+\frac{1}{2}\right) \pi\right)$

## Numerical methods

Trapezium rule: $\int_{a}^{b} y \mathrm{~d} x \approx \frac{1}{2} h\left\{\left(y_{0}+y_{n}\right)+2\left(y_{1}+y_{2}+\ldots+y_{n-1}\right)\right\}$, where $h=\frac{b-a}{n}$
The Newton-Raphson iteration for solving $\mathrm{f}(x)=0$ : $x_{n+1}=x_{n}-\frac{\mathrm{f}\left(x_{n}\right)}{\mathrm{f}^{\prime}\left(x_{n}\right)}$

## Probability

$\mathrm{P}(A \cup B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A \cap B)$
$\mathrm{P}(A \cap B)=\mathrm{P}(A) \mathrm{P}(B \mid A)=\mathrm{P}(B) \mathrm{P}(A \mid B)$ or $\mathrm{P}(A \mid B)=\frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$
Standard deviation
$\sqrt{\frac{\sum(x-\bar{x})^{2}}{n}}=\sqrt{\frac{\sum x^{2}}{n}-\bar{x}^{2}}$ or $\sqrt{\frac{\sum f(x-\bar{x})^{2}}{\sum f}}=\sqrt{\frac{\sum f x^{2}}{\sum f}-\bar{x}^{2}}$

## The binomial distribution

If $X \sim \mathrm{~B}(n, p)$ then $\mathrm{P}(X=x)=\binom{n}{x} p^{x}(1-p)^{n-x}$, mean of $X$ is $n p$, variance of $X$ is $n p(1-p)$

## Hypothesis test for the mean of a normal distribution

If $X \sim \mathrm{~N}\left(\mu, \sigma^{2}\right)$ then $\bar{X} \sim \mathrm{~N}\left(\mu, \frac{\sigma^{2}}{n}\right)$ and $\frac{\bar{X}-\mu}{\sigma / \sqrt{n}} \sim \mathrm{~N}(0,1)$

## Percentage points of the normal distribution

If $Z$ has a normal distribution with mean 0 and variance 1 then, for each value of $p$, the table gives the value of $z$ such that $\mathrm{P}(Z \leqslant z)=p$.

| $p$ | 0.75 | 0.90 | 0.95 | 0.975 | 0.99 | 0.995 | 0.9975 | 0.999 | 0.9995 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $z$ | 0.674 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 2.807 | 3.090 | 3.291 |

## Kinematics

Motion in a straight line
Motion in two dimensions
$v=u+a t$

$$
\mathbf{v}=\mathbf{u}+\mathbf{a} t
$$

$s=u t+\frac{1}{2} a t^{2}$
$\mathbf{s}=\mathbf{u} t+\frac{1}{2} \mathbf{a} t^{2}$
$s=\frac{1}{2}(u+v) t$
$\mathbf{s}=\frac{1}{2}(\mathbf{u}+\mathbf{v}) t$
$v^{2}=u^{2}+2 a s$
$s=v t-\frac{1}{2} a t^{2}$
$\mathbf{s}=\mathbf{v} t-\frac{1}{2} \mathbf{a} t^{2}$

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