

Electromagnetism

Magnetic flux density

Force on a current-carrying wire when field is perpendicular to current, $F = BIl$

Fleming's left hand rule can be used to work out the direction in which the wire will move:

- Thumb: thrust
- First finger: Field
- Second finger: Current

The strength of a magnetic field is given by its **flux density**, B , which is measured in Tesla, T.

Flux density is a vector - its direction is along a tangent to the field line at that point. Its magnitude is represented by the density of magnetic field lines.

One Tesla is defined as "the flux density of the field that produces a force of 1 N on a unit length of conductor carrying a current of 1 A perpendicular to the field."

- Note that for a magnetic field, field lines always run North to South
- The **right hand grip** rule gives the direction of field lines where the thumb points in the direction of current flow
 - \otimes represents current flowing into the page, and \odot current flowing out of the page (Imagine a dart flying)
 - Magnetic flux passing through area A perpendicular to a field, $\phi = BA$ unit: Webers, Wb
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If area is not perpendicular to the field then $\phi = BA \cos \theta$ where θ is the angle to the normal to the area, or $\phi = BA \sin \theta$ where θ is to the plane of the area. Consider the graphs and where you would expect a maximum to occur.

magnetic flux linkage = $N\phi$ where N is the number of turns cutting the flux

Charges in a magnetic field

Force on charged particles moving in a magnetic field, $F = BQv$ when field perpendicular to v .

Charged particles in a magnetic field follow a circular path. The direction of the force on a positive charge is given by Fleming's LHR, and the force = centripetal force required to maintain this motion.

For a charged particle in a magnetic field, $BQv = \frac{mv^2}{r} = mr\omega^2$

The **cyclotron** is an application of this phenomenon (magnetic deflection). Two D-shaped electrodes are separated by a small gap in an evacuated chamber placed in the uniform magnetic field of a large electromagnet.

Charged particles produced by an ion source at the centre enter one D and move in a circular path due to the field. A high-frequency alternating current is connected between the Ds, with frequency such that its polarity reverses at the same rate as the particles cross from one D to another. The energy of the particle increases every time it crosses from one D to the other, the radius of orbit increases as energy increases and the beam finally emerges tangentially from the cyclotron.

At radius r , magnetic force = centripetal force, so $r \propto v$

Electromagnetic induction

When a wire cuts through magnetic field lines, an electromotive force is induced in the wire.

- Lenz's law: *"The direction of the induced current is such that it opposes the motion producing it"*
- Faraday's law: *"The magnitude of the induced emf is proportional to the rate of change of flux linked with that circuit, or the rate at which magnetic flux is cut"*

These laws combined: $\epsilon_{ind} = - \frac{\Delta}{\Delta t}$ (flux linkage)

Flux linkage: $N\phi$, $\epsilon_{ind} = \frac{\Delta}{\Delta t}(N\phi)$

Given $\phi = BA$, the flux linking a coil = BAN and:

$$\epsilon_{ind} = \frac{\Delta}{\Delta t}(BAN)$$

Or average $\epsilon_{ind} = \frac{\text{change in } BAN}{\text{time taken}}$

Both of these assume the plane of the coil is perpendicular to the coil. Otherwise flux linking coil = $BAN \cos\theta$ or $BAN \sin\theta$ (consider maxima)

emf induced in a moving conductor:

$$\epsilon_{ind} = Blv$$

Fleming's right hand rule gives the direction of the induced current if a complete circuit. If asked to label emf, consider the conductor just as any other source - in a wire connected between the terminals, current would flow from positive to negative.

emf induced in a rotating coil:

For θ between the normal to the coil and the field, flux linking the coil is given by $\varphi = BAN\cos\theta$. For a constant rate of rotation $\theta = \omega t$ where ω is the angular speed in rad s^{-1} .

Therefore $\varphi = BAN\cos(\omega t)$.

$$\text{Combining with Faraday's Law, } \varepsilon_{ind} = BAN\omega\sin(\omega t)$$

The induced emf is therefore a sine wave with peak value $BAN\omega$. The faster the coil is rotated, the greater the peak. This very much depends on when timing starts however, so consider maxima. Note that maximum ε_{ind} occurs where $\frac{d\varphi}{d\theta}$ is greatest (i.e. wires are cutting the most field lines).