

Force, Energy and Momentum

Vectors

- Scalars have magnitude only, whereas vectors have both magnitude and direction.
- Resolving vectors by calculation:
 - Horizontal component: $X = R \cos \theta$
 - Vertical component: $Y = R \sin \theta$
- Finding the resultant vector
 - $\theta = \arctan \frac{Y}{X}$
 - $R = \sqrt{X^2 + Y^2}$
- Free-body diagrams should contain all forces acting on an object but not any forces exerted by the object itself
- Three coplanar forces acting on a body in equilibrium will form a closed loop - **triangle of forces**.
- On an inclined plane the weight of the object acts straight down, but the normal reaction at a right angle to the plane. Friction acts against the object sliding down the plane. Note that the angle between mg and the normal to the plane (i.e. the reaction force) is the same as the slope angle.

Moments

- moment = force \times perpendicular distance from the line of action of the force to the pivot.
Unit: N m

The principle of moments states "for a body to be in equilibrium, the sum of the clockwise moments about any point must equal the sum of the anticlockwise moments about that point"

- A **couple** is a pair of coplanar forces of **equal size** acting **parallel** to each other but in **opposite** directions
 - moment of couple = $F \times$ distance between forces
- The **centre of mass** of a body is the point through which a single force on the body has no turning effect.
 - If a body is in stable equilibrium, when displaced then released it returns to its original position because c.o.m. is directly below the point of support when the body is at rest.
 - A plank on a drum is in unstable equilibrium — if displaced slightly then released the plank will roll off the drum because the c.o.m. is directly above the point of support when in equilibrium — so weight acts to take it further away from equilibrium position.

- An object will topple if the line of action of its weight passes beyond the pivot
in order for an object to tilt: $\text{moment} > mg \times \frac{\text{width of base}}{2}$

Motion

- On displacement - time graphs gradient = velocity
- On velocity - time graphs gradient = acceleration & area under graph = displacement

Equations of uniform acceleration:

$$v = u + at$$

$$s = \frac{u + v}{2}t$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

Projectile motion:

- In suvat equations, $a = g$. g always acts downwards (i.e. negative)
- Horizontal and vertical components of motion must be thought of separately
 - vertical motion - constant a due to g being constant
 - horizontal motion - constant speed until projectile lands
- If a projectile is launched at an angle we need to resolve the initial velocity into vertical and horizontal components

$$X = R \cos \theta$$

$$Y = R \sin \theta$$

- resistance \propto speed
- Friction/drag acts in the opposite direction to the motion of the object and converts kinetic energy to heat and sound energy. It increases with speed
- Lift acts perpendicular to fluid flow
- The **terminal velocity** occurs where the driving force is constant and there is a resistance force which increases with speed
 - Maximum speed is affected by the magnitude of the driving force, and the magnitude of the resistance force.

Newton's laws of motion

1. Velocity of an object will not change unless a resultant force acts upon it
2. Acceleration of an object \propto the magnitude of the resultant force
 - Force is the rate of change of momentum

- $F = ma = m \frac{\Delta v}{\Delta t} = \frac{\Delta(mv)}{t}$ therefore $F\Delta t = \Delta(mv)$
- The impulse, defined as $F \times t$, thus equals the change in momentum. It is also the area under a force-time graph.
- All objects fall at the same rate regardless of mass (but resistance does play a part)

3. "If body A exerts a force on body B, then body B exerts an equal but opposite force on body A" - aka every action has an equal and opposite reaction.

- **Momentum** is always conserved, assuming no external forces act.

$$\text{momentum, } p = mv, \text{ unit: kg m s}^{-1}$$

- In an **elastic** collision, both momentum and energy are conserved.
- In an **inelastic** collision, energy is not conserved.

Energy:

$$\text{work done} = \text{force} \times \text{distance moved}$$

- Note that force is not always in the same direction as the movement (e.g. for a sled being pulled by a string, only the horizontal force causes the motion).

$$\text{horizontal: } W = Fs \cos \theta$$

$$\text{vertical: } W = F s \sin \theta$$

- Area under a force—displacement graph tells us the work done

$$P = \frac{\Delta W}{\Delta t} = \frac{E}{t} = Fv$$

Conservation of energy:

- "Energy cannot be created nor destroyed, it can be transferred from one form to another, but the total energy in a closed system cannot change"

$$E_k = \frac{1}{2}mv^2$$

$$E_p = mgh$$