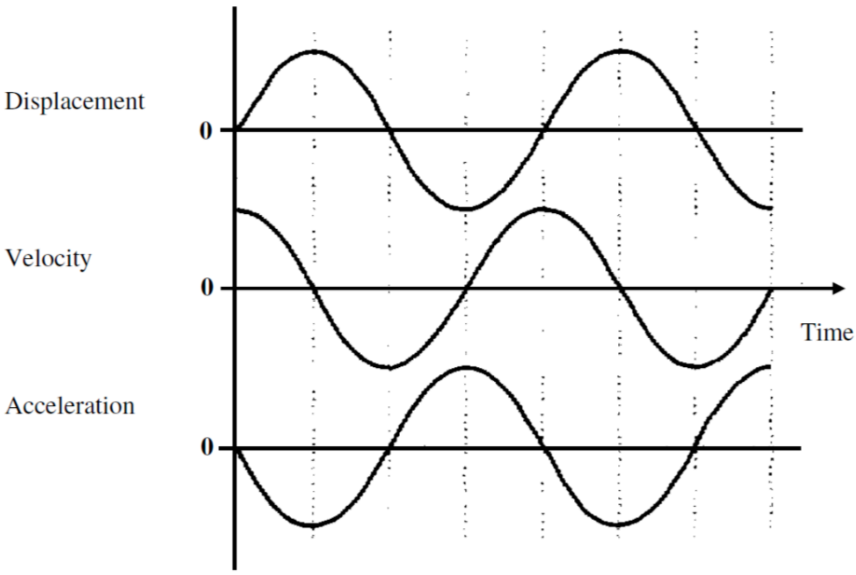
Simple harmonic motion

Simple harmonic motion

Phase difference in radians = $2\pi \frac{\Delta t}{T}$

Δt is the time between successive instants where the two objects are at maximum displacement in the same direction.



For a body executing SHM, these graphs are true.

- SHM is **oscillating motion** where the **acceleration** is:
  - Proportional to displacement
  - In the **opposite** direction to displacement
- Therefore, the graphs of displacement and acceleration are in anti-phase
- The definition of SHM leads to $a = -\omega^2 x$ where a = amplitude and x = displacement
  - General solution: $x = A sin(\omega t + \varphi)$ where φ is phase difference between t=0 and x=0
  - If timing starts at the centre (i.e.$x = 0$, $x = A sin(\omega t)$
  - If timing starts at $x = +A$, then $x = A cos(\omega t)$ works

$$v = \pm \omega \sqrt{A^2 = x^2}$$

Mass-spring system

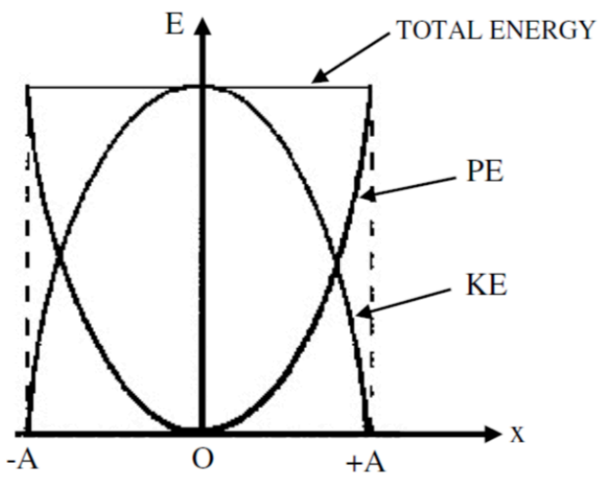
$$T = 2\pi\sqrt{\frac{m}{k}}$$

T is increased by adding mass or using a weaker spring. Note that it does not depend on g.

Simple pendulum

$$T = 2\pi\sqrt{\frac{l}{g}}$$

T is increased by increasing the length of the pendulum. Note the "small angle approximation" - the angle of swing must be less than 10°.



Variation of energy with displacement

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$$E_p = \frac{1}{2}kx^2$$

$$E_k = \frac{1}{2}k(A^2 - x^2)$$

$$E_{total} = \frac{1}{2}kA^2$$

Damping

- **Light damping** - T independent of amplitude so T remains constant as amplitude decreases. Amplitude gradually decreases by the same fraction each cycle.

- **Critical damping** - the system returns to equilibrium in the shortest possible time without overshooting

- **Heavy damping** - so strong that the displaced object returns to equilibrium much more slowly than if the system is critically damped - no oscillation occurs.

Forced vibrations and resonance

When a system oscillates without a periodic (driving) force applied, it oscillates at its natural frequency. Forced vibrations occur when a periodic force is applied to a system.

- As the applied frequency increases from 0:
  - Amplitude of oscillation increases until a maximum is reached at a particular frequency - this is the **resonant frequency** - and the amplitude then decreases again.
  - Phase difference between the displacement and the periodic force increases from 0 to $\frac{\pi}{2}$ at the maximum amplitude, and then from $\frac{\pi}{2}$ to π as frequency increases further.

When the system oscillates with maximum amplitude the phase difference between the displacement and the periodic force is $\frac{\pi}{2}$. The periodic force is then exactly in phase with the velocity of the system and resonance occurs.

- The lighter the damping
  - The greater the maximum amplitude at resonance
  - The closer the resonant frequency to the natural frequency
  - Hence the peak on a resonance curve will be much sharper with lighter damping.

- As the applied frequency becomes much larger than the resonant frequency:
  - Amplitude of oscillations decreases more and more
  - Phase difference between displacement and periodic force increases from $\frac{\pi}{2}$ until the displacement is π out of phase with the force.

- For an oscillating system with little to no damping, at resonance the applied frequency of the periodic force = the natural frequency of the system.